Efficient Tomographic Analysis of a Six Photon State

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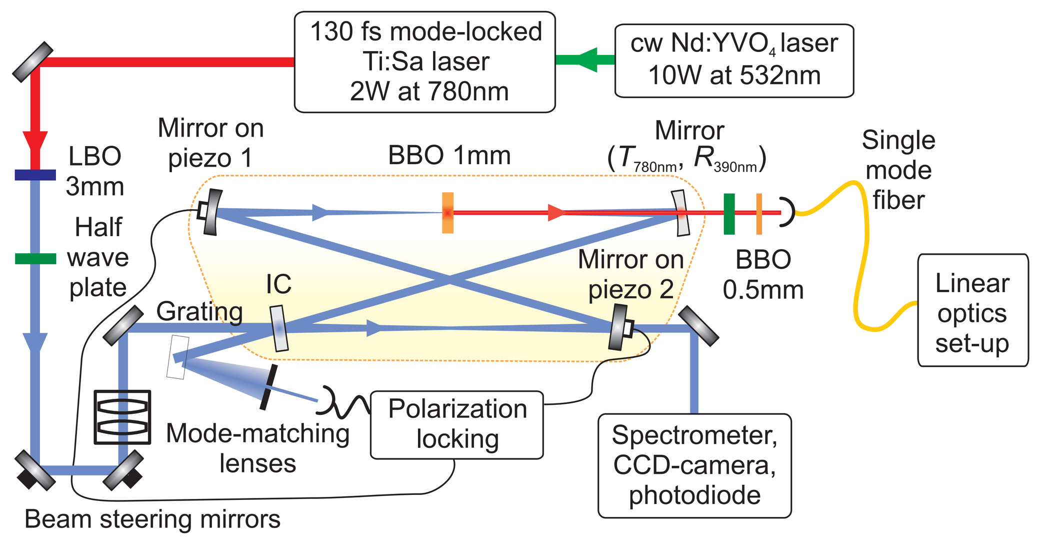
Abstract

Multi-partite entangled quantum states play a crucial role for quantum information processing. Yet, their characterization is exponentially costly. Tomography under the assumption of permutational invariance (PI) enables polynomial scaling of the measurement effort as well as of the numerical resources [1,2]. This is of great importance since many prominent quantum states like, for example, GHZ states, W states or symmetric Dicke states are permutationally invariant. Also a series of operators like, for example, collective spin operators are permutationally invariant and thus now require only polynomially increasing resources for their evaluation. Here, we present experimental results of the tomographic analysis of a photonic six qubit symmetric Dicke state, as obtained from parametric down-conversion. For such systems full tomography is extremely challenging due to an exceedingly high number of 3^6=729 measurement settings and low count rates. In comparison permutationally invariant tomography needs only 28 settings. For low rank states, the scheme can be further optimized by combining it with compressed sensing [3,4]. Our experiments show that even a tomographic reconstruction of states with an exceedingly high number of qubits is in principle feasible, when restricting to the permutationally invariant subspace.

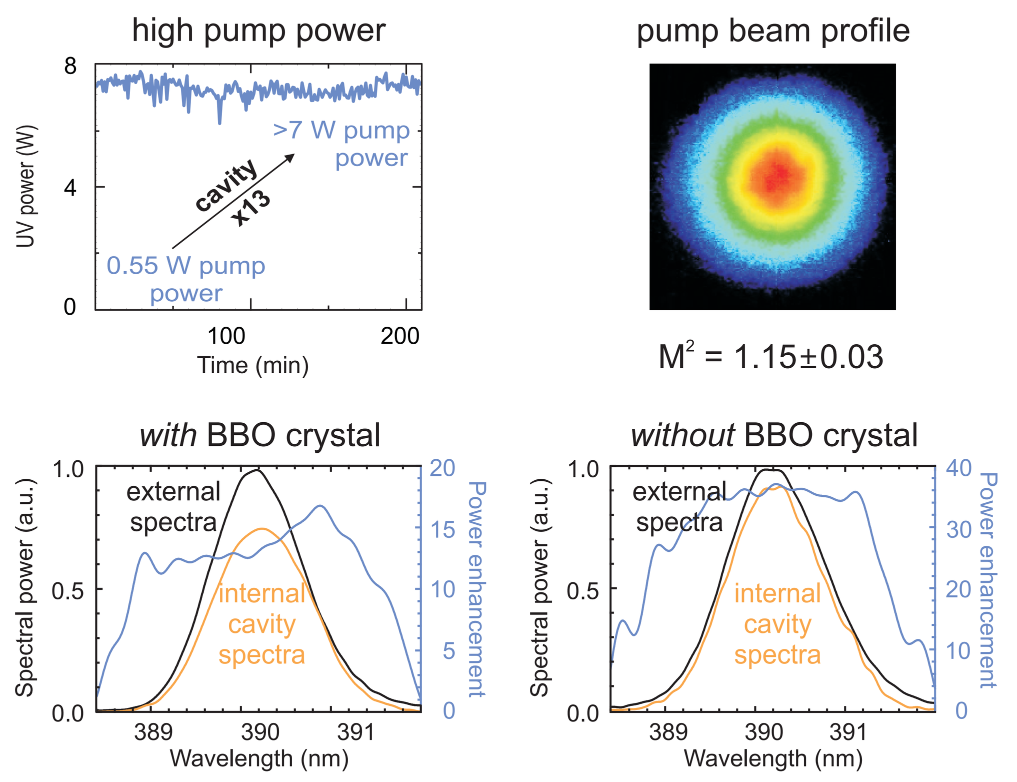
New SPDC Pump Source

fs enhancement cavity in the UV

- schematic



- characterization of the cavity

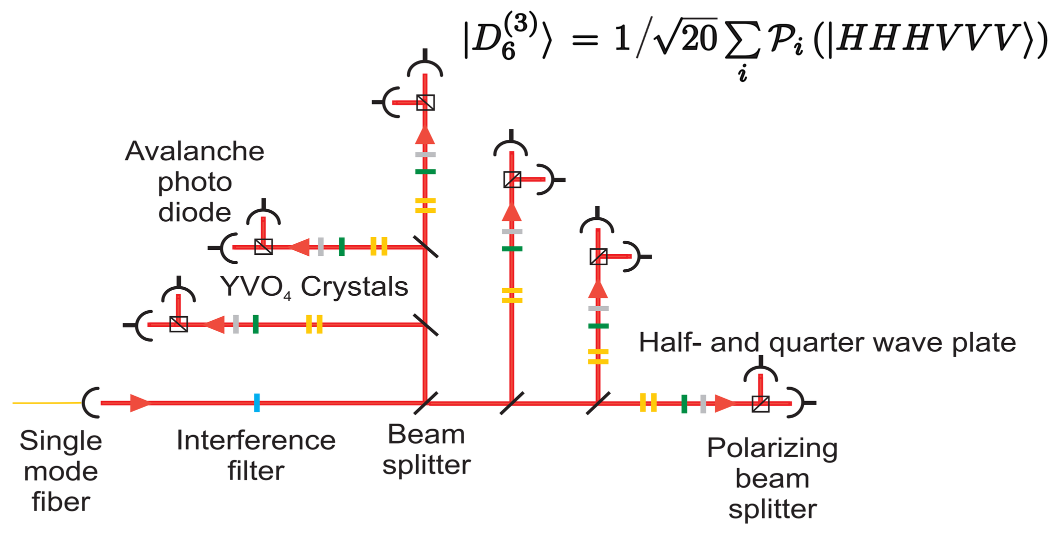


power enhancement x13 | power enhancement x37

Linear optical setup

- six photon Dicke setup to generate the state $|D_6^{(3)}\rangle$

$$|D_6^{(3)}\rangle = 1/\sqrt{20}\sum_i \mathcal{P}_i(|HHHVVV\rangle)$$

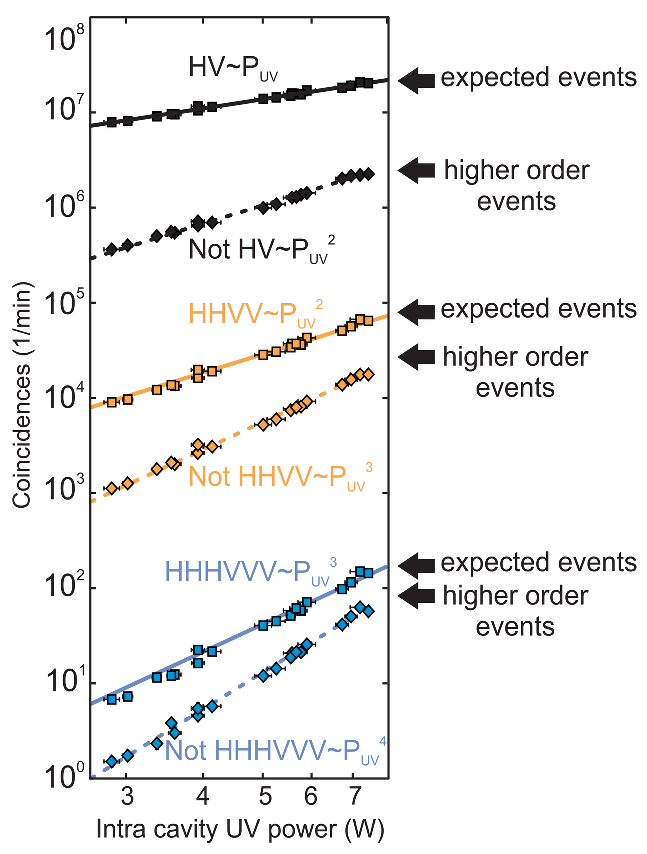


input state from the photon source is split up into spacial modes
success rate of the setup is 1.3 % for six photons
conditional detection scheme is applied

Higher orders

- SPDC source delivers all orders at the same time

$$|\Psi_{SPDC}\rangle \propto e^{ic(\hat{a}_H^\dagger \hat{a}^\dagger_V)}|vac\rangle \propto |vac\rangle + \underbrace{ic(\hat{a}_H^\dagger \hat{a}^\dagger_V)|vac\rangle}_{1st\ order} - \underbrace{\frac{c^2}{2}(\hat{a}_H^\dagger \hat{a}^\dagger_V)^2|vac\rangle}_{2nd\ order} - \underbrace{\frac{ic^3}{6}(\hat{a}_H^\dagger \hat{a}^\dagger_V)^3|vac\rangle}_{3rd\ order} + \dots$$



investigation of six-partite entangled symmetric Dicke states requires *reasonable* count rates

high pump power of up to 8.6 W leads to a strong increase in higher order events but *also* to an increase in colored noise

Six Qubit Tomography

Permutationally invariant tomography

- Efficient tomography: number of measurement settings scales only ***quadratically*** with the number of qubits
- Applicable for permutationally invariant states like GHZ, W or symmetric Dicke states
- Three setting test measurement $X^{\otimes N}$, $Y^{\otimes N}$ and $Z^{\otimes N}$ suffices to determine the overlap with the symmetric subspace
- For each measurement the same local setting is applied to all qubits, i.e. ***global*** measurements are performed

Comparison of PI and full tomography

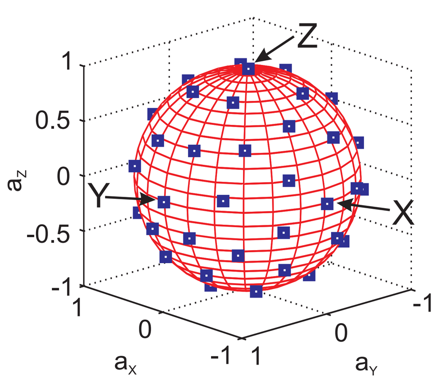
- UV pump level of 8.4 W
- Mean six-fold count rate of 58/min
- Average counts per basis setting >230
- 28 basis settings versus 729 settings for full tomography

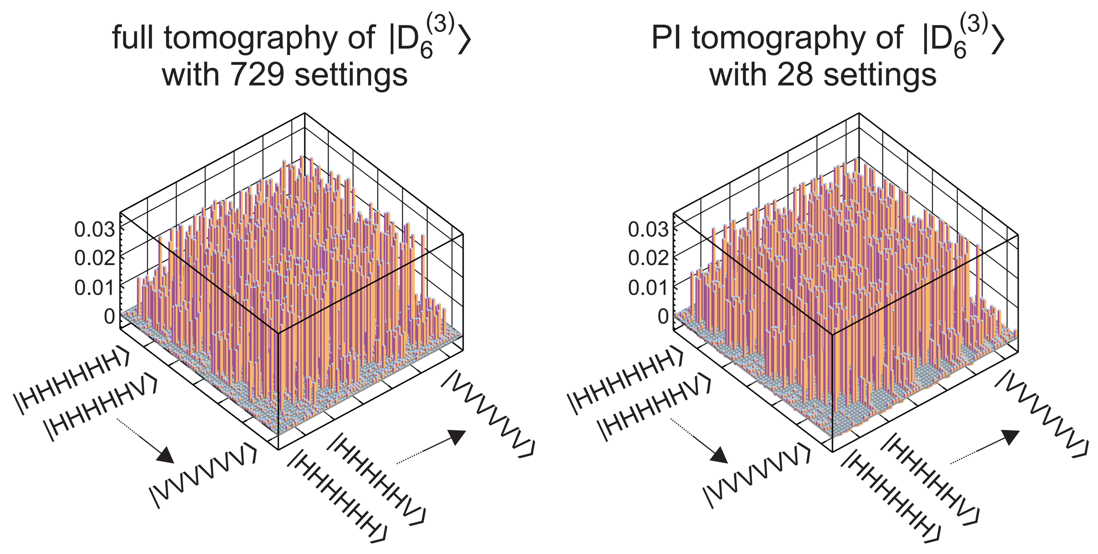
- Overlap with the symmetric subspace can be determined ***a priori*** from ***only three*** basis measurements

→ overlap with the symmetric subspace >92.2%

→ permutationally invariant tomography makes sense

- measurement directions for PI tomography





Overlap between full and PI tomography is 0.922

Efficient state reconstruction algorithm

- Reduction of the number of degrees of freedom due to symmetry

Full tomography: 4^N = 4096

symmetry ↓

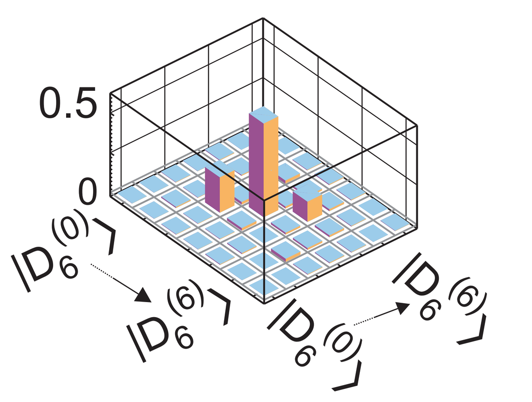
PI tomography: $\binom{N+3}{N}$ = 84

- Change to spin basis

$$\rho_{PI} = \bigoplus_{j=j_{min}}^{N/2} \frac{\mathbb{1}_{d_j}}{d_j} \otimes p_j \rho_j$$

$$\rho_{PI} = \begin{pmatrix} \tilde{\rho}_{N/2} & & & \\ & \tilde{\rho}_j & 0 & \\ & 0 & \tilde{\rho}_j & \\ & & & \tilde{\rho}_0 \end{pmatrix} \quad \mathbb{1}_{d_j}\otimes\tilde{\rho}_j$$

- Convex optimization
- Maximum likelihood and least squares methods
- State reconstruction of a 20 qubit state in 10 minutes on a standard desktop PC

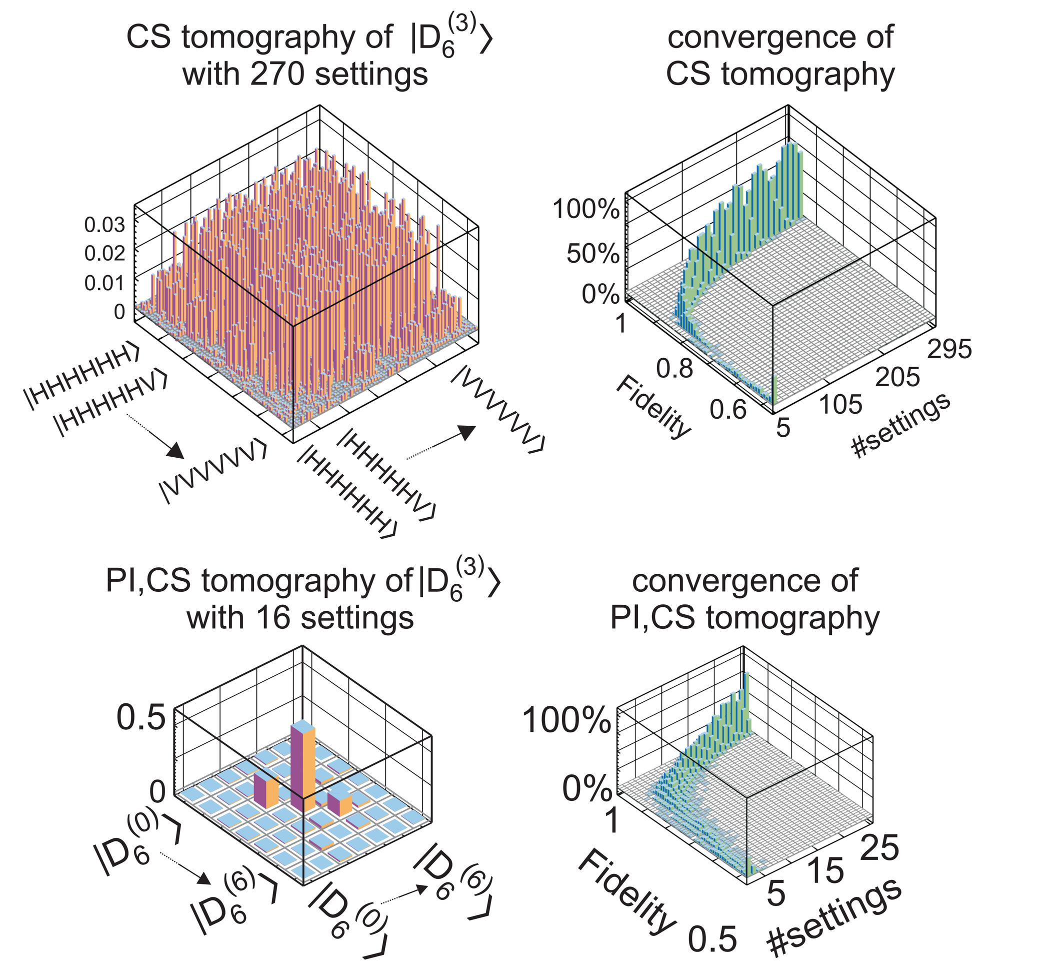


- Symmetric subspace obtained with PI tomography, the central bar can be associated with the target state and the small bars next to it originate from higher order noise

Analysis of Low Rank States

Efficient analysis of low rank states

- Compressed sensing enables a square root improvement for the tomographic analysis of low rank states



- comparison of tomography schemes

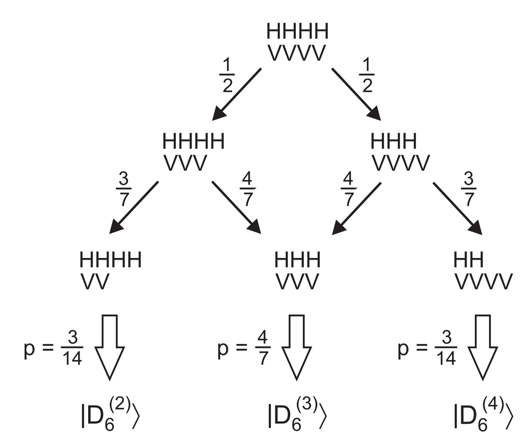
Fidelities w.r.t. the symmetric Dicke states

State	Full	PI	CS	PI,CS
$\|D_6^{(0)}\rangle$	0.001	0.001	0.001	0.002
$\|D_6^{(1)}\rangle$	0.005	0.008	0.011	0.006
$\|D_6^{(2)}\rangle$	0.197	0.222	0.181	0.207
$\|D_6^{(3)}\rangle$	0.604	0.590	0.615	0.592
$\|D_6^{(4)}\rangle$	0.122	0.127	0.118	0.119
$\|D_6^{(5)}\rangle$	0.003	0.004	0.003	0.005
$\|D_6^{(6)}\rangle$	0.000	0.003	0.001	0.004
Σ	0.933	0.954	0.929	0.935

- Combining PI tomography and compressed sensing reduces the measurement effort by about a factor of 50 without significantly changing the parameters specifying the state

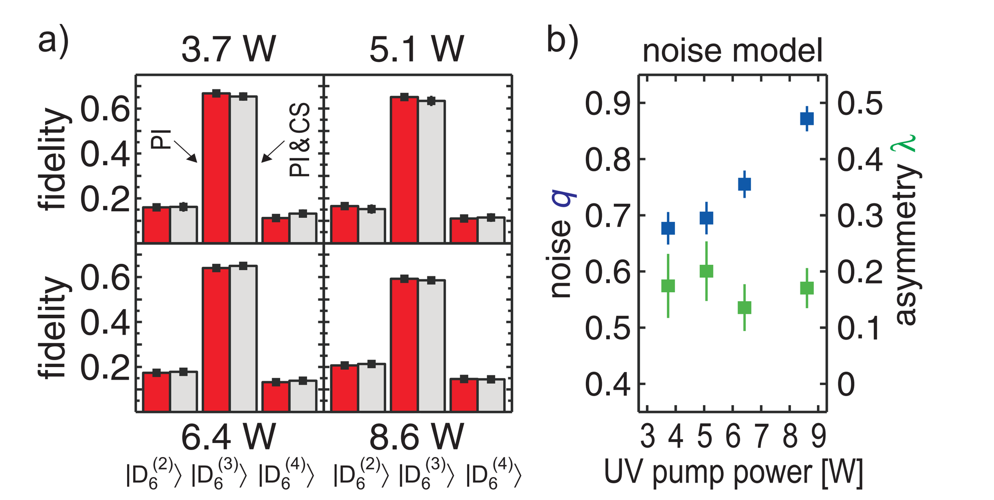
Analysis of higher order noise

- Analysis of higher order noise originating from eight-photon events with loss of two photons at four different UV pump levels from 3.7W to 8.6W
- For low UV pump levels full tomography is not possible within reasonable measurement times



$$\rho_{exp} = (1-q)\rho_{|D_6^{(3)}\rangle} + q\rho_{noise}$$

$$\rho_{noise} = \frac{4}{7}\rho_{|D_6^{(3)}\rangle} + \frac{3}{14}[(1+\lambda)\rho_{|D_6^{(2)}\rangle} + (1-\lambda)\rho_{|D_6^{(4)}\rangle}]$$



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Outlook

- Check applicability of permutationally invariant tomography for process tomography
- Overcomplete PI tomography with more settings but less measurement time per setting
- Analysis of systematic errors of the state reconstruction