Activation of metrologically useful genuine multipartite entanglement, arXiv:2203.05538

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Simple example

- Let us consider $M = 2$ copies of the 3-qubit state

$$\varrho_p = p|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| + (1-\mathrm{p})\frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|),$$

with $p = 0.8$.

- Then, we have

$$\mathcal{F}_Q[\varrho, H_2] = 28.0976, \qquad \text{(2 copies)}$$

while for $M = 1$ we have

$$\mathcal{F}_Q[\varrho, H_1] = 23.0400. \qquad \text{(1 copy)}$$

- In both cases,

$$\mathcal{F}_Q^{(\mathrm{sep})}(H_k) = 12,$$

hence for the metrological gain

$$g_1 = 1.92 < g_2 = 2.34.$$

Simple example II

- Considering the state

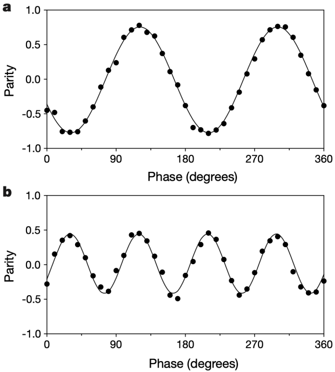
$$\varrho_p = p|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| + (1-\mathrm{p})\frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|),$$

  we took care of phase flip errors.

- We can also correct bitflip errors in the usual way, if the state is outside of the $\{|000\rangle, |111\rangle\}$ subspace.

Simple example III

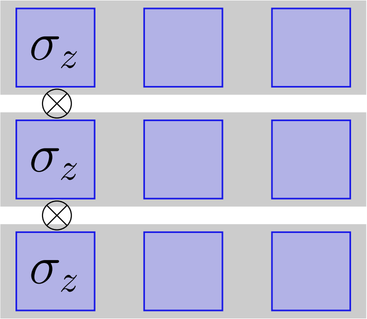
- Directly relevant to experiments with GHZ states!
- One can obtain maximal visibility.



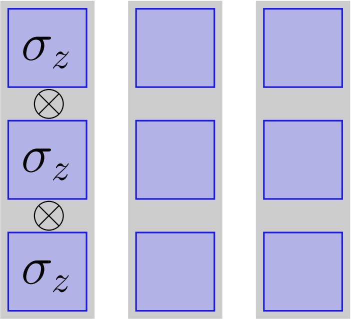
$N = 2$ and $N = 4$ particles, Sackett *et al.*, Experimental entanglement of four particles, Nature (2000).

Comparison to error correction

$M = 3$ copies, $N = 3$ qubits



3 logical qubits,
1 logical qubit=3 physical qubits



W. Dür, M. Skotiniotis, F. Fröwis, B. Kraus, Phys. Rev. Lett. (2014).

Comparison to error correction II

- How do we store a three-qubit GHZ state?
- Multicopy metrology:

$$|\mathrm{GHZ}\rangle = \tfrac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \otimes \tfrac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \otimes \tfrac{1}{\sqrt{2}}(|000\rangle + |111\rangle),$$
$$H = \sigma_z^{(1)}\sigma_z^{(4)}\sigma_z^{(7)} + \sigma_z^{(2)}\sigma_z^{(5)}\sigma_z^{(8)} + \sigma_z^{(3)}\sigma_z^{(6)}\sigma_z^{(9)}.$$

Improves performance without syndrome measurements.

- Error correction for bit-flip code (phase-flip code is similar):

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\,000\,000\rangle + |111\,111\,111\rangle),$$
$$H = \sigma_z^{(1)}\sigma_z^{(2)}\sigma_z^{(3)} + \sigma_z^{(4)}\sigma_z^{(5)}\sigma_z^{(6)} + \sigma_z^{(7)}\sigma_z^{(8)}\sigma_z^{(9)},$$

+ error syndrome measurements + error correction.

Comparison to error correction III

- Let us see our scheme for $M = 3, N = 3$.

- Let be ϱ some mixture of the states with at most 1 copy with a phase error

$$
\begin{aligned}
|\Psi_{+++}\rangle &= |\mathrm{GHZ}+\rangle \otimes |\mathrm{GHZ}+\rangle \otimes |\mathrm{GHZ}+\rangle, \\
|\Psi_{-++}\rangle &= |\mathrm{GHZ}-\rangle \otimes |\mathrm{GHZ}+\rangle \otimes |\mathrm{GHZ}+\rangle, \\
|\Psi_{+-+}\rangle &= |\mathrm{GHZ}+\rangle \otimes |\mathrm{GHZ}-\rangle \otimes |\mathrm{GHZ}+\rangle, \\
|\Psi_{++-}\rangle &= |\mathrm{GHZ}+\rangle \otimes |\mathrm{GHZ}+\rangle \otimes |\mathrm{GHZ}-\rangle,
\end{aligned}
$$

  where

$$|\mathrm{GHZ}\pm\rangle = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle).$$

  We still obtain

$$\mathcal{F}_Q[\varrho, H] = \max.$$