# Generalized spin squeezing in the vicinity of Dicke states.

G. Vitagliano[1]

**collaboration with**

I. Apellaniz[1], P. Hyllus[1], I.L. Egusquiza[1], G. Tóth[123] (Bilbao)
B. Lücke[4], J. Peise[4], J. Arlt[4], L. Santos[4], C. Klempt[4] (Hannover)

[1]Theoretical Physics, University of the Basque Country UPV/EHU, E-48080 Bilbao, Spain

[2]IKERBASQUE, Basque Foundation for Science, Bilbao, Spain

[3]Wigner Research Centre for Physics, H-1525 Budapest, Hungary

[4]Institut für Quantenoptik, Leibniz Universitt Hannover, Hannover, Germany

DPG Meeting-March 24 2015, Heidelberg

# Contents

# Motivations

- The idea

1. **Define a generalized spin squeezing parameter that detects a wider class of states**
2. **Use the same quantities appearing in the parameter to detect multipartite entanglement**

- Motivations

1. We can detect multiparticle entanglement with just collective measurements.
   $\longrightarrow$ necessary for many-body systems.
2. A figure of merit for new entangled states, e.g. Dicke states is defined.
   $\longrightarrow$ As spin squeezed states might be useful for quantum enhanced metrology.
3. Experimentally one has to measure only $(\Delta J_k)^2$, $\langle J_k \rangle$ to verify entanglement and its depth.

# Definition of separable state

- Let us consider a system composed of $N$ particles.
- The following state

$$\rho = \sum_i p_i \rho_i^{(1)} \otimes \cdots \otimes \rho_i^{(N)},$$

  where $p_i > 0$ and $\sum_i p_i = 1$ is called separable.
- The $\rho_i^{(M)}$ are single particle states.

Any state that is not separable is called **entangled**.

# Definition of $k$-producible state

- Consider a state of $N$ particles of the form

$$\rho = \sum_i p_i \rho_i^{(1)} \otimes \cdots \otimes \rho_i^{(M_i)}, \qquad (1)$$

where $p_i > 0$ and $\sum_i p_i = 1$.

- A state is called $k$-producible if it can be written as (1) with $\rho_i^{(n)}$ states of **at most** $k$ **particles**

Any state that is not $k$-producible is called $(k+1)$**-entangled**.

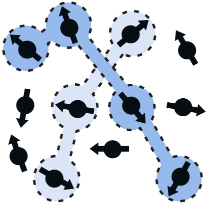


Figure: **A $k$-producible state is separable in groups of $k$ particles.**

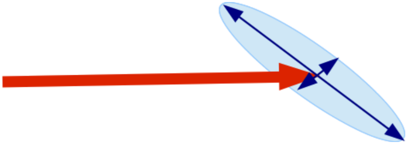
# What is a spin squeezed state?

- Let's consider an $N$-particle system and the collective spins

$$J_k := \sum_{n=1}^{N} j_k^{(n)}$$

- Spin Squeezed States satisfy:

$$\boxed{(\Delta J_x)^2_{\mathrm{SSS}} < (\Delta J_x)^2_{\mathrm{SQL}} = \frac{1}{2}|\langle J_z \rangle|}$$



Spin squeezed states are very useful for high precision metrology.

[M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993);
D.J. Wineland, J. J. Bollinger, and W. M. Itano, Phys. Rev. A **50**, 67 (1994). ]

# Spin squeezing and multiparticle entanglement

- For spin-$\frac{1}{2}$ systems, any separable state must satisfy

$$\xi_s^2 = \frac{N(\Delta J_x)^2}{\langle J_y\rangle^2+\langle J_z\rangle^2} \geq 1$$

- A definition of spin squeezing is given by $\xi_s^2 < 1$.

**Every spin squeezed state of spin-$\frac{1}{2}$ particles is also entangled.**

[A. Sørensen, L.M. Duan, J.I. Cirac, and P. Zoller, Nature **409**, 63 (2001)]

# A complete set of entanglement criteria for qubits

$$\langle J_x^2\rangle + \langle J_y^2\rangle + \langle J_z^2\rangle \leq \frac{N(N+2)}{4} \qquad \text{(2a)}$$

$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq \frac{N}{2} \qquad \text{(2b)}$$

$$(N-1)\left[(\Delta J_x)^2 + (\Delta J_y)^2\right] - \langle J_z^2\rangle \geq \frac{N(N-2)}{4} \qquad \text{(2c)}$$

$$\boxed{(N-1)\left[(\Delta J_x)^2\right] - \langle J_y^2\rangle - \langle J_z^2\rangle \geq -\frac{N}{2}} \qquad \text{(2d)}$$

- Any state that violates one of Eq. (2) is entangled.

[G. Tóth, C. Knapp, O. Gühne and H.J. Briegel, PRL 99, 250405 (2007);
G. Tóth, C. Knapp, O. Gühne and H.J. Briegel, PRA 79,042334 (2009).]

# Generalized spin squeezing

- A separability (spin squeezing) inequality is

$$\xi_{\rm os}^2 := (N-1)\frac{(\Delta J_x)^2}{\langle J_y^2\rangle+\langle J_z^2\rangle-\frac{N}{2}} \geq 1$$

- $\xi_{\rm os}^2 < 1$ is a generalized definition of spin squeezing.

- It means that $(\Delta J_x)^2$ is small compared to $\langle J_y^2\rangle + \langle J_z^2\rangle - \frac{N}{2}$.

- It can detect also states such that $\langle J_x\rangle^2 + \langle J_y\rangle^2 + \langle J_z\rangle^2 = 0$

**[GV, I. Apellaniz, I.L. Egusquiza, and G. Tóth, PRA 89, 032307 (2014)]**

# The original spin squeezing criterion for the entanglement depth

- A condition for $k$-producibility is

$$(\Delta J_z)^2 \geq J_{\max} F_{\frac{k}{2}} \left( \frac{\sqrt{\langle J_x \rangle^2 + \langle J_y \rangle^2}}{J_{\max}} \right)$$

- Every state that violates it is for sure $k+1$-entangled.

- The function $F_j(x)$ is defined as

$$F_j(X) := \frac{1}{j} \min_{\frac{\langle j_x \rangle}{j} = X} (\Delta j_z)^2$$

[A.S. Sørensen and K. Mølmer, Phys. Rev. Lett. 86, 4431 (2001); experimental test: C. Gross, T. Zibold, E. Nicklas, J. Esteve, M. K. Oberthaler, Nature 464, 1165 (2010).]

# A generalized spin squeezing criterion for the entanglement depth

- An other condition for $k$-producibility is

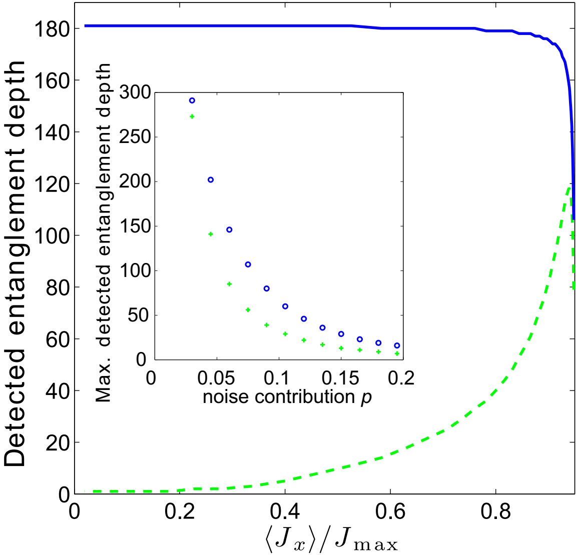
$$(\Delta J_z)^2 \geq J_{\max} F_{\frac{k}{2}} \left( \frac{\sqrt{\langle J_x^2 + J_y^2 \rangle - J_{\max}\left(\frac{k}{2}+1\right)}}{J_{\max}} \right)$$

- Every state that violates it is for sure $k+1$-entangled.

- The function $F_j(x)$ is the same as for Sørensen-Mølmer's criterion.

**[B. Lücke, J. Peise, GV, J. Arlt, L. Santos, G. Tóth and C. Klempt, Phys. Rev. Lett. 112, 155304 (2014), editors' suggestion.]**

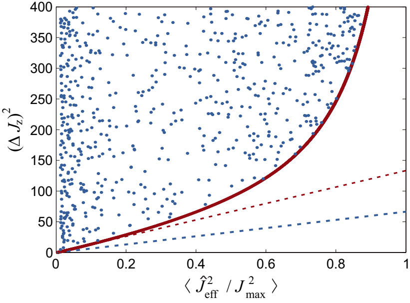
# In many practical situations our condition is more sensitive and more tolerant to white noise.




[B. Lücke, J. Peise, GV, J. Arlt, L. Santos, G. Tóth and C. Klempt, Phys. Rev. Lett. 112, 155304 (2014), editors' suggestion.]

# Experimental detection of 28-particle entanglement

The boundary to our criterion is the red line




The blue dashed line is the condition given in [L.-M. Duan, PRL 107, 180502 (2011)]

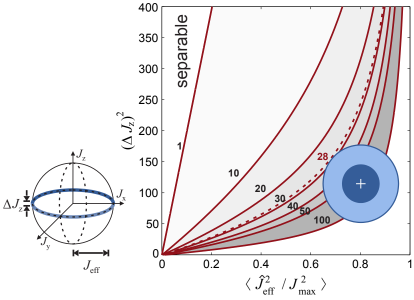



Figure: Left: ideal Dicke state in the Bloch sphere, $J_{\rm eff}$ is maximal, $(\Delta J_z)^2 = 0$.
Right: experimental data; the point witnesses 68-particle entanglement; the circles are 1 and 2 $\sigma$.

The state is also such that $\xi_{\rm os}^2 = -11.4(5)dB$, while $\xi_{\rm s}^2 \to +\infty$.

# Conclusion

- We have defined a new spin squeezing parameter $\xi_{\mathrm{os}}^2$
  1. detects a wider class, including Dicke states of states and is more tolerant to noise.
- We derived also new criterion for the entanglement depth.
  1. It detects more states than the Sørensen-Mølmer criterion and is more tolerant to noise.
- The criteria have been implemented to detect the entanglement depth of a state close to a Dicke state.
  1. They measured $k = 68$ ($k = 28$ within 2 sigmas) and $\xi_{\mathrm{os}}^2 = -11.4(5)dB$, while $\xi_{\mathrm{s}}^2 \to +\infty$.

THANKS FOR YOUR ATTENTION!

GV, P. Hyllus, I.L. Egusquiza, and G. Tóth, **PRL 107, 240502 (2011)**

GV, I. Apellaniz, I.L. Egusquiza, and G. Tóth, **PRA 89, 032307 (2014)**

B. Lücke, J. Peise, GV, J. Arlt, L. Santos, G. Tóth and C. Klempt, **PRL 112, 155304 (2014), editors' suggestion** (Open access) *featured in* **physics.aps.org** *and* **Revista Española de Fisica.**

Group Home Page

www.gedentqopt.eu