Current topics in foundations of quantum mechanics

Bell inequalities, Kochen-Specker theorem, and generalized probabilistic models

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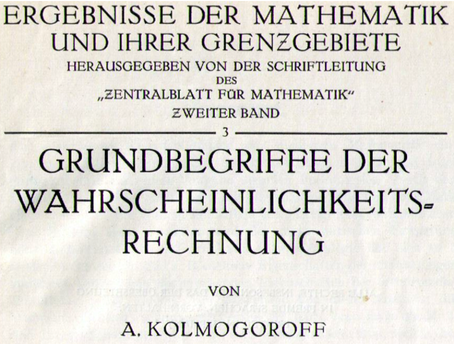
P A R T 1: Contextuality

1. axioms of Kolmogorov vs. Specker's contextuality
2. a proof of the Kochen-Specker theorem
3. connection to Bell-inequalities

Ernst Specker, Simon Kochen, Adán Cabello

Rolling dice



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GRUNDBEGRIFFE DER WAHRSCHEINLICHKEITS-RECHNUNG

VON

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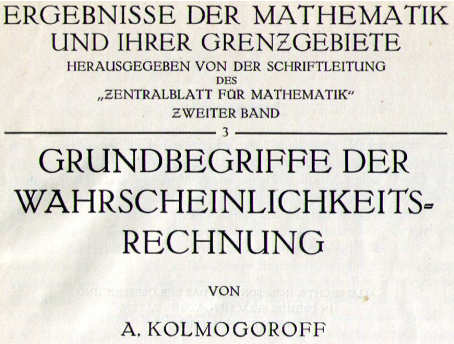
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GRUNDBEGRIFFE DER
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- the sample space Ω contains all outcomes, e.g. $\Omega = \{1, 2, 3, 4, 5, 6\}$
- the event space is $\mathcal{F} = \{ A \mid A \subset \Omega \}$
- the probability $P\colon \mathcal{F} \to [0, 1]$ obeys $P(\Omega) = 1$ and $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$ for disjoint sets.

“The logic of non-simultaneously decidable propositions”

- sample space Ω
- events $\mathcal{F} = \{ A \mid A \subset \Omega \}$
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What happens if $\mathcal{F} \subsetneq \{ A \mid A \subset \Omega \}$?

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Specker's parable of the over-protective seer

- $\Omega = \{\, 1, 2, 3 \,\}$,
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- $P(\{\ \}) = 0$, $P(\{\, i \,\}) = \frac{1}{2}$, and $P(\{\, i, j \,\}) = 1$.

[Specker, Dialektika (1960)]

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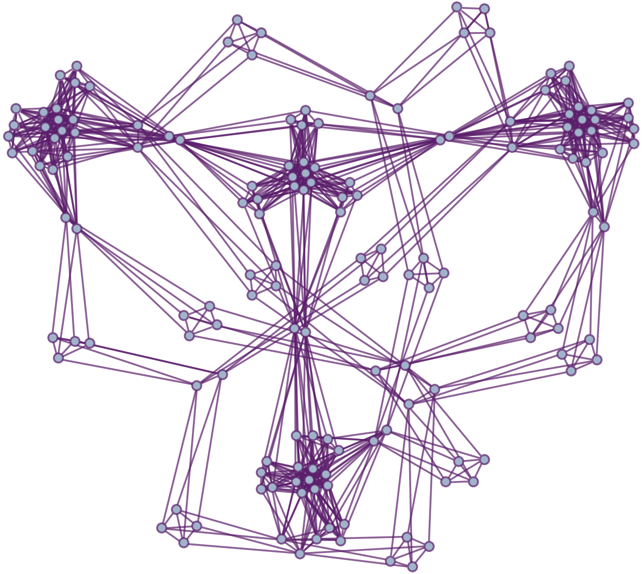
…Does it?

Quantum mechanics predicts contextual correlations

[Original proof: Kochen and Specker, J. Math. Mech. (1967)]



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Peres-Mermin square

$$\begin{bmatrix} A & B & C \\ a & b & c \\ \alpha & \beta & \gamma \end{bmatrix}$$

- A,B, etc. have outcomes $\{-1, +1\}$.
- Only values within one row or one column can be accessed simultaneously.

$$\chi = \langle ABC \rangle + \langle abc \rangle + \langle \alpha\beta\gamma \rangle + \langle Aa\alpha \rangle + \langle Bb\beta \rangle - \langle Cc\gamma \rangle$$

[Cabello, Phys. Rev. Lett. (2008)]

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[Cabello, Phys. Rev. Lett. (2008)]

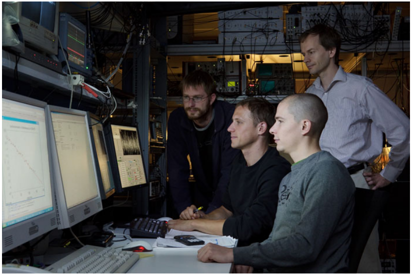
- using probability theory $\chi \leq 4.$
- in quantum mechanics $\chi = 6:$

  $\langle ABC \rangle = \langle abc \rangle = \langle \alpha\beta\gamma \rangle = \langle Aa\alpha \rangle = \langle Bb\beta \rangle = 1$ but $\langle Cc\gamma \rangle = -1$.

Are there contextual correlations in Nature?

Experimental result

$$\chi = 5.46 \pm 0.04 > 4$$



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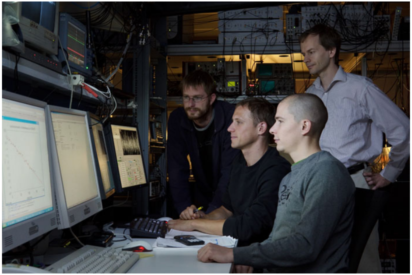

nature Vol 460 | 23 July 2009 | doi:10.1038/nature08172

LETTERS

State-independent experimental test of quantum contextuality

Why is it called *contextuality*?

Kolmogorov:

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- events $\mathcal{F} = \{ A \mid A \subset \Omega \}$
- probability P

Specker:

- $\Omega = \{ 1, 2, 3 \}$,
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Saving Kolmogorov's axioms

- three sample spaces $\Omega_A = \{ 1, 2 \}$, $\Omega_B = \{ 1, 3 \}$, $\Omega_C = \{ 2, 3 \}$.
- each outcome $\{ 1, 2, 3 \}$ participates in two **contexts**, $\Omega_A, \Omega_B \ni 1$, $\Omega_A, \Omega_C \ni 2$, and $\Omega_B, \Omega_C \ni 3$.

$\hookrightarrow$ global sample space $\Omega = \{ 1_A, 1_B, 2_A, 2_C, 3_B, 3_C \}$.

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Are we forced to identify $1_A \equiv 1_B \equiv 1$?

An open debate

Are we forced to identify $1_A \equiv 1_B \equiv 1$?

- finite precision problem [Meyer, Phys. Rev. Lett. (1999); Cabello, Phys. Rev. A (2002)]
- non-disturbance [Gühne, MK, Cabello, et. al., Phys. Rev. A (2010)]
- non-contextual noise [Szangolies, MK, Gühne, Phys. Rev. A (2013)]
- memory cost [MK, Gühne, Portillo, et. al., New J. Phys. (2011)]

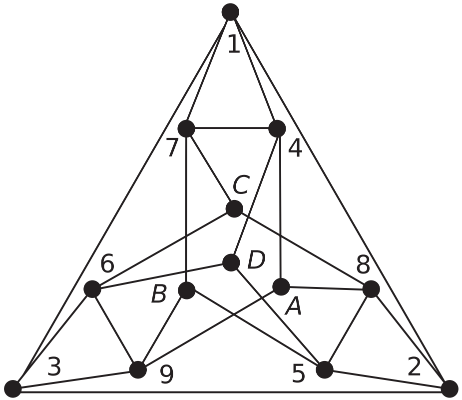
...

What is the simplest inequality?

Record holder: 13 rays in $\mathbb{C}^3$.

[Yu, Oh, Phys. Rev. Lett. (2012)
MK, Budroni, Larsson, et al., Phys. Rev. Lett. (2012)

Cabello, MK, Budroni, *preprint* (2015)]



Spacial separation: Bell inequalities

The CHSH-inequality:

$$\chi = \langle A \otimes a \rangle + \langle A \otimes b \rangle + \langle B \otimes a \rangle - \langle B \otimes b \rangle$$

[Bell, Physics (1964); Clauser, Horne, Shimony, Holt, Phys. Rev. Lett. (1969)]

classical value: $\chi \leq 2$ quantum value: $\chi \leq 2\sqrt{2}$.

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Spacial separation:
A, B and a, b are measured in different laboratories.

Ongoing experiments.

P A R T 2: Generalized probabilistic models

1. driving question: Why is quantum mechanics so particular?
2. quantum mechanics
3. generalized probabilistic models
4. quantum mechanics as an emergent theory
5. the triple slit experiment

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- Why is quantum mechanics better?
- Why $2\sqrt{2}$ but not 4?

Quantum mechanics

The underlying structure is a complex Hilbert space $\mathcal{H}$.

Measurements

A measurement with outcomes $(1, 2, \dots)$ is described by operators $(E_1, E_2, \dots)$ on $\mathcal{H}$ with $E_k \geq 0$ and $\sum_k E_k = \mathbb{1}$.

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A state is a linear map $\omega\colon \mathcal{B}(\mathcal{H}) \to \mathbb{C}$ with $\omega(\mathbb{1}) = 1$ and $\omega(E) \geq 0$ for all operators $E \geq 0$.

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Example

Let $\mathcal{H} = \mathbb{C}^2$ and define

$$A_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \text{ and } B_+ = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

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CHSH-inequality:

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attains the value

$$\omega(X) \text{ with } X = A \otimes a + A \otimes b + B \otimes a - B \otimes b$$

Example (continued)

Remember: $\omega(E) \geq 0$ for all $E \geq 0$ and $\omega(\mathbb{1}) = 1$.
Hence, $\chi \leq \sup\{\,\omega(X) \mid \omega\,\} = \|X\| = 2\sqrt{2}$.

Example (continued)

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Theorem (Tsirelson)

For any choice of measurements and any separable Hilbert space,

$$|\tfrac{1}{2}\chi| \leq k_{\mathbb{R}}(2),$$

where $k_{\mathbb{R}}(2) = \sqrt{2}$ is Grothendieck's constant.

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- Assumes Connes' embedding conjecture (for von-Neumann algebras), which implies that $[A, B] = 0$ only if $A = A' \otimes \mathbb{1}$ and $B = \mathbb{1} \otimes B'$.

Beyond quantum mechanics

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Why?

Beyond quantum mechanics

"The underlying structure is a complex Hilbert space $\mathcal{H}\ldots$"

Drop!

Assume a real (Archimedean) order-unit vector space $(V, \leq, e)$:

- V is a real vector space
- $\leq$ is a partial ordering
- for any a, $a \leq re$ for some $r \in \mathbb{R}^+$.

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Interpretation: $\omega(f_k)$ is the probability to obtain outcome k.

Examples of order-unit vector spaces

1. $V = C(X)$, $f \geq 0$ if $f(X) \subset \mathbb{R}^+$, and $e\colon x \mapsto 1$.
   - order lattice
   - all order lattices are of this form (Stone, Kakutani, Krein, and Yosida)
   - the set of states is a simplex
   - corresponds to Kolmogorovian probability theory
   - all order-unit vector spaces can be embedded into $C(X)$ (Kadison)

2. $V = \mathcal{B}(\mathcal{H})$, $E \geq 0$, and $e = \mathbb{1}$.
   - this is quantum mechanics

3. $V = \mathbb{R} \times \mathbb{R}^2$, $(t, \mathbf{x}) \geq 0$ if $t \geq \|\mathbf{x}\|_1$, and $e = (1, \mathbf{0})$.
   - achieves $\chi = 4$
   - called "Popescu-Rohrlich" box

Quantum correlations are the emergent correlations

Theorem (Dvoretzky)

If $\eta\colon S^{n-1} \to \mathbb{R}$ *is a Lipschitz function with constant* L *and central value* 1*, then for every* $\varepsilon > 0$*, if* $E \subset \mathbb{R}^n$ *is a random subspace of dimension* $k \leq k_0 = c_0 \varepsilon^2 n / L^2$*, we have, that*

$$P\left[\sup_{S^{n-1}\cap E} |\eta(\vec{x}) - 1| > \varepsilon\right] \leq c_1 \mathrm{e}^{-c_2 k_0},$$

where c_0*,* c_1*, and* c_2 *are absolute constants.*

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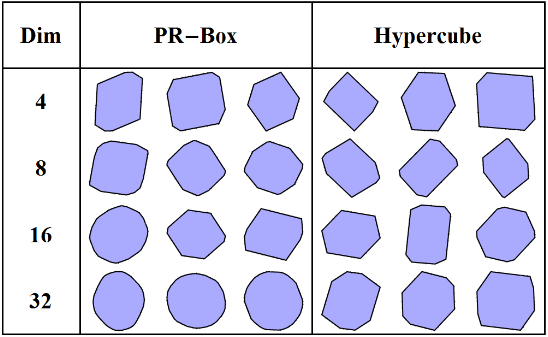

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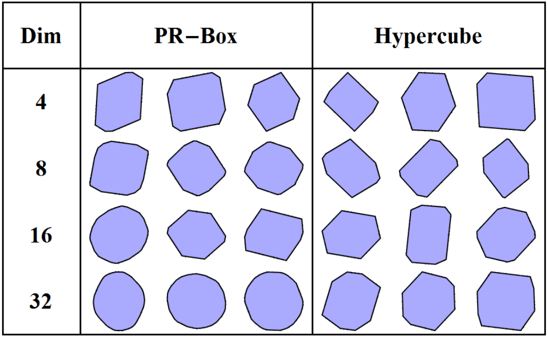
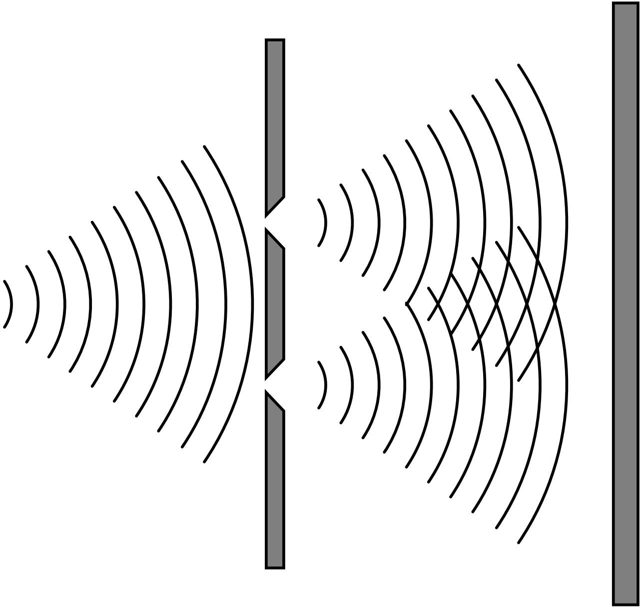


Theorem

For a bipartite scenario, if the local measurements are chosen from a typical section of all possible measurements then, with a high degree of accuracy, the predicted correlations agree with quantum predictions.

[MK, Osborne, Scholz, Werner, Phys. Rev. Lett. (2013)]

Sequential measurements: the double slit experiment



Sequential measurements: the triple slit experiment

The screen

- segment the screen into discrete intervals $\{1, 2, \dots\}$
- finding a particle in interval k corresponds to an outcome f_k

$\hookrightarrow$ measurement $(f_1, f_2, \dots)$.

The slits

- opening one, two, or three of the slits $\{1, 2, 3\}$ changes the measurement according to $\phi_\alpha \colon V \to V$, $\alpha \subset \{1, 2, 3\}$.
- double slit correlations:
  $\psi_{1,2} = \phi_{\{1,2\}} - (\phi_{\{1\}} + \phi_{\{2\}})$
- triple slit correlations:
  $\psi_{1,2,3} = \phi_{\{1,2,3\}} - (\phi_{\{1\}} + \phi_{\{2\}} + \phi_{\{3\}})$

Theorem (Sorkin)

In quantum mechanics there are no triple-slit (or higher order) correlations, $\psi_{1,2,3} = \psi_{1,2} + \psi_{1,3} + \psi_{2,3}$.

Sequential measurements in generalized models

In quantum mechanics, the action of the slits ϕ_α is given by Lüders' rule:

$$\phi_\alpha \colon E \mapsto \Pi_\alpha E \Pi_\alpha, \text{ where}$$

- Π_α is a projection
- $\Pi_{\alpha\cup\beta} = \Pi_\alpha + \Pi_\beta$ for disjoint sets

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Definition

For order-unit vector spaces, a Lüders' rule $\phi \colon V \to V$ obeys

1. $\phi(a) \geq 0$ for all $a \geq 0$
2. $\phi(e) \leq e$
3. if $0 \leq g \leq \phi(e)$, then $\phi(g) = g$.

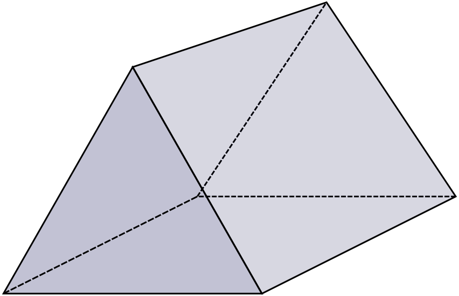
[MK, J. Phys. A (2014)]

Example: triple-slit correlations

There exists a generalized probabilistic model, so that

- $\psi_{k,j} = 0$ for all $k \neq j$,
- but $\psi_{1,2,3} \neq 0$.

$\hookrightarrow$ strong triple-slit correlations



set of states

Summary

- Classical probability theory is insufficient to describe general correlations.
- Nature did not choose to obey Kolmogorov's axioms.
- Quantum mechanics is a very particular theory.
- But its correlation are emergent from any generalized model.