Generation of macroscopic singlet states in atomic ensembles

Géza Tóth[1,2,3] and Morgan W. Mitchell[4]

[1]Theoretical Physics, The University of the Basque Country, Bilbao, Spain
[2]Ikerbasque - Basque Foundation for Science, Bilbao, Spain
[3]Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, Budapest
[4]ICFO-The Institute of Photonic Sciences, Barcelona, Spain

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Outline

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1. Motivation
2. Spin squeezing and entanglement
3. Spin squeezing with atomic ensembles
4. Von Neumann measurement

Motivation

- In many quantum control experiments the qubits cannot be individually addressed. We still would like to create and detect entanglement.

- Entanglement creation and detection is possible through spin squeezing. We will use the ideas behind the spin squeezing approach in order to
  - Create and detect entanglement between particles with arbitrarily large spin
  - Engineer quantum states other than the classical spin squeezed state with a large spin, that is, unpolarized states.
  - Generalize the Gaussian approach for describing the dynamics leading to such states.

Outline

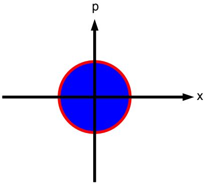
From squeezing to spin squeezing

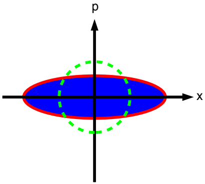
- The variances of the two quadrature components are bounded

$$(\Delta x)^2(\Delta p)^2 \geq const.$$

- Coherent states saturate the inequality.

- Squeezed states are the states for which one of the quadrature components have a smaller variance than for a coherent state.





- Can one use similar ideas for spin systems?

Spin squeezing

Definition

The variances of the angular momentum components are bounded

$$(\Delta J_x)^2(\Delta J_y)^2 \geq \frac{1}{4}|\langle J_z\rangle|^2,$$

where the mean spin points to the z direction. If $(\Delta J_x)^2$ is smaller than the standard quantum limit $\frac{|\langle Jz\rangle|}{2}$ then the state is called spin squeezed.

- In practice this means that the angular momentum of the state has a small variance in one direction, while in an orthogonal direction the angular momentum is large.

[M. Kitagawa and M. Ueda, PRA **47**, 5138 (1993).]

Entanglement

Definition

Fully separable states are states that can be written in the form

$$\rho = \sum_l p_l \rho_l^{(1)} \otimes \rho_l^{(2)} \otimes \ldots \otimes \rho_l^{(N)},$$

where $\sum_l p_l = 1$ and $p_l > 0$.

Definition

A state is entangled if it is not separable.

The standard spin-squeezing criterion

Definition

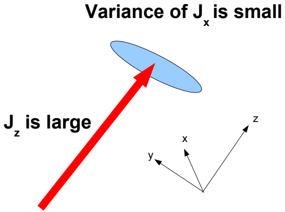
The spin squeezing criterion for entanglement detection is

$$\frac{(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq \frac{1}{N}.$$

If it is violated then the state is entangled.

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature **409**, 63 (2001).]

- Note that this criterion is for spin-1/2 particles.
- States violating it are like this:



A generalized spin squeezing entanglement criterion

Separable states of N spin-j particles must fulfill

$$\xi_s^2 := (\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq Nj.$$

It is maximally violated by a many-body singlet, e.g., the ground state of an anti-ferromagnetic Heisenberg chain.

[GT, PRA **69**, 052327 (2004);GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL **99**, 250405 (2007).]

- For such a state
$$\langle J_k^m \rangle = 0.$$
- $N\xi_s^2$ gives an upper bound on the number of unentangled spins.
- ξ_s^2 characterizes the sensitivity to external fields acting as $U = \exp(i\phi J_{\mathbf{n}})$. $\xi_s = 0$ corresponds to complete insensitivity.

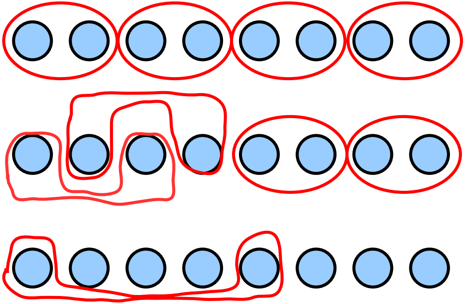
Many-body singlet states

Many-body singlet states have been studied a lot in condensed matter physics and quantum information science. They can be created typically in Heisenberg lattices.

- Here we realize singlets without two-spin interactions or waiting for a Heisenberg system to settle in ground state.
- Ours is the permutationally invariant singlet. For the qubit case, the bipartite entanglement of such a state is known.
- Surprisingly, this state appears even in quantum gravity calculations of black hole entropy. [E.R. Livine, and D.R. Terno, Phys. Rev. A. **72**, 022307 (2005).]
- Such singlet states might be used in cases when it is important that the system is insensitive to the effect of the homogenous fields. (e.g., measuring field gradient, storing information in the decoherence free subspace).

Permutationally invariant singlet

- Our singlet is the equal mixture of all permutations of a pure singlet state.
- For qubits, it is the mixture of all chains of two-qubit singlets:



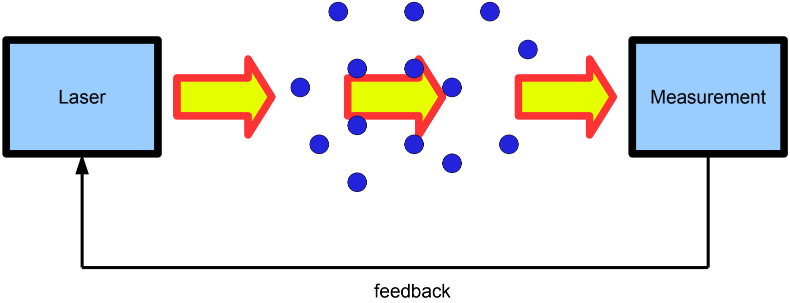
- Such a state has intriguing properties ...

Outline

The physical system: atoms + light

- We consider atoms interacting with light. [B. Julsgaard, A. Kozhekin, and E.S. Polzik, Nature **413**, 400 (2001); S.R. de Echaniz, M.W. Mitchell, M. Kubasik, M. Koschorreck, H. Crepaz, J. Eschner, and E.S. Polzik, J. Opt. B **7**, S548 (2005); J. Appel, P.J. Windpassinger, D. Oblak, U.B. Hoff, N. Kjaergaard, and E.S. Polzik, arXiv:0810.3545.]

- The light is then measured and the atoms are projected into an entangled state.



Quantum non-demolition measurement (QND) of the ensemble

The steps the the QND measurement of J_k :

- 1. Set the light to
$$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$
- 2. The atoms interact with the light for time t
$$H = \Omega J_k S_z$$
- 3. Measurement of S_y.

- The most obvious effect of such a measurement is the decrease of $(\Delta J_k)^2$.
- The timescale of the dynamics, for $J := Nj$, is
$$t \sim \tau := \frac{1}{\Omega \sqrt{S_0 J}}.$$

The proposed protocol

1. Initial state
   - Atoms
   $$\varrho_0 := \frac{\mathbb{1}}{(2j+1)^N}$$
   - Light
   $$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$
2. Measurement of J_x + feedback or postselection.
3. Measurement of J_y + feedback or postselection.
4. Measurement of J_z + feedback or postselection.

- We consider 10^6 spin-1 ^{87}Rb atoms and $S_0 = 0.5 \times 10^8$.
- Initial state of the atoms has $(\Delta J_k)^2 \sim N$ for $k = x, y, z$.
- After squeezing, we obtain $\xi_s < 1$.
- Thus, we get a state close to a singlet state.

Gaussian states

- Gaussian states are quantum states for which all third and higher order correlations are trivial.

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- Continuous variable systems: The dynamics of Gaussian systems can be followed by writing down dynamical equations for the covariance matrix and the expectation values of x_k and p_k. For a single mode, this matrix looks like

$$\Gamma_{xp} \propto \begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle xp + px \rangle/2 - \langle x \rangle\langle p \rangle \\ \langle xp + px \rangle/2 - \langle x \rangle\langle p \rangle & \langle p^2 \rangle - \langle p \rangle^2 \end{pmatrix}$$

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- Spin systems: Such ideas can be used if one of the spin components is large. Then the other two components behave like x and p operators.
- We extend this approach to states for which the spin is not large. Our covariance matrix for a single spin is

$$\Gamma_J \propto \begin{pmatrix} \langle \Delta J_x \Delta J_x \rangle & \langle (\Delta J_y \Delta J_x)_{\mathrm{sym}} \rangle & \langle (\Delta J_z \Delta J_x)_{\mathrm{sym}} \rangle \\ \langle (\Delta J_x \Delta J_y)_{\mathrm{sym}} \rangle & \langle \Delta J_y \Delta J_y \rangle & \langle (\Delta J_z \Delta J_y)_{\mathrm{sym}} \rangle \\ \langle (\Delta J_x \Delta J_z)_{\mathrm{sym}} \rangle & \langle (\Delta J_y \Delta J_z)_{\mathrm{sym}} \rangle & \langle \Delta J_z \Delta J_z \rangle \end{pmatrix}.$$

Covariance matrix

- We define the set of operators

$$R = \{\frac{J_x}{\sqrt{J}}, \frac{J_y}{\sqrt{J}}, \frac{J_z}{\sqrt{J}}, \frac{S_x}{\sqrt{S}}, \frac{S_y}{\sqrt{S}}, \frac{S_z}{\sqrt{S}}\}$$

and covariance matrix as

$$\Gamma_{mn} := \langle R_m R_n + R_n R_m \rangle / 2 - \langle R_m \rangle \langle R_n \rangle.$$

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$$\Gamma_{mn} := \langle R_m R_n + R_n R_m \rangle / 2 - \langle R_m \rangle \langle R_n \rangle.$$

- For short times, the dynamics of an operator O_0 is given by

$$O_P = O_0 - it[O_0, H],$$

  where we assumed $\hbar = 1$.

Covariance matrix II

- Consider dynamics for $t \sim \tau := \frac{1}{\Omega\sqrt{JS_0}}$.
- Hence, for the unitary dynamics one arrives to

$$\Gamma_P = M\Gamma_0 M^T, \tag{1}$$

where M is the identity matrix, apart from $M_{5,1} = \frac{\langle S_x \rangle}{S_0}\kappa$, and $\kappa := t/\tau = \Omega t\sqrt{JS_0}$.

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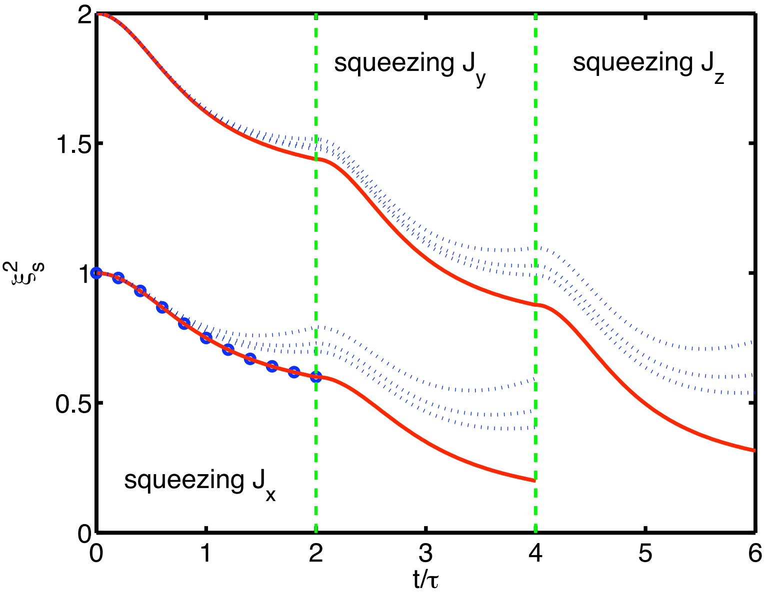
- The measurement of the light can be modeled with a projection

$$\Gamma_M = \Gamma_P - \Gamma_P (P_y \Gamma_P P_y)^{\mathrm{MP}} \Gamma_P^T, \tag{2}$$

where MP denotes the Moore-Penrose pseudoinverse, and P_y is $(0, 0, 0, 0, 1, 0)$. [G. Giedke and J.I. Cirac, Phys. Rev. A **66**, 032316 (2002).]

Spin squeezing dynamics (top curve, solid)



Modeling losses

The dynamics of the covariance matric for the case of losses

$$\Gamma'_P = (\mathbb{1} - \eta D) M \Gamma_0 M^T (\mathbb{1} - \eta D) + \eta(2 - \eta) D \Gamma_{\rm noise},$$

where $D = \mathrm{diag}(1, 1, 1, 0, 0, 0)$ and $\Gamma_{\rm noise} = \mathrm{diag}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 0, 0)$.

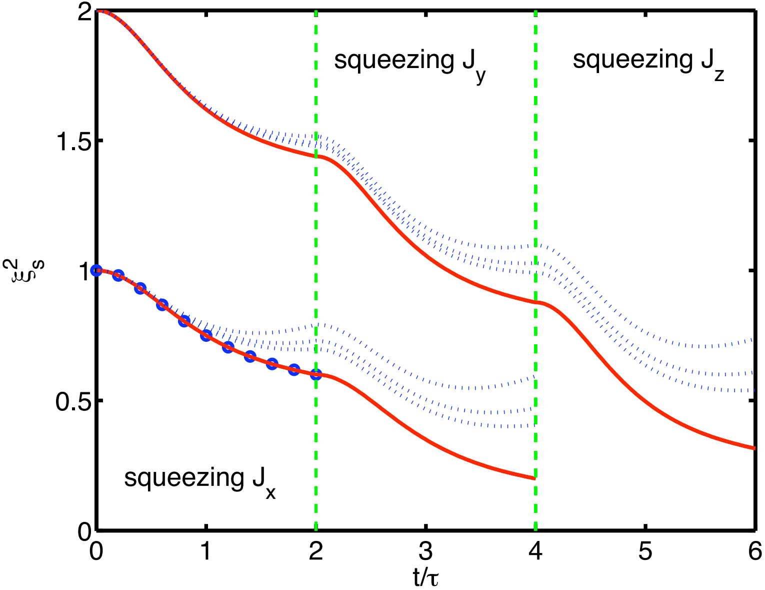
- η the fraction of atoms that decoherence during the QND process.
- The losses are connected to κ through

$$\eta = Q\kappa^2/\alpha,$$

where α is the resonant optical depth of the sample and $Q = \frac{8}{9}$

[L.B. Madsen and K. Mølmer, Phys. Rev. A **70**, 052324 (2004).]

Spin squeezing dynamics: $\alpha = 50, 75, 100$ (dotted)



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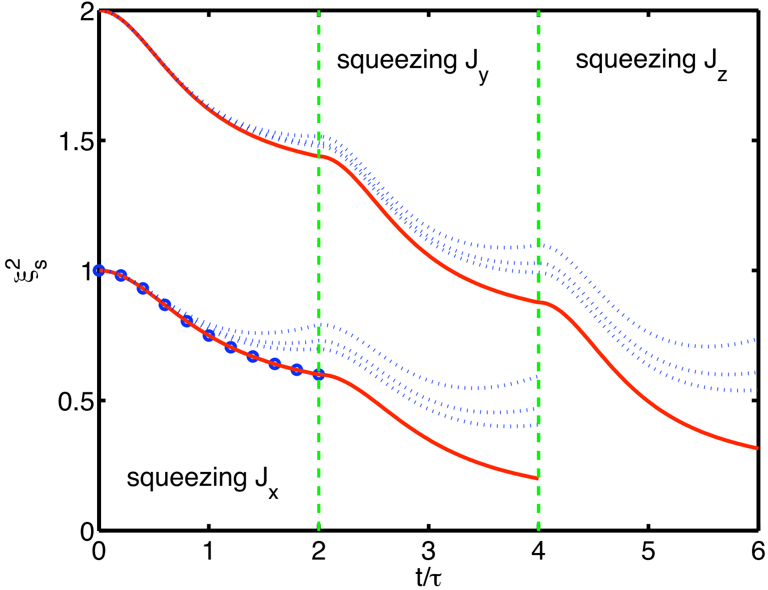
Exact model

Results: for $t \sim \tau \times N^{\frac{1}{4}}$ the variances decrease to $\sim \sqrt{N}$, while for $t \sim \tau \times \sqrt{N}$ the variances reach ~ 1, which we call the von Neumann limit.

- Straightforward simulation of the quantum dynamics of million atoms is not possible.
- However, in the large N limit, a formalism can be obtained that replaces sums by integrals.
- This approach works also for the regime in which the Gaussian approximation is no more valid.
- Comparison with exact model is possible for an initial state for which half of the spins are in the $|+1\rangle_x$ state, half of them are in the $|-1\rangle_x$ state.

Spin squeezing dynamics (bottom curve, dots)



Conclusions

- We presented a method for creating and detecting entanglement in an ensemble of atoms with spin $j > \frac{1}{2}$.
- Our experimental proposal aims to create a many-body singlet state through squeezing the uncertainties of the collective angular momenta.
- We showed how to use an extension of the usual Gaussian formalism for modeling the experiment.
- Presentation based on: GT and M.W. Mitchell, arxiv:0901.4110.

*** THANK YOU ***