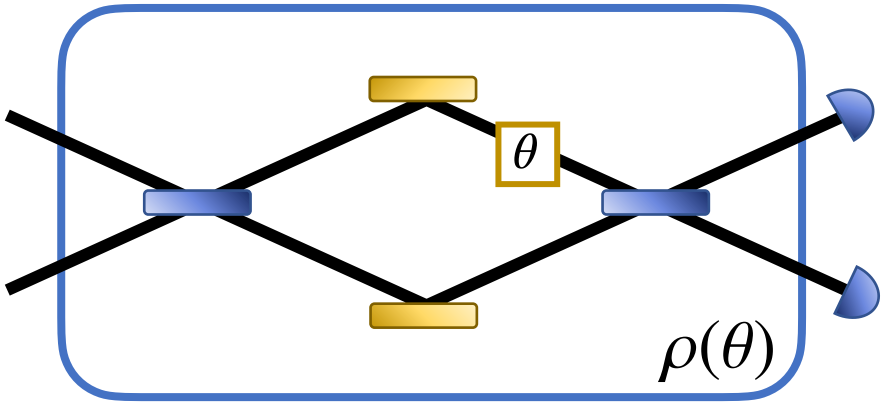
Quantum-Enhanced Estimation of Mode Parameters

Manuel Gessner
IFIC-Institut de Física Corpuscular
Universitat de València

Bilbao
25/01/2023

Quantum Parameter Estimation



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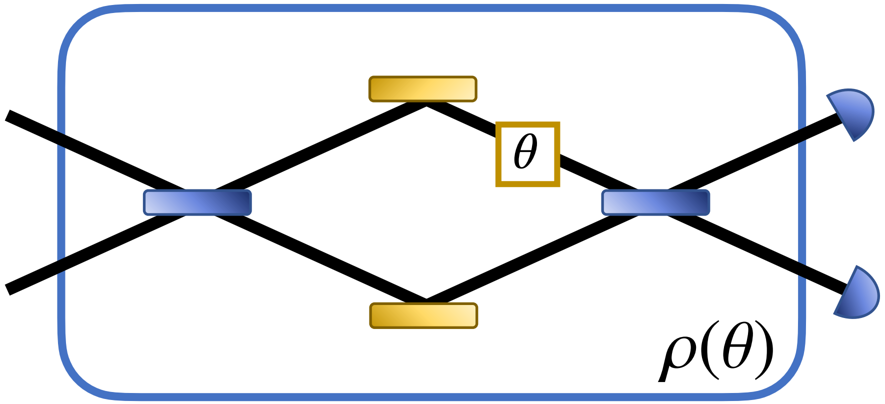
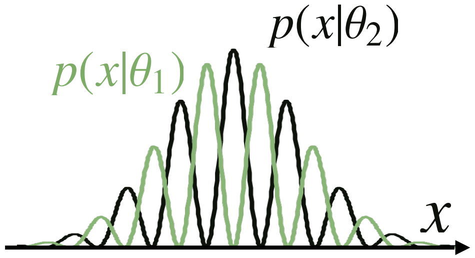


Measurement

Results:

$x_1, x_2, \ldots, x_\mu$

Distribution: $p(x|\theta)$



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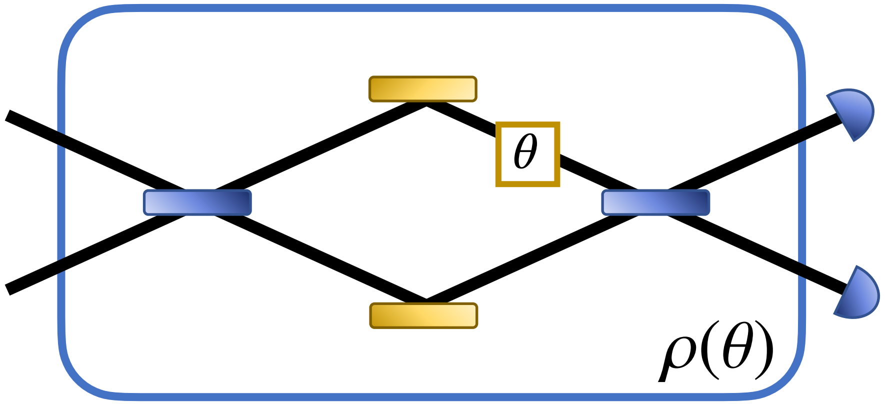

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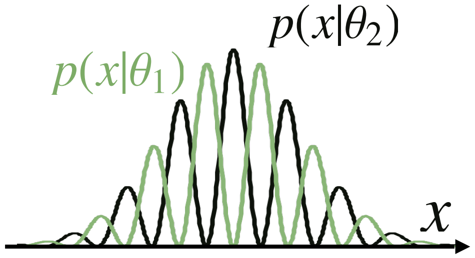

Estimation

$\theta_{\text{est}}(x_1, x_2, \ldots, x_\mu)$

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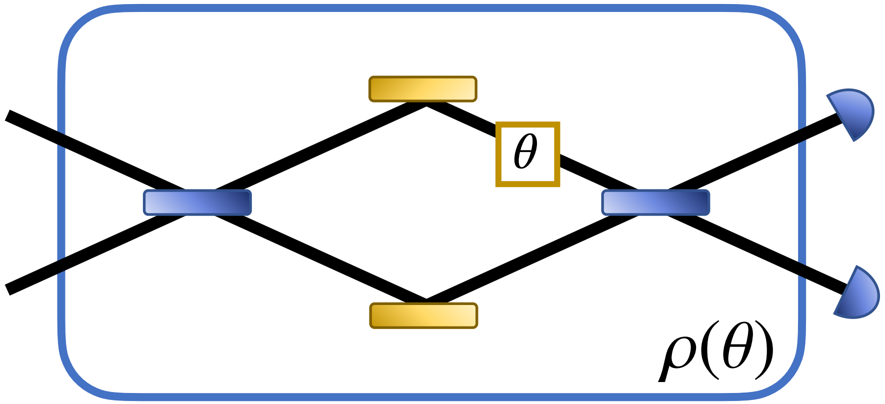

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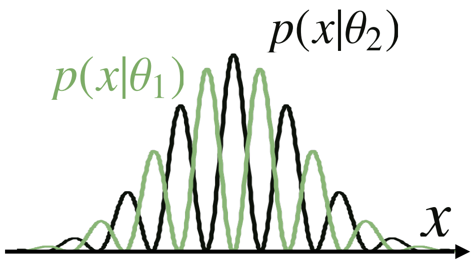

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$\langle \theta_{\text{est}} \rangle = \theta$

Minimize $(\Delta\theta_{\text{est}})^2$

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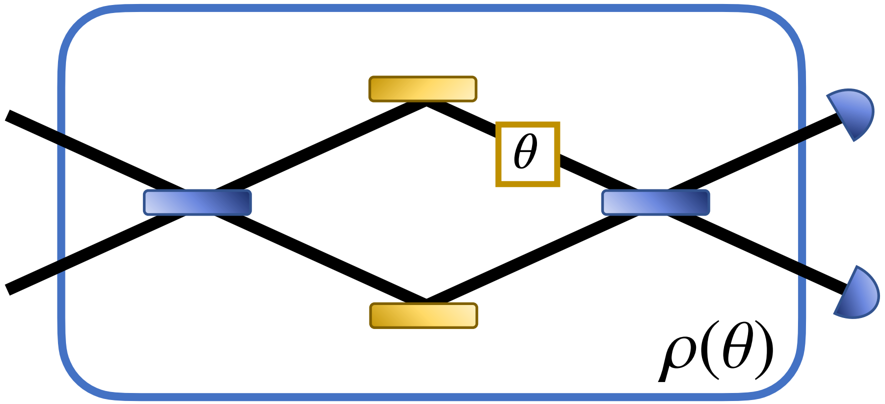

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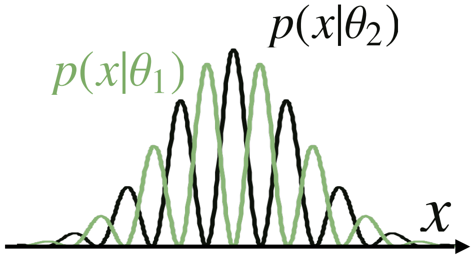

Quantum system:

$$p(x|\theta) = \mathrm{Tr}\{\rho(\theta)\Pi_x\}$$

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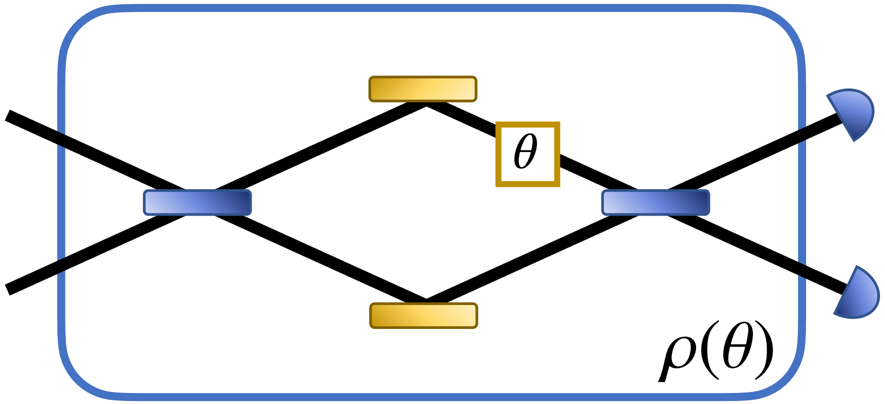
Quantum Parameter Estimation

State preparation

ρ

Parameter imprinting



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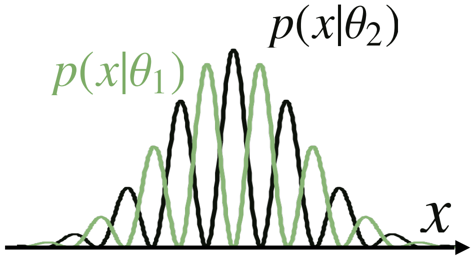

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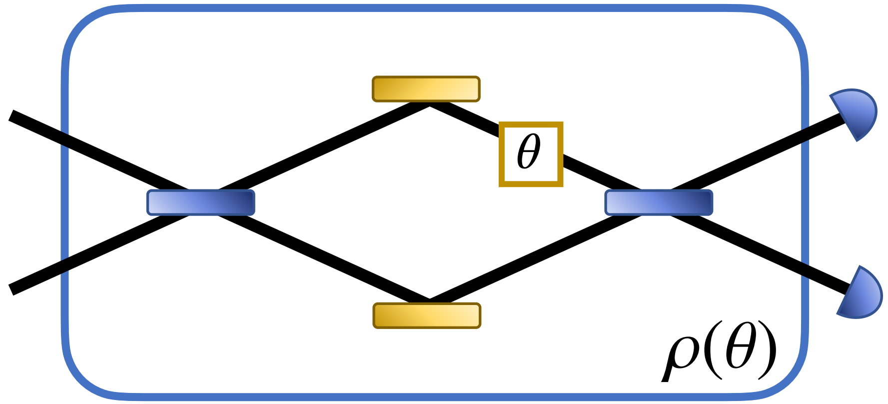

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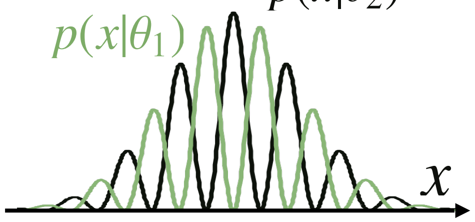

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- initial state

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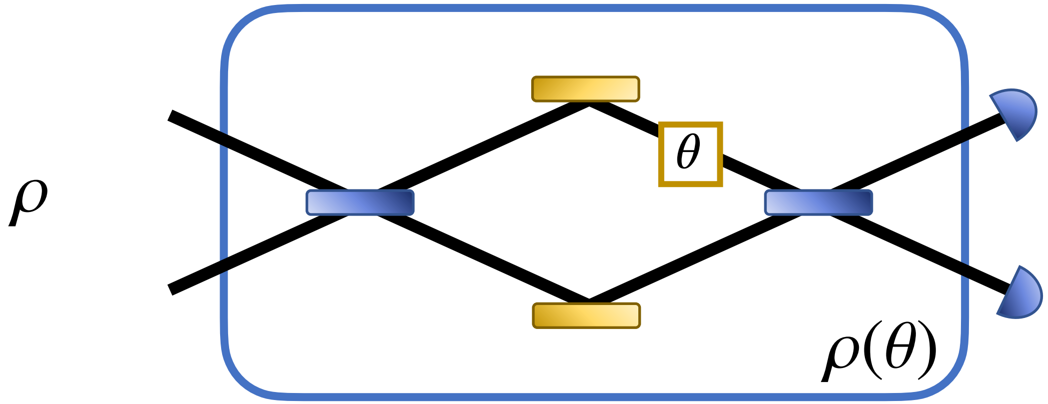

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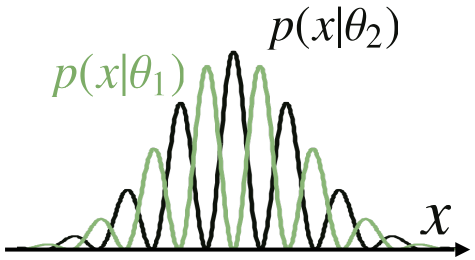

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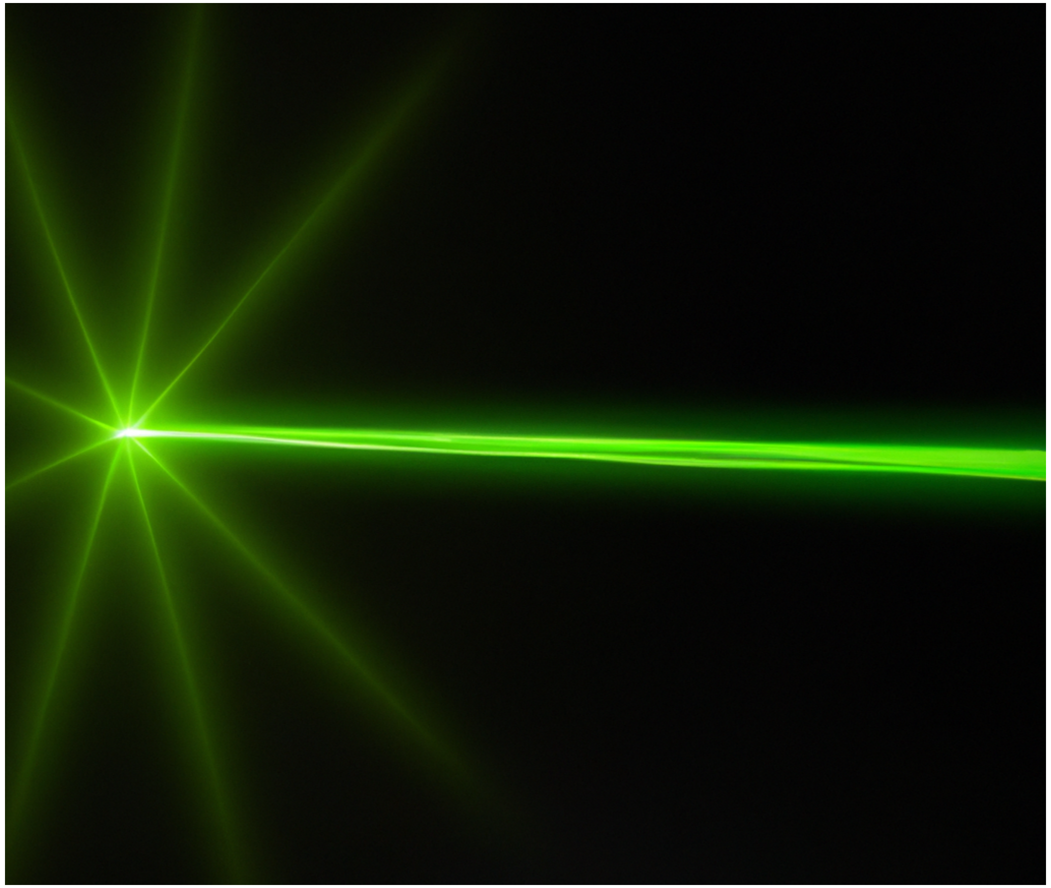
Fundamental limitation:
Quantum fluctuations

MODES AND STATES IN QUANTUM OPTICS

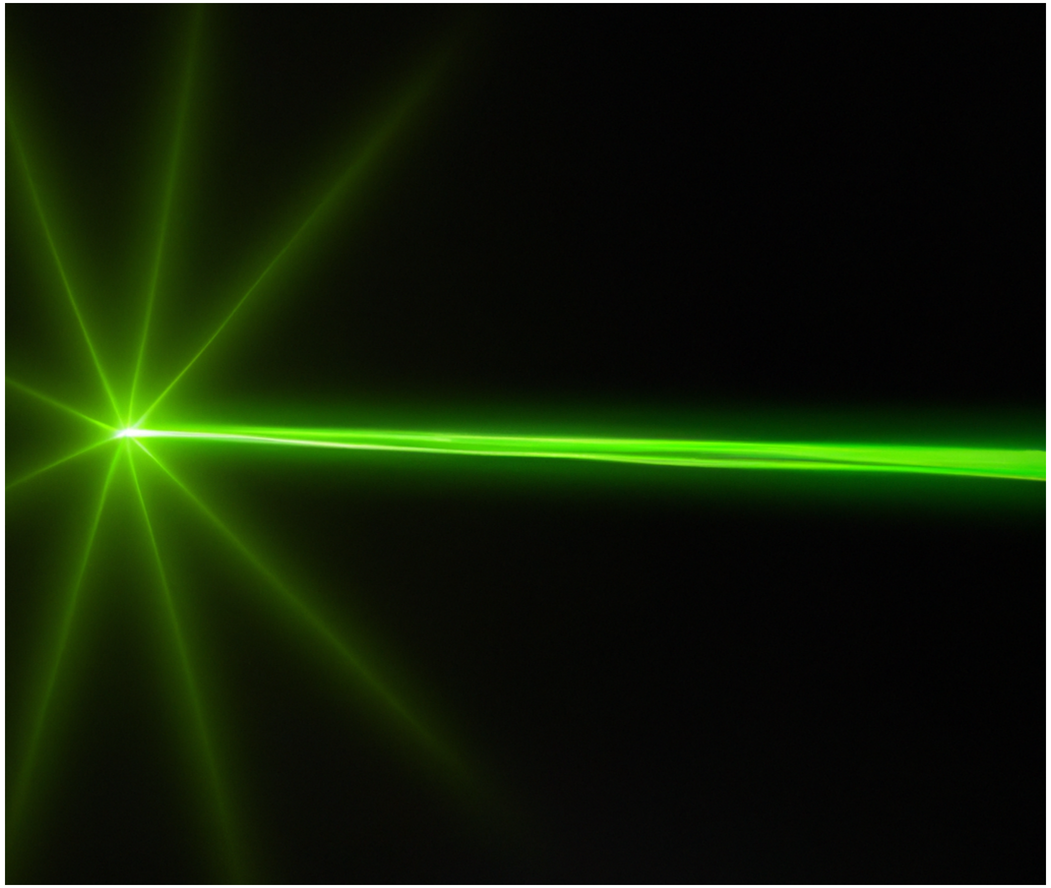


$$|\alpha\rangle = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle$$

"where $\hat{a}^\dagger$ creates a photon."

What defines the quantum state of light?

Modes and States in Quantum Optics



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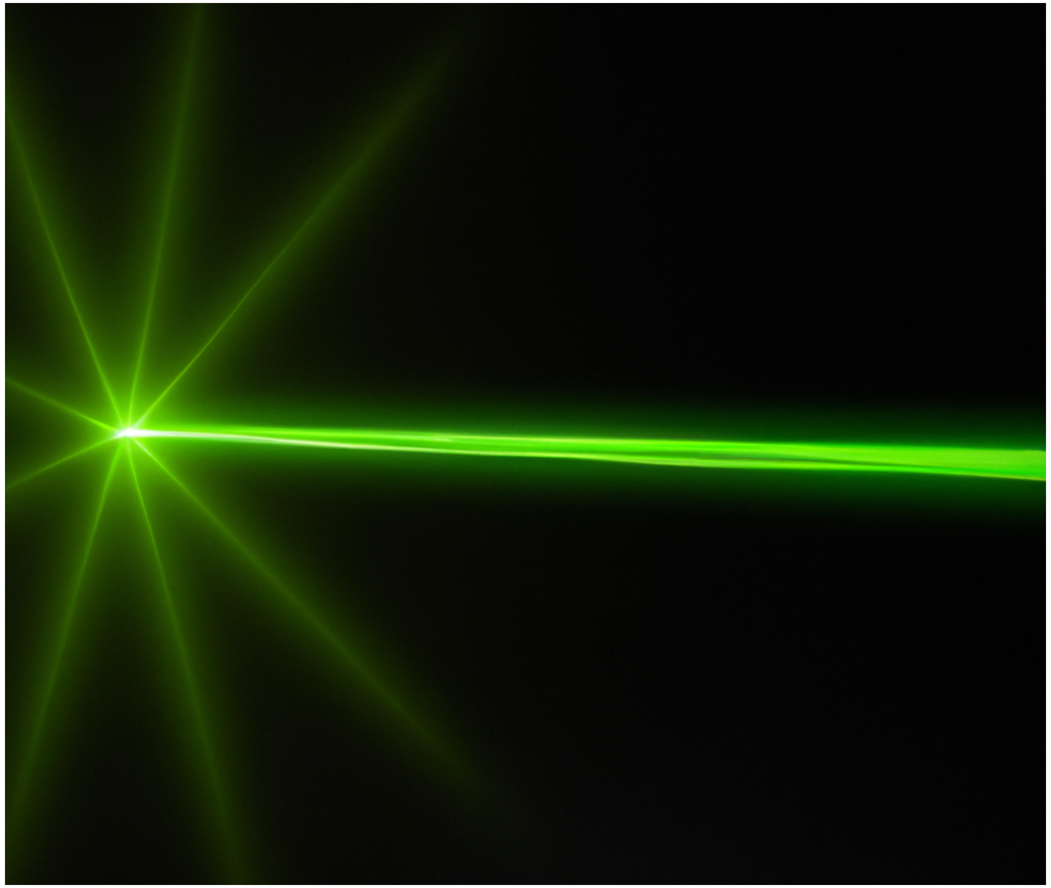

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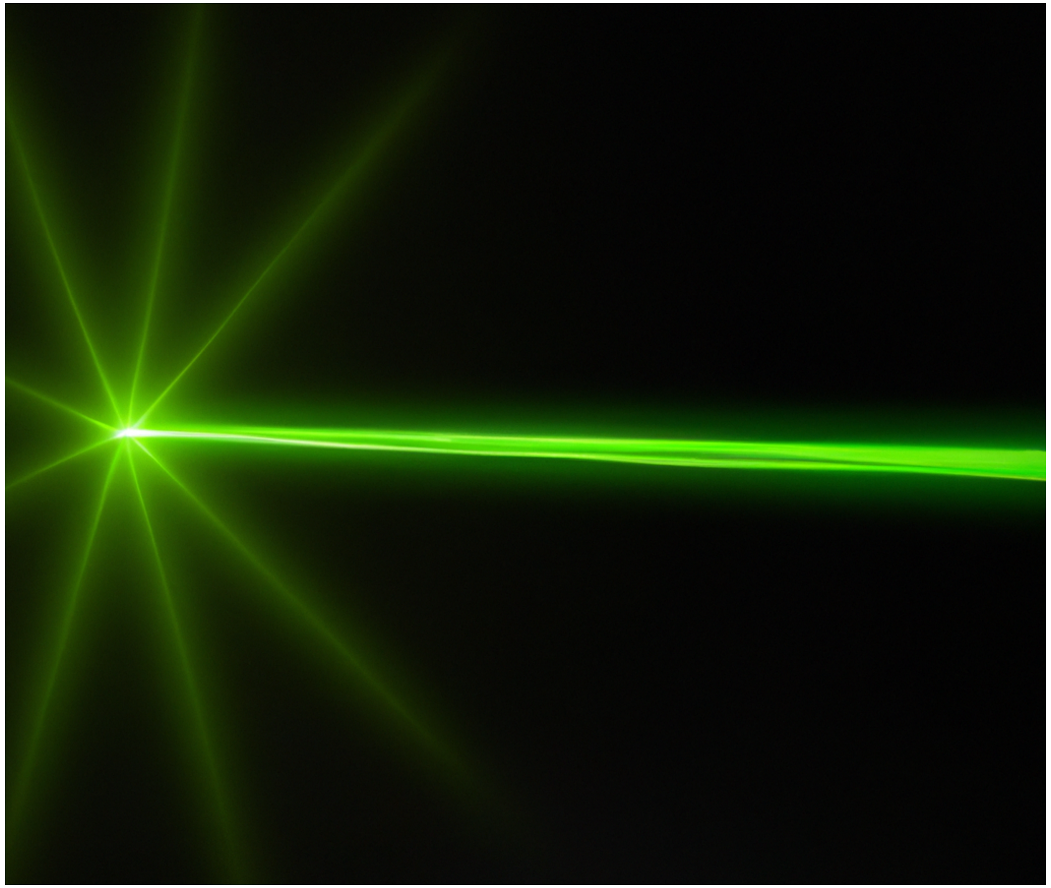

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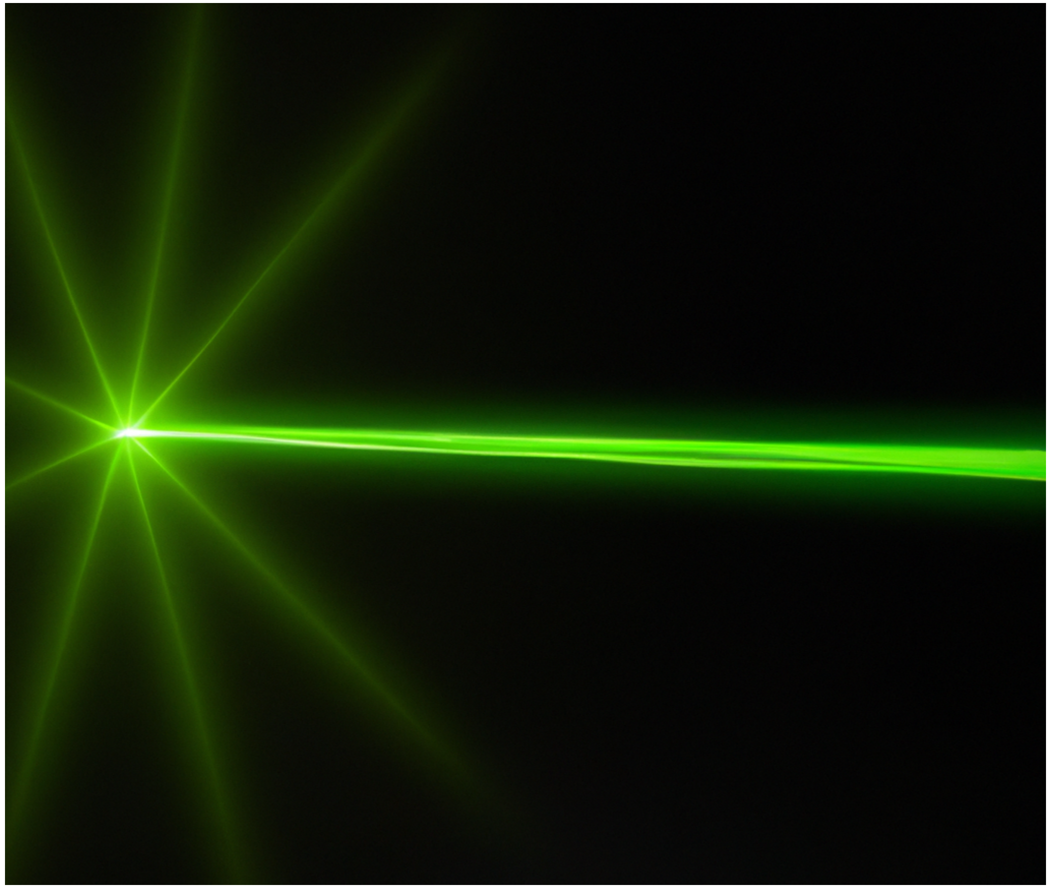

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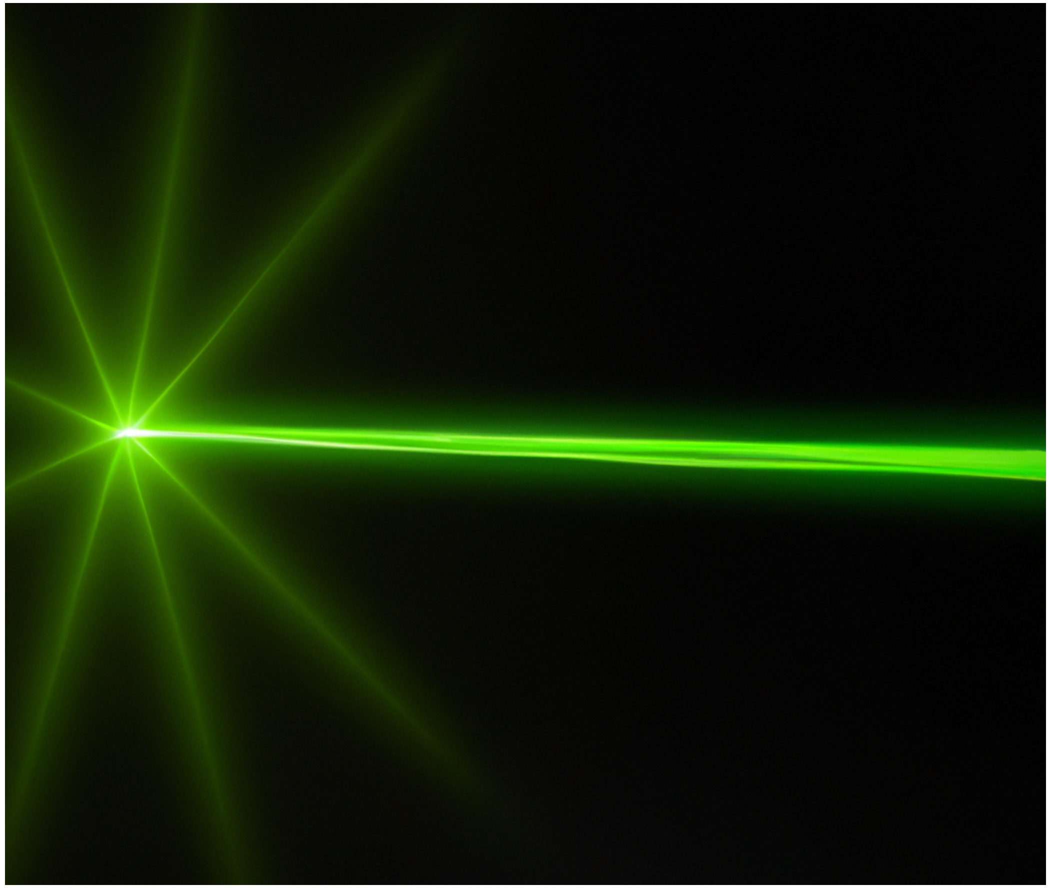

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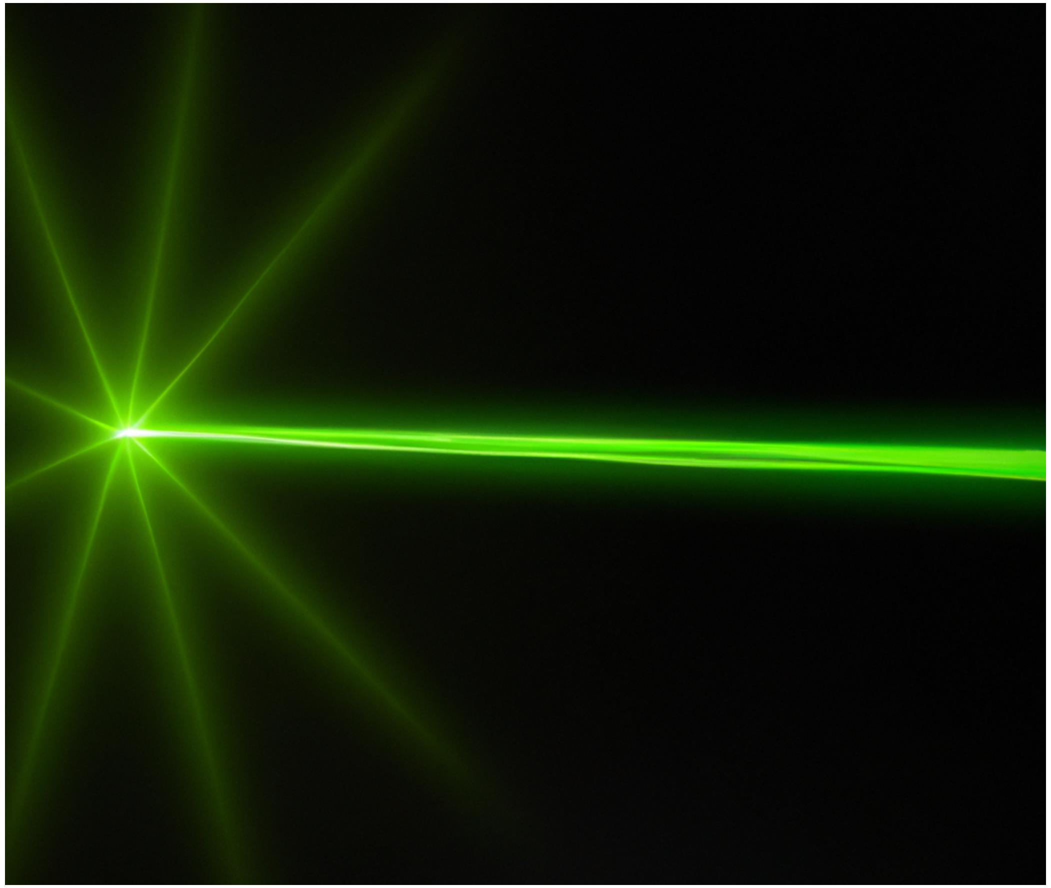

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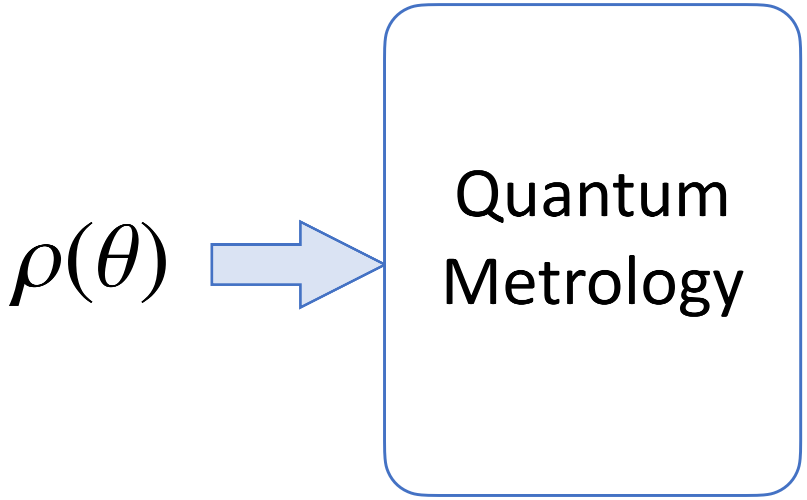
Goal: Identify quantum precision limits and optimal strategies for the estimation of a mode parameter

Quantum Metrology



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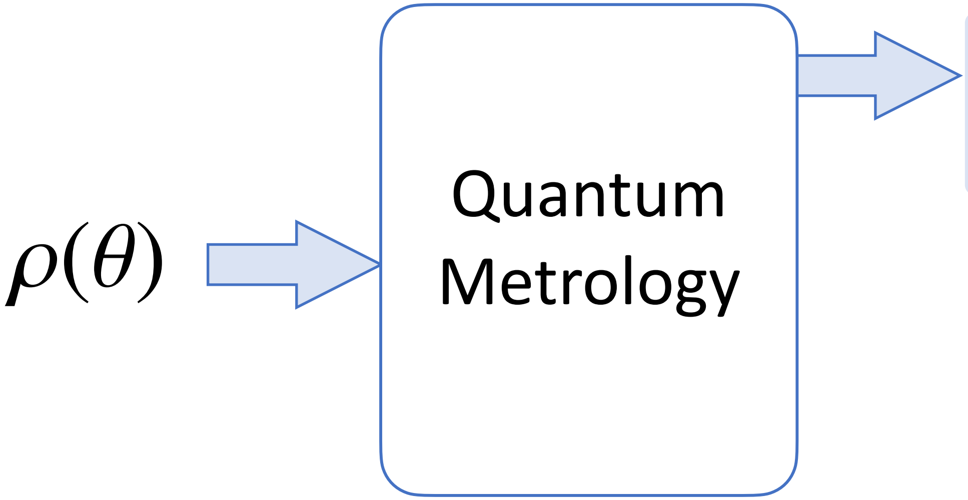

Sensitivity limit:
quantum Cramér-Rao bound

$$(\Delta\theta_{\mathrm{est}})^2 \geq \frac{1}{F_Q[\rho(\theta)]}$$

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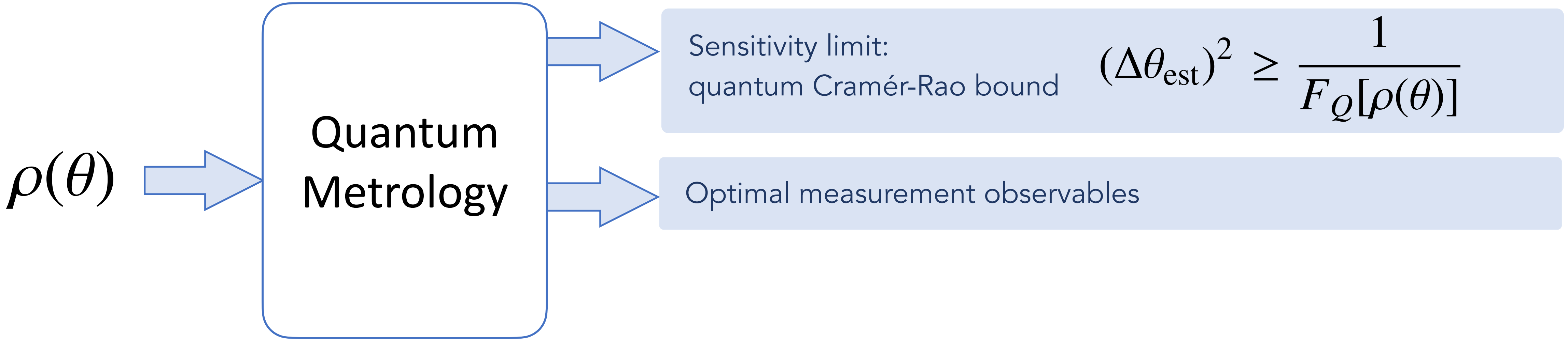

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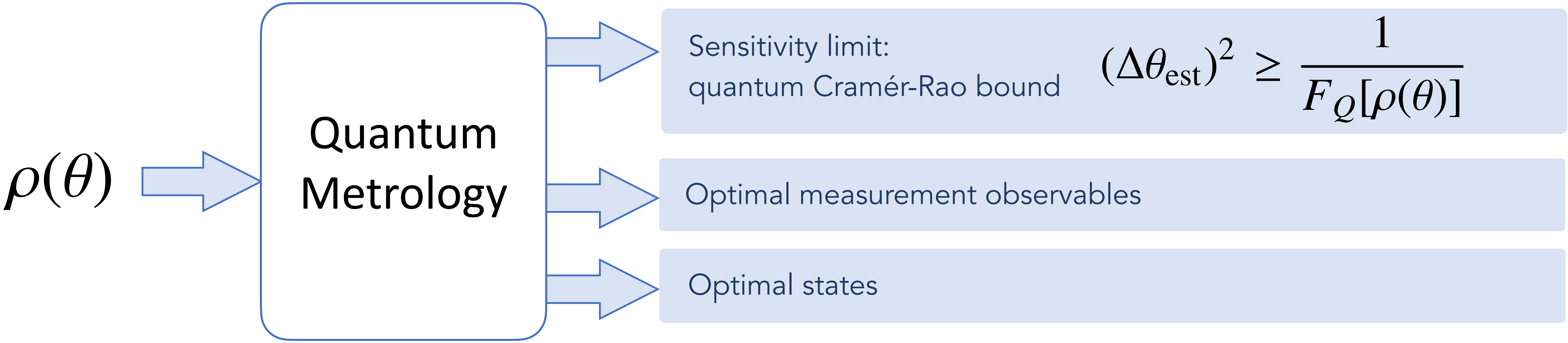

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QUANTUM METROLOGY

$\rho(\theta)$ → **Quantum Metrology** →

- Sensitivity limit: quantum Cramér-Rao bound $(\Delta\theta_{\text{est}})^2 \geq \dfrac{1}{F_Q[\rho(\theta)]}$
- Optimal measurement observables
- Optimal states

Arbitrary quantum state

$$\rho(\theta) = \sum_k p_k |k\rangle\langle k|$$

Quantum Fisher information

$$F_Q[\rho(\theta)] = \sum_{k,l} \frac{2}{p_k + p_l} |\langle k|\partial_\theta \rho(\theta)|l\rangle|^2$$

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Unitary evolution

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Pure state

$$F_Q[\rho, H] = 4(\Delta H)^2_{\rho(\theta)}$$

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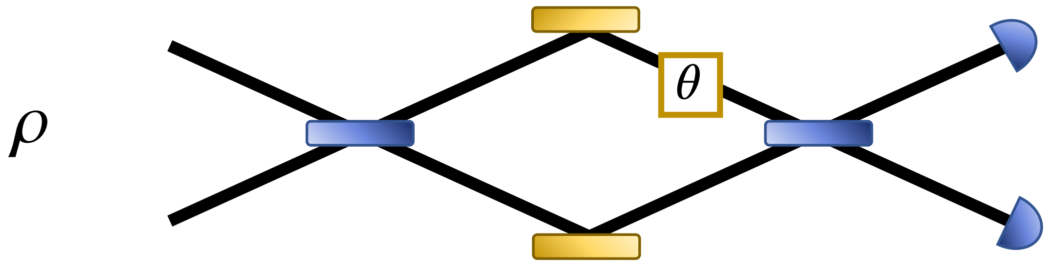

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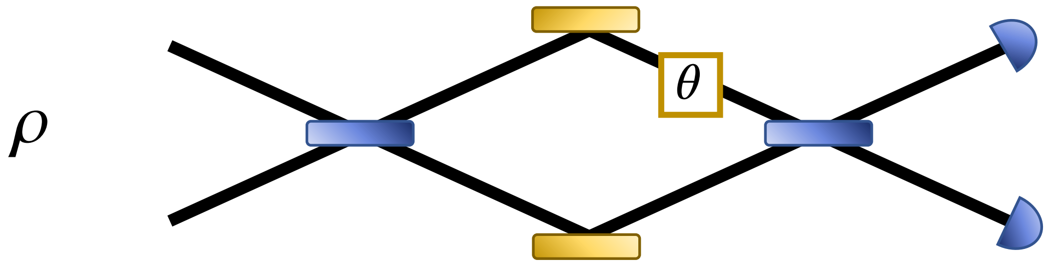

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N: average number of probe particles (photons, atoms, …)

Classical source state ρ

$$F_Q \leq N \quad \Rightarrow \quad (\Delta\theta_{\mathrm{est}})^2 \geq \frac{1}{N}$$

Standard quantum limit (SQL)
Fluctuations of the vacuum

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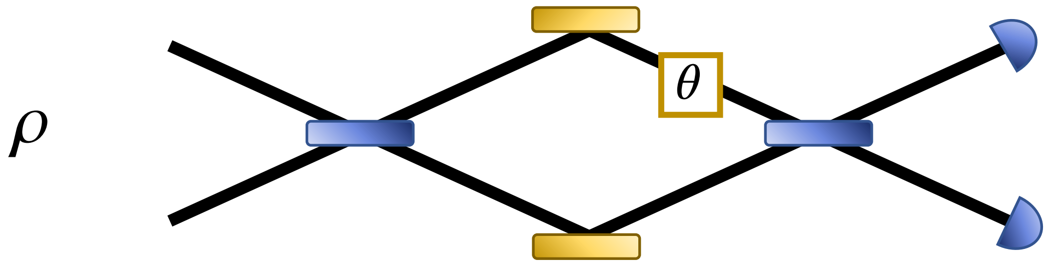

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Heisenberg limit (HL)

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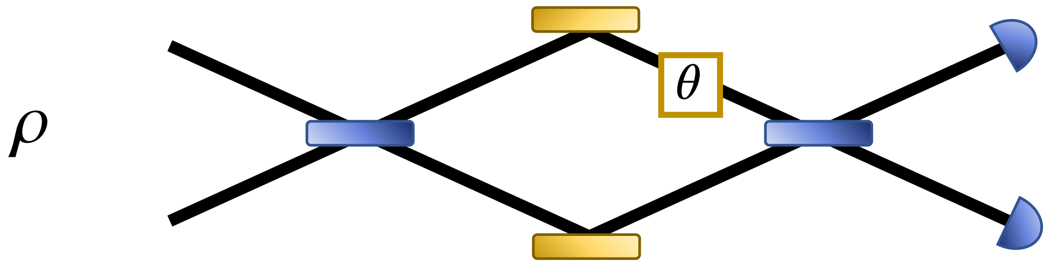

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Gravitational wave detectors, atomic clocks and interferometers, …

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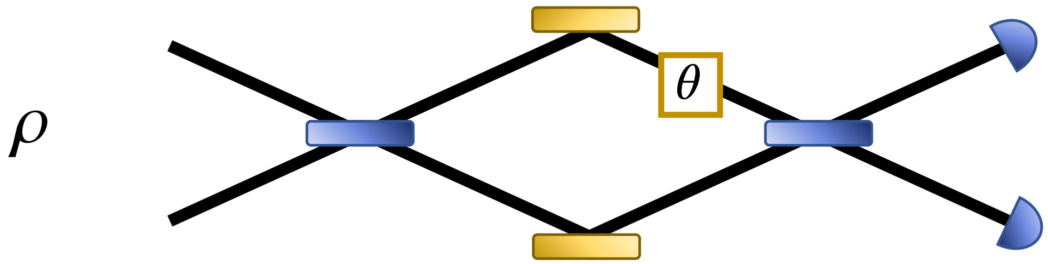

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Can we achieve scaling enhancements also for the estimation of mode parameter?
Possible applications: displacement sensing, imaging, timing, spectroscopy, etc

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Modes and States in Quantum Optics

Optical mode $f(r, t)$

- Vector field that depends on space and time
- Normalized solution for
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C. Fabre and N. Treps, Rev. Mod. Phys. **92**, 035005 (2020)

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Classical electromagnetic field

$$E^{(+)}(r, t) = \sum_m \epsilon_m f_m(r, t)$$

coefficients → ϵ_m; basis of modes → f_m

C. Fabre and N. Treps, Rev. Mod. Phys. **92**, 035005 (2020)

MODES AND STATES IN QUANTUM OPTICS

Optical mode $f(r,t)$

- Vector field that depends on space and time
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$$g_n = \sum_m u_{mn} f_m$$

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A change of the mode basis

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unitary matrix

changes the creation operators in the exact same way:

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creates a photon in the mode g_m

C. Fabre and N. Treps, Rev. Mod. Phys. **92**, 035005 (2020)

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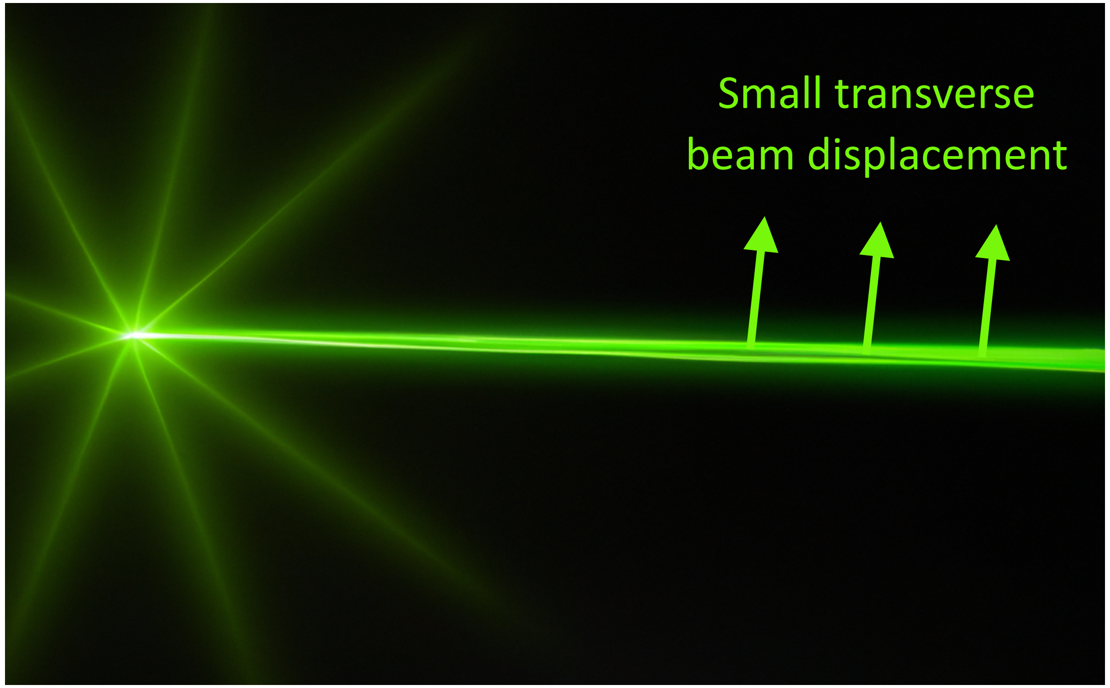
C. Fabre, J. B. Fouet, and A. Maître, Opt. Lett. **25**, 75 (2000)



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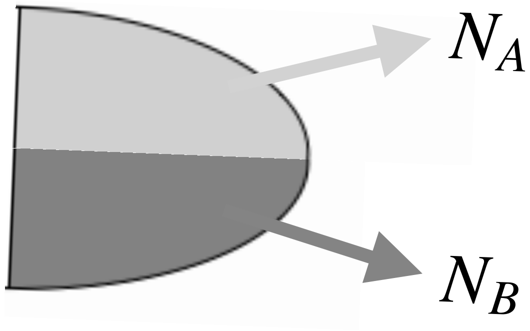

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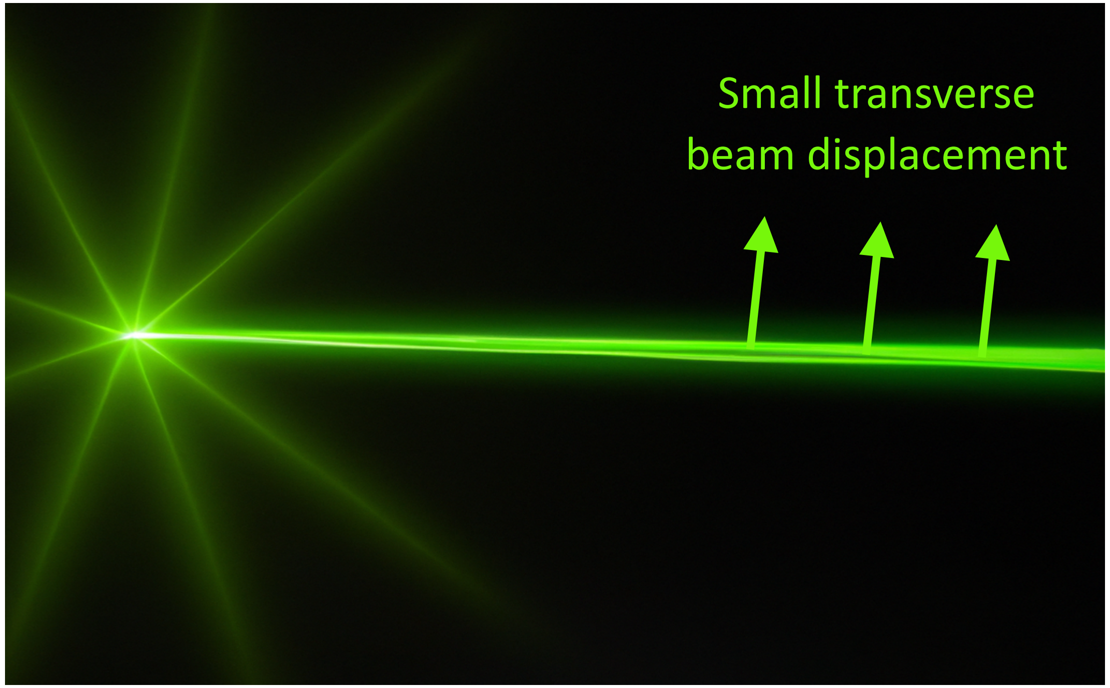

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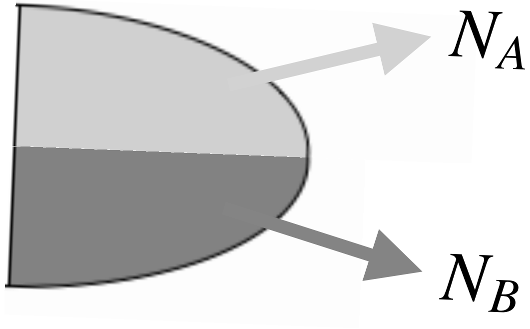

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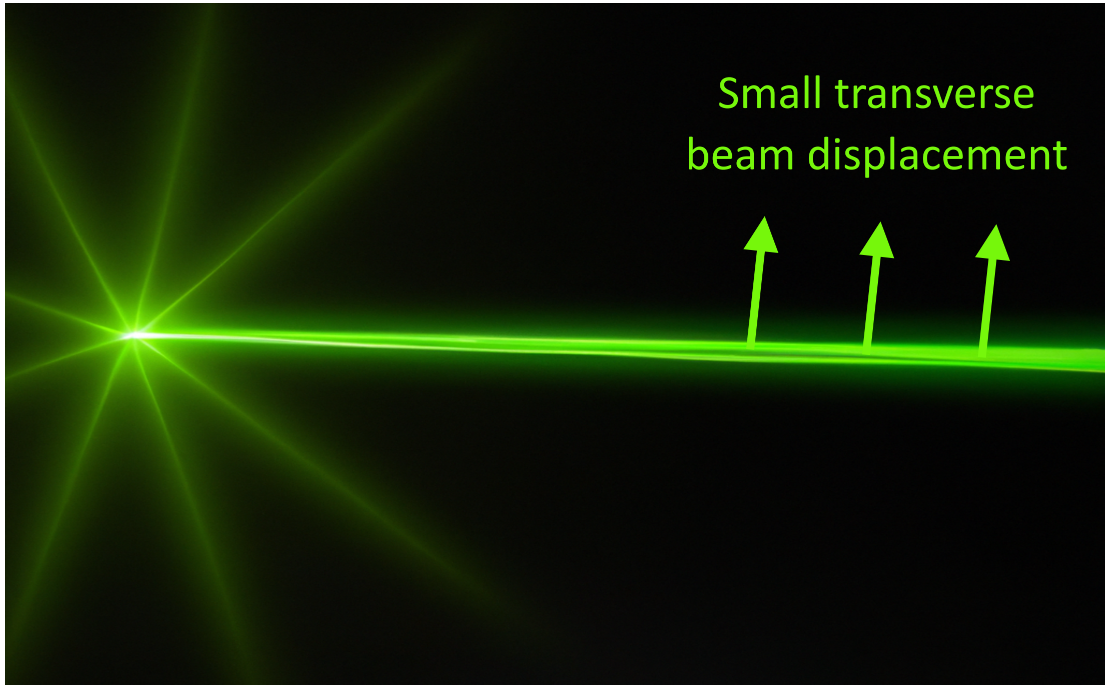

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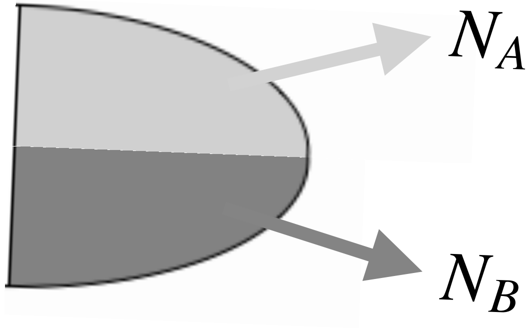

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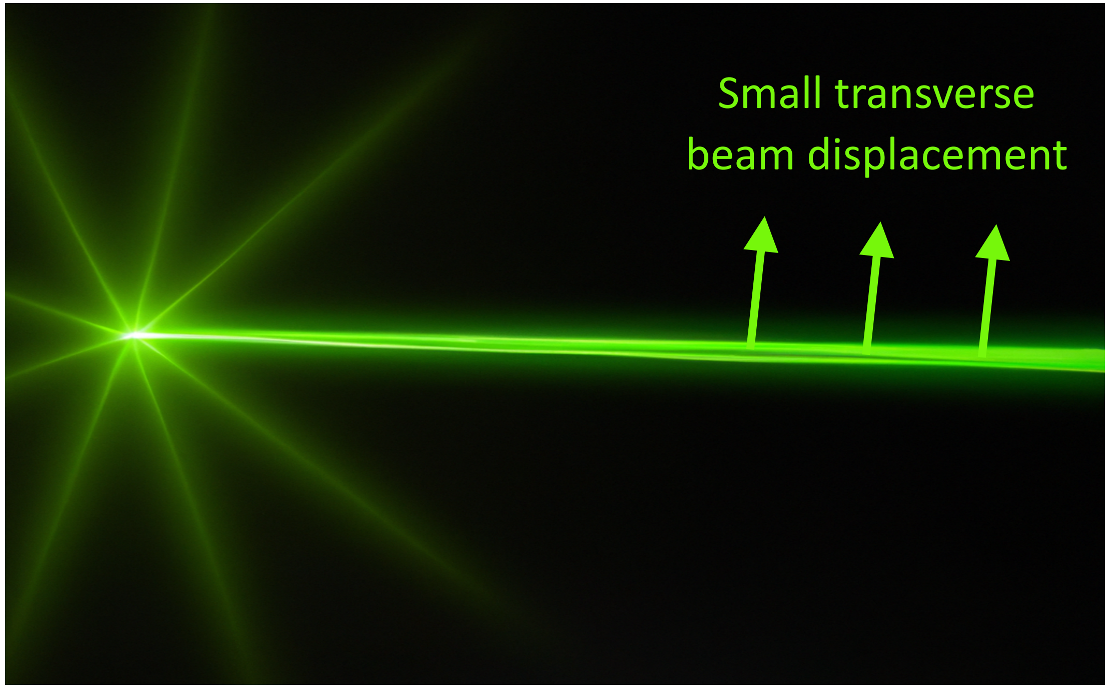

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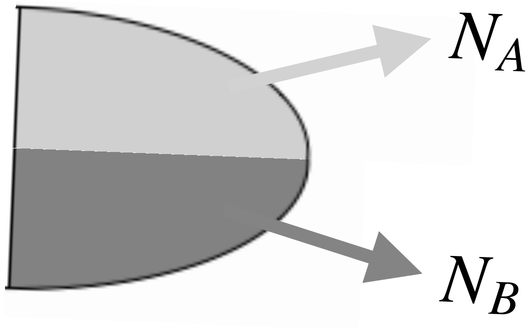

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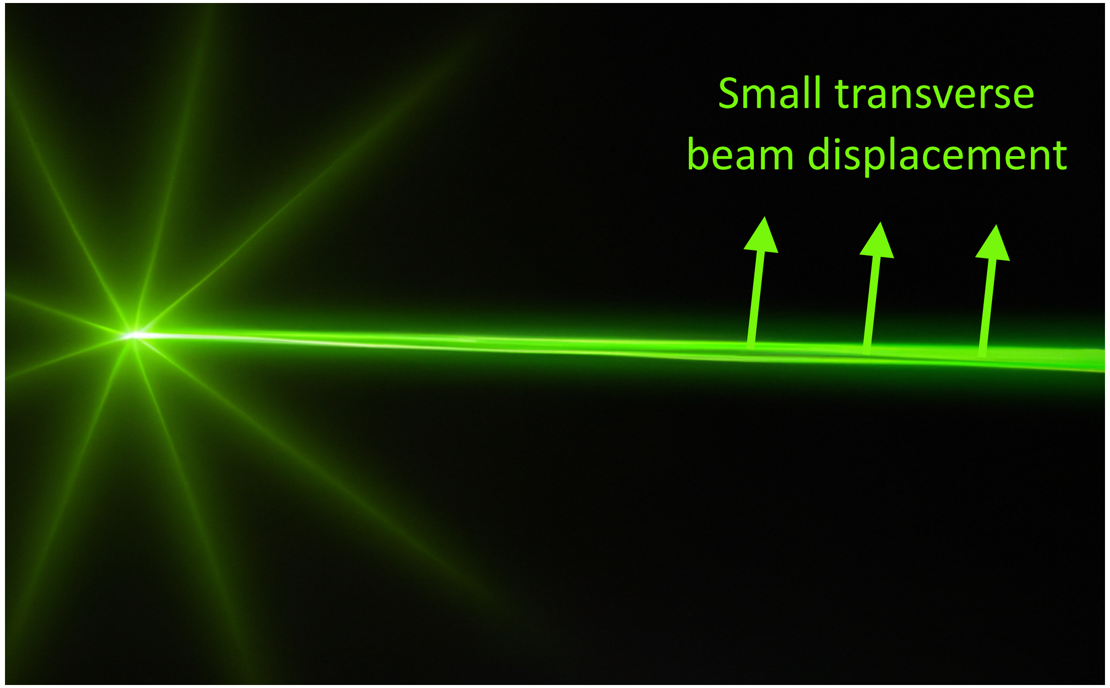

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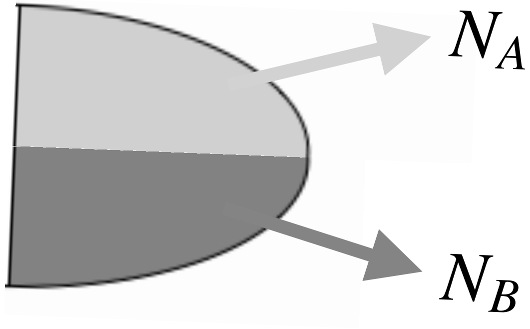

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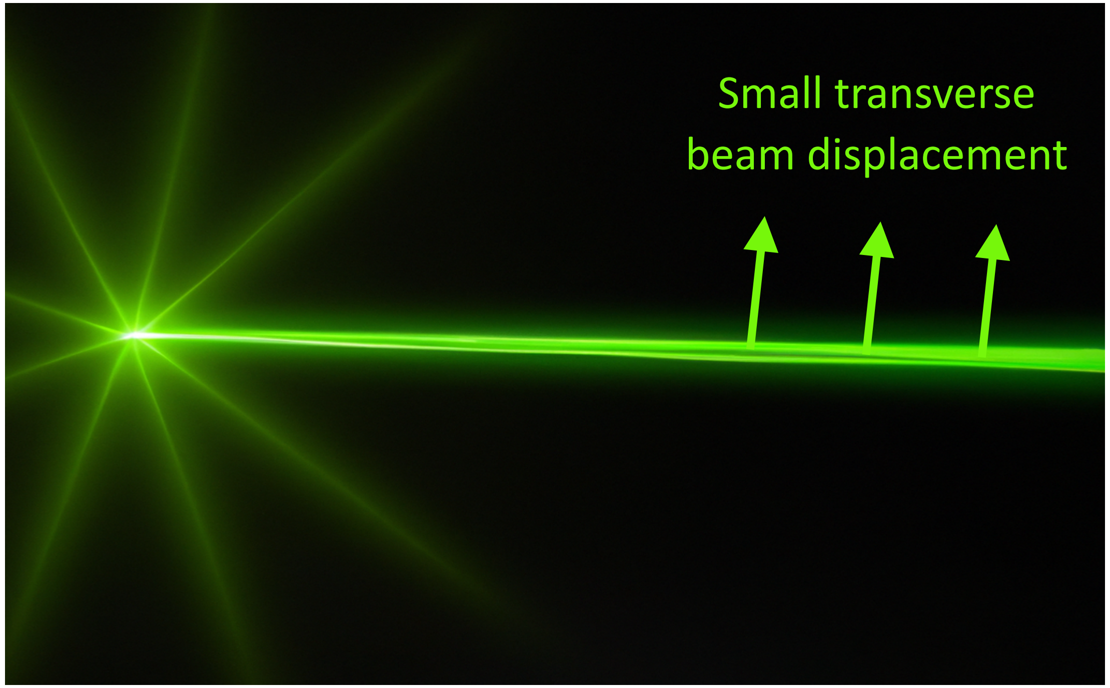

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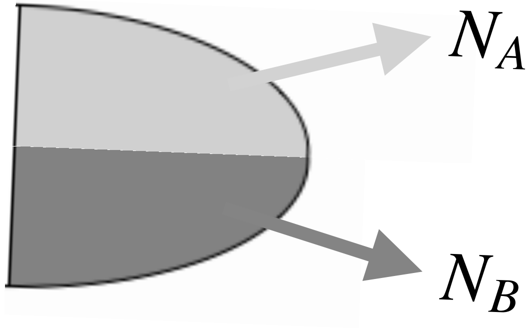

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Analysis either limited to specific measurements and estimators or to a specific family of states

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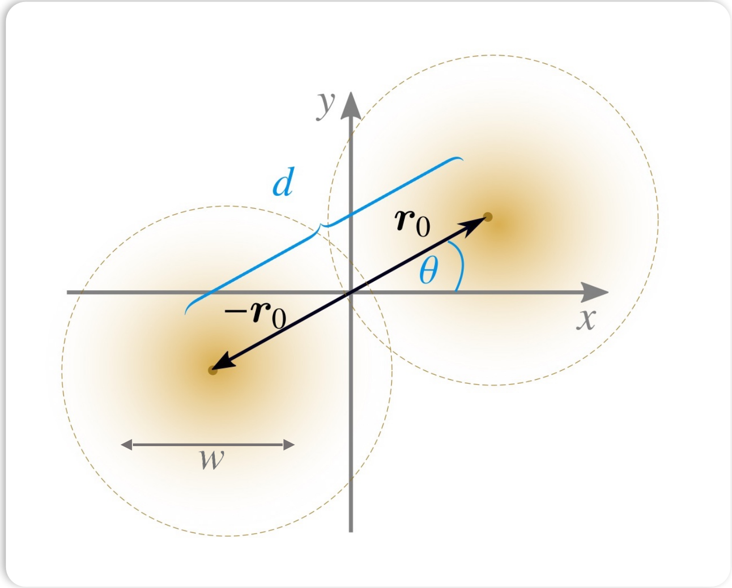

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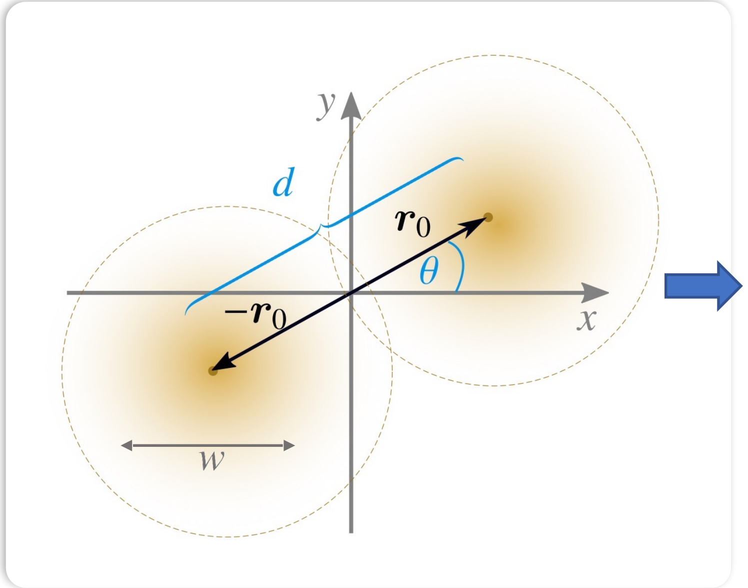

Direct intensity measurement

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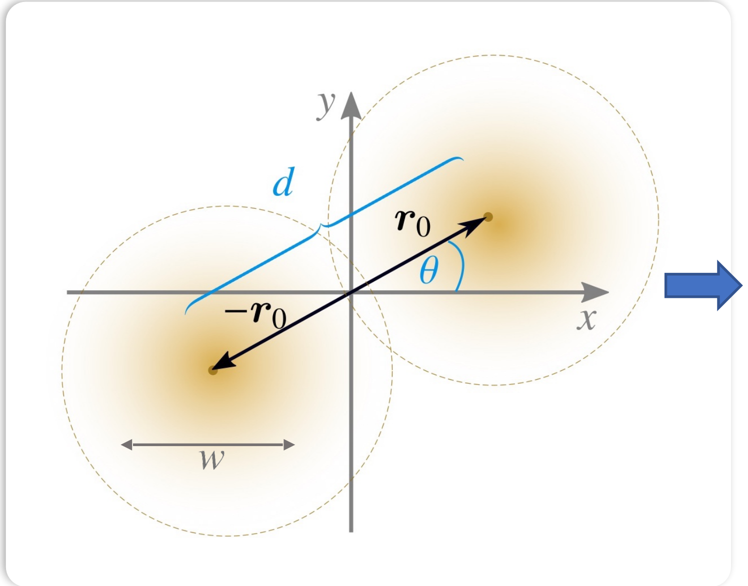

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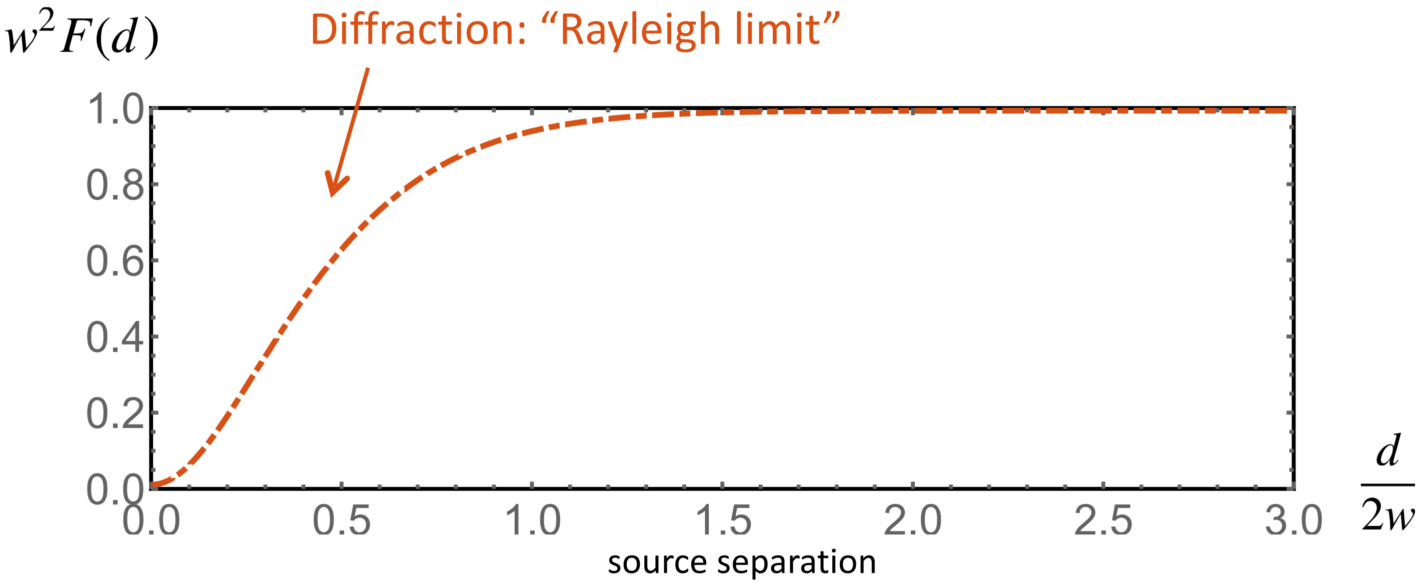

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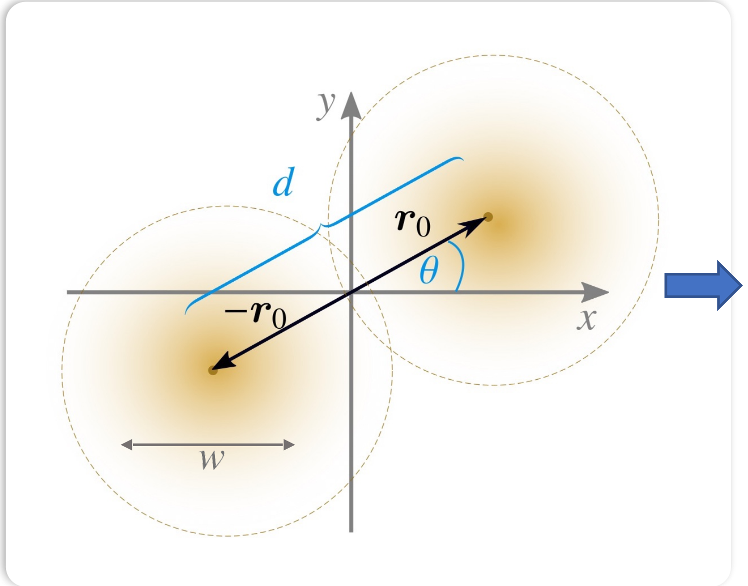

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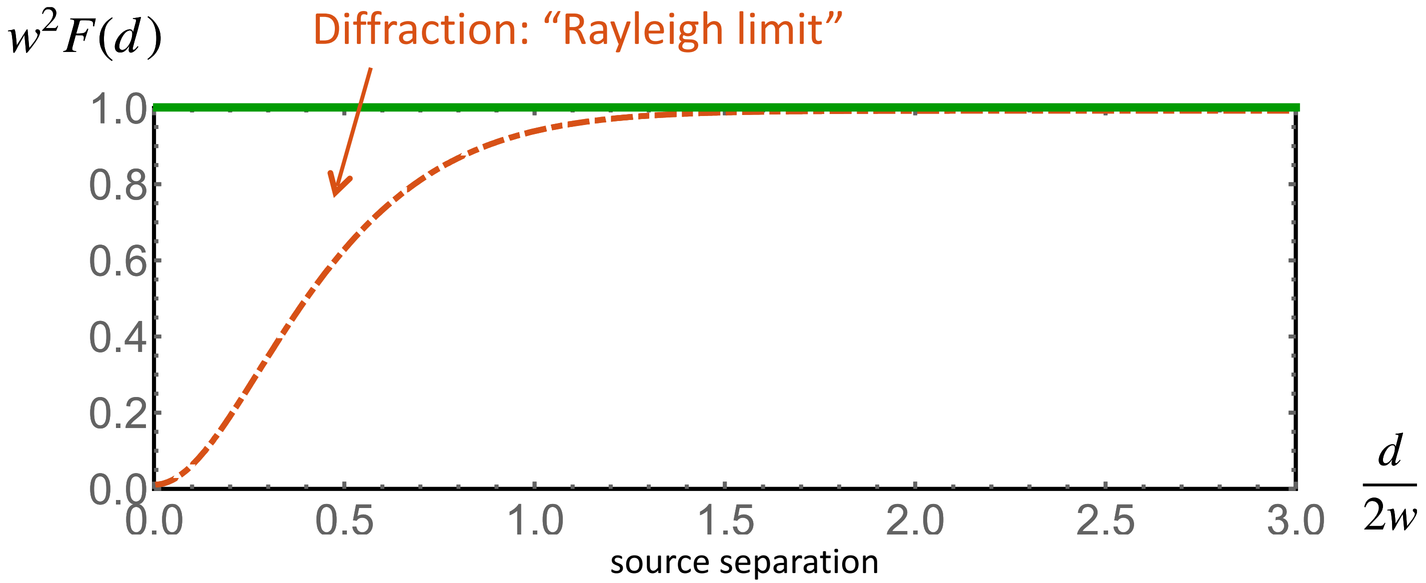

M. Tsang, R. Nair, X.-M. Lu, PRX **6**, 031033 (2016).

QFI for weak thermal light (per photon) $F_Q(d) = w^{-2}$

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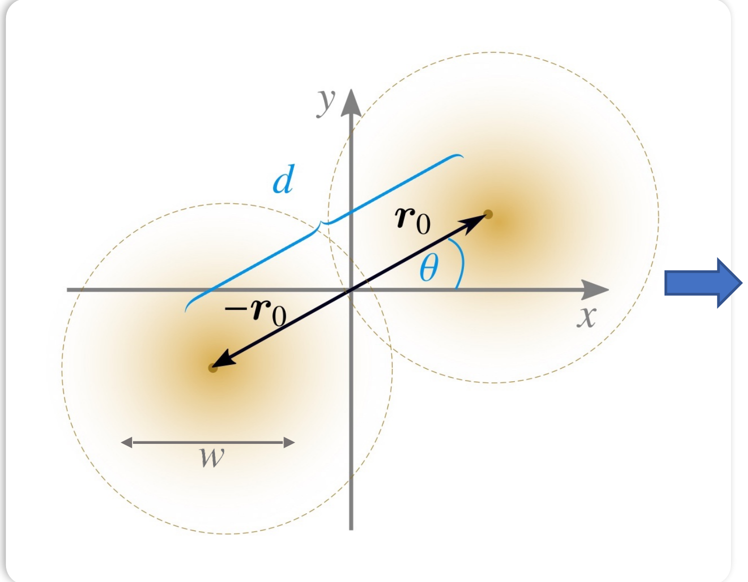
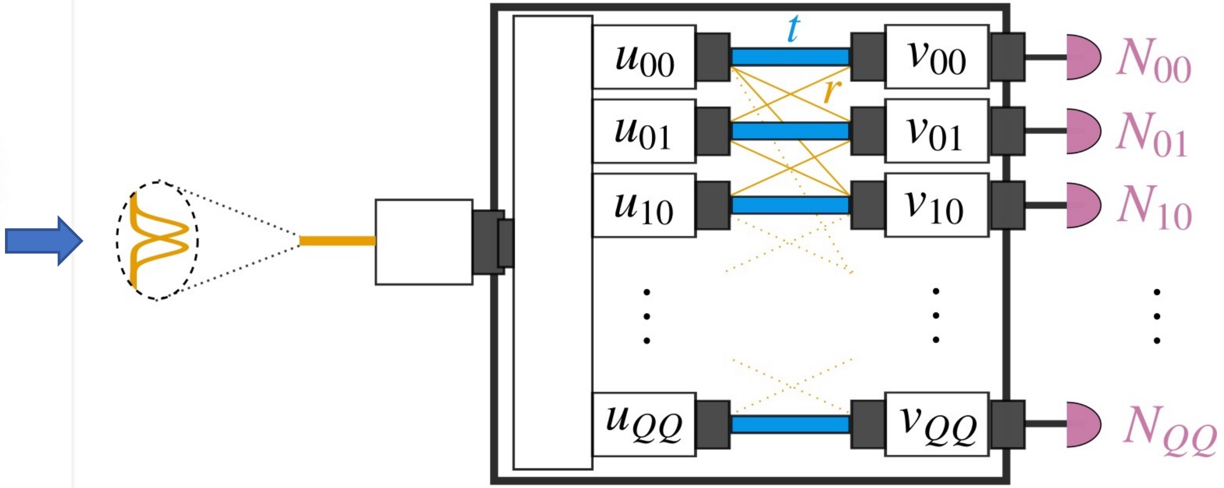
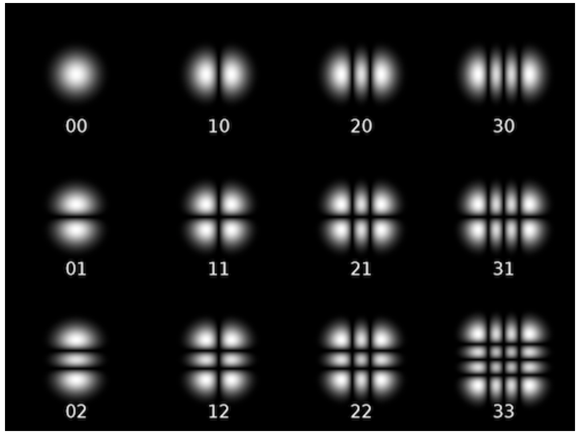
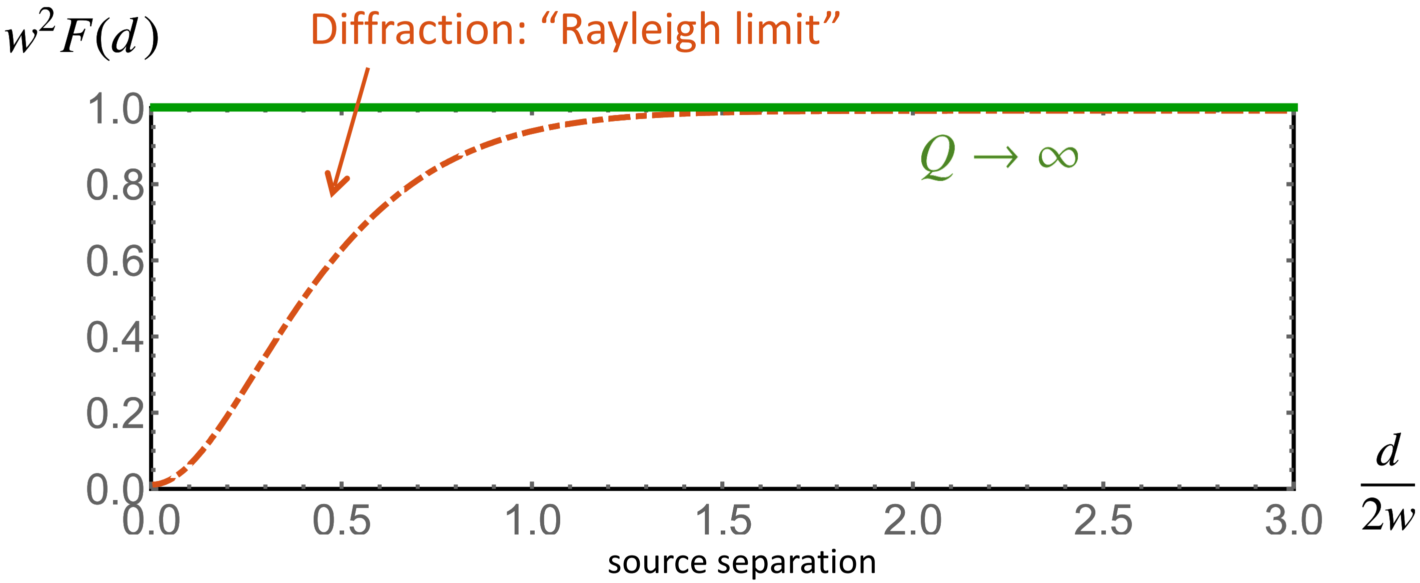




Hermite-Gauss modes





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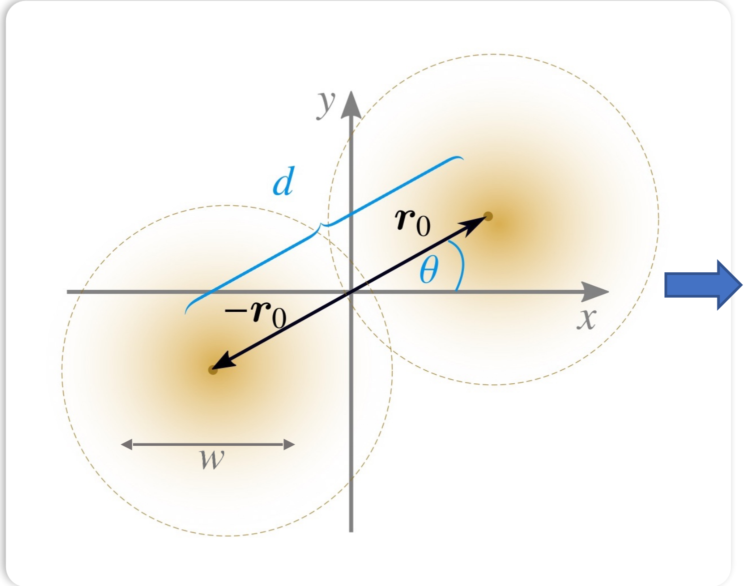

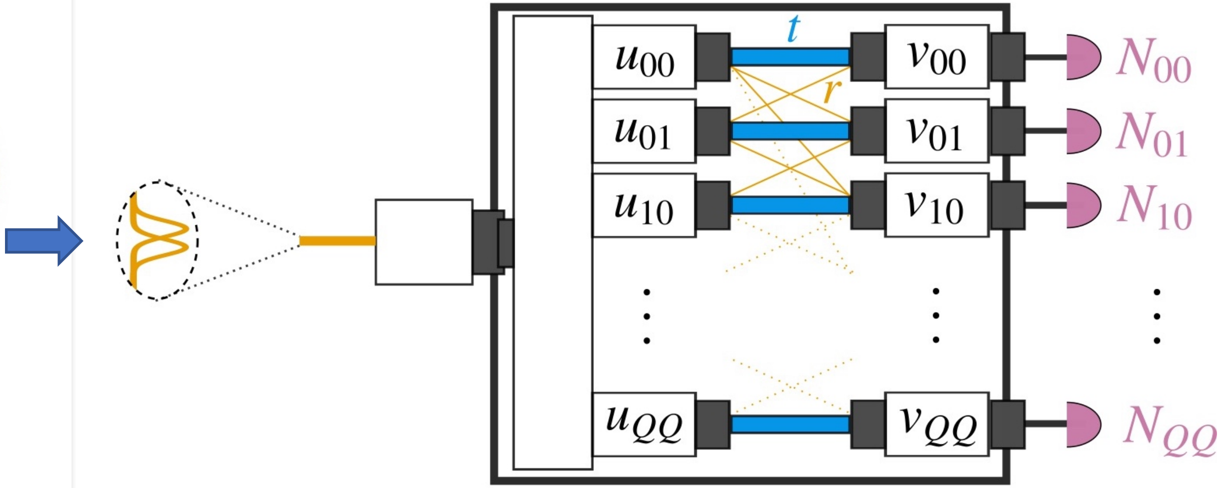

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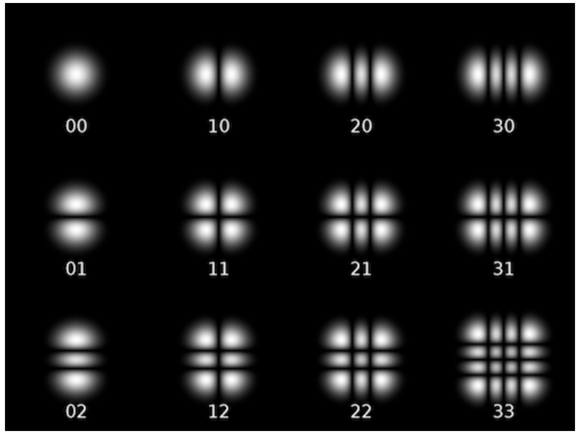

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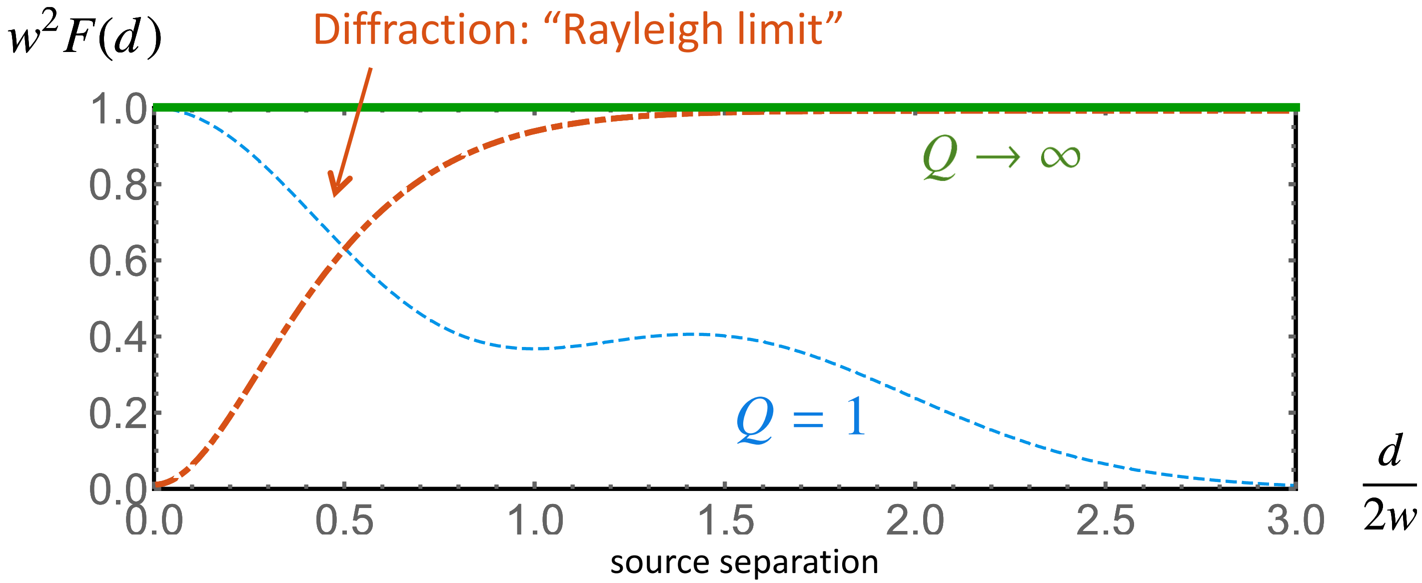

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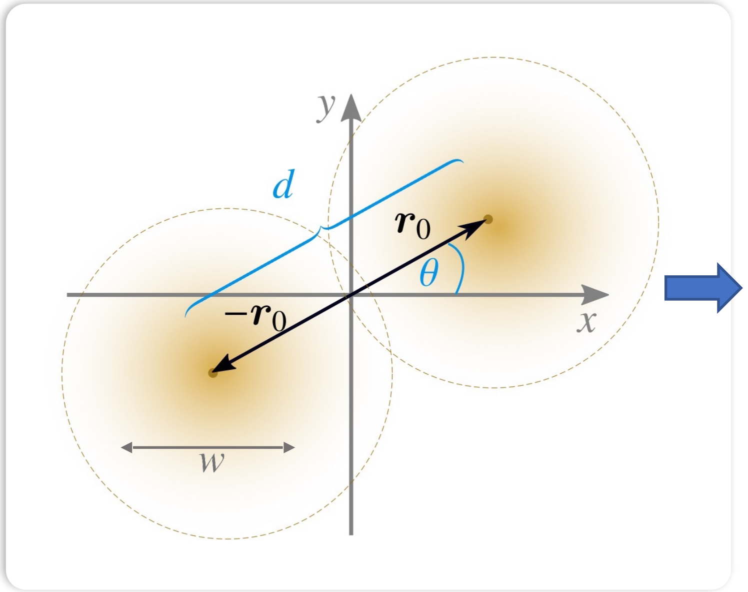

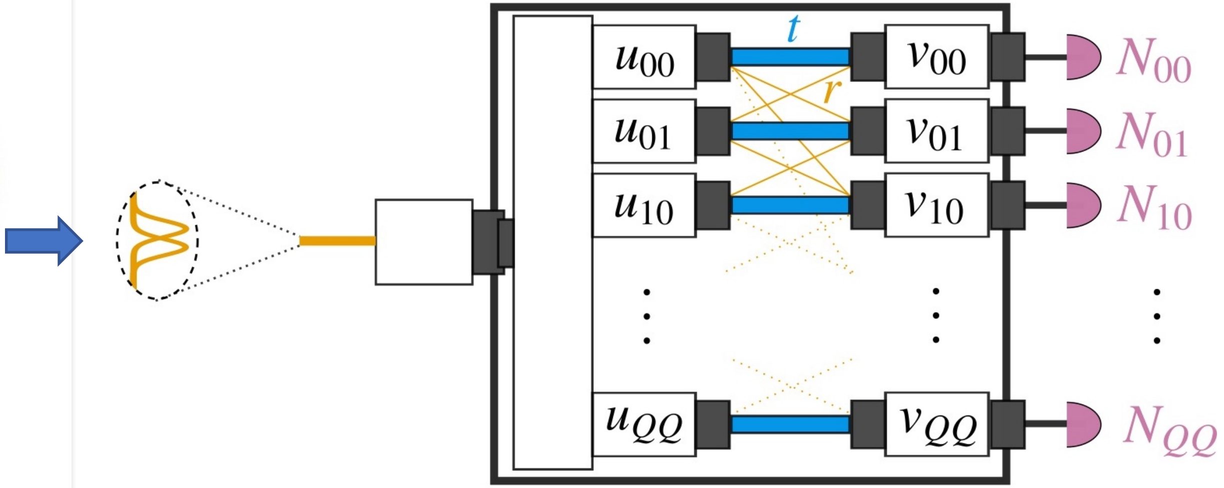

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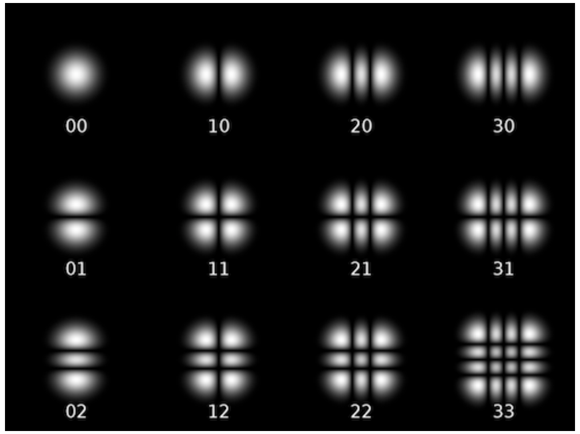

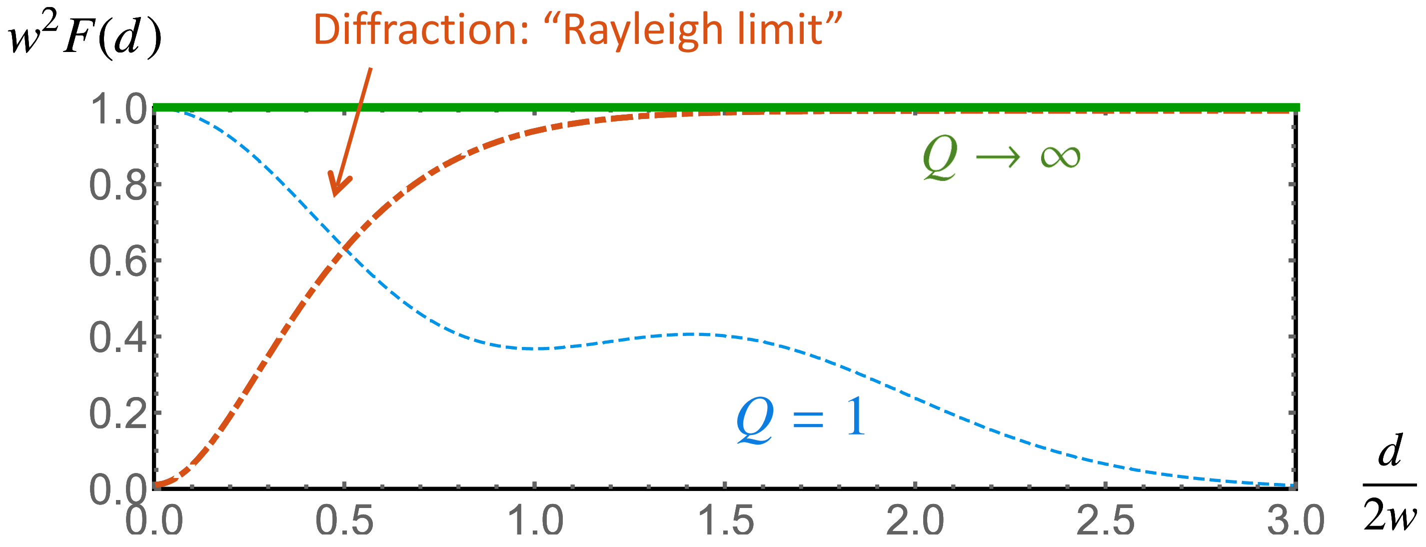

M. Tsang, R. Nair, X.-M. Lu, PRX **6**, 031033 (2016).

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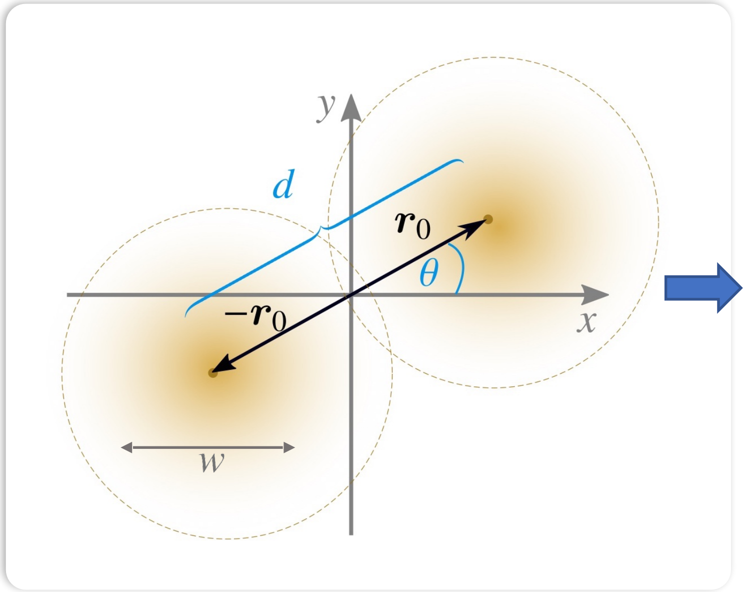
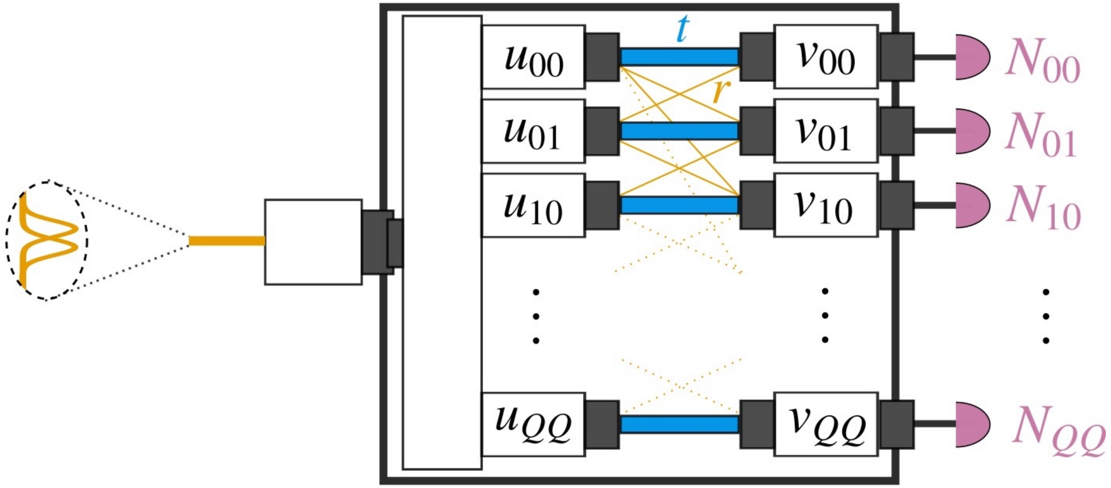
for *N* photon state $F_Q(d) = w^{-2} N$

PREVIOUS WORK: SUPERRESOLUTION IMAGING





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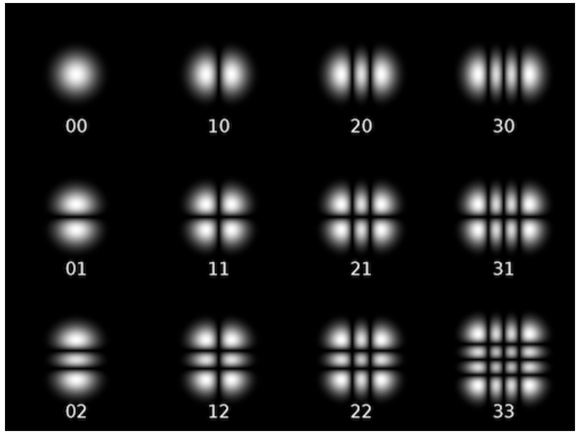

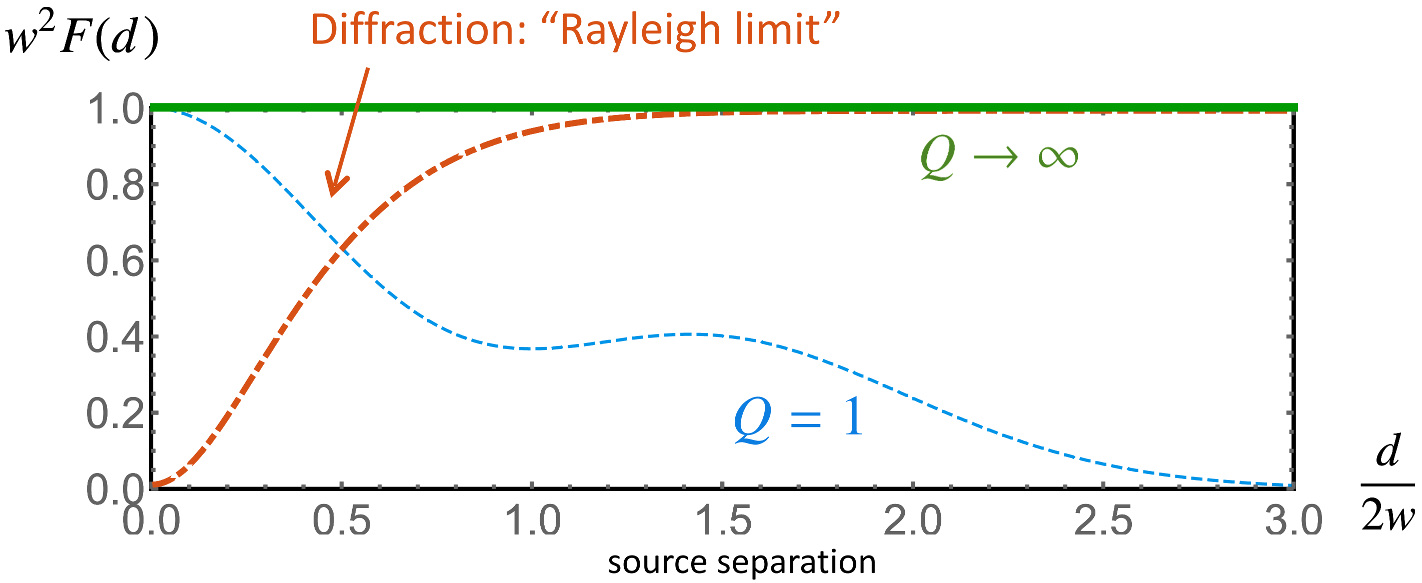

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C. Lupo and S. Pirandola, PRL **117**, 190802 (2016).

Upper limit on QFI of *N*-photon state $F_Q(d) \leq cN$

const.

Quantum Theory of Mode Parameter Estimation

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$\rho(\theta)$ quantum state defined on a θ-dependent basis of modes

$\hat{a}_m^\dagger$ creates a photon in the mode f_m

$\{f_m\}$ basis of modes, parametrized by θ

M. Gessner, N. Treps, C. Fabre, arXiv:2201.04050

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$$\frac{\partial}{\partial \theta}\rho = -i[H,\rho]$$

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$$H = i\sum_{jk}(f_j|f_k')\hat{a}_j^\dagger\hat{a}_k$$

$$(f_j|f_k') = \int dx f_j^*(x) f_k'(x)$$

Unitary evolution

Hamiltonian depends on shape and derivative of the modes

M. Gessner, N. Treps, C. Fabre, arXiv:2201.04050

Quantum sensitivity limits for mode parameter estimation

Determined by QFI for a unitary parameter imprinting with a beam-splitter-like Hamiltonian

Sensitivity: $F_Q[\rho, H]$ with $H = i \sum_{jk} (f_j | f'_k) \hat{a}_j^\dagger \hat{a}_k$

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$$= F_Q[\rho, H_I] + \langle O \rangle$$

$H_I = i\sum_{jk\in I}(f_j|f_k')\hat{a}_j^\dagger \hat{a}_k$ *I:* set of populated modes

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$$O = 4\sum_{kl\in I}\left[(f_k'|f_l') - \sum_{j\in I}(f_k'|f_j)(f_j|f_l')\right]\hat{a}_k^\dagger\hat{a}_l$$

$$= 4\sum_{kl\in I}(f_k'|\Pi_{\mathrm{vac}}|f_l')\hat{a}_k^\dagger\hat{a}_l \quad \text{with} \quad \Pi_{\mathrm{vac}} = \sum_{j\neq I}|f_j)(f_j|$$

M. Gessner, N. Treps, C. Fabre, arXiv:2201.04050

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Independent of the fluctuations
→ No quantum advantage from this term

M. Gessner, N. Treps, C. Fabre, arXiv:2201.04050

Origin of Quantum Sensitivity Enhancements

Interpretation

Effective beam splitter moves information about the parameter into derivative modes

Depends on the fluctuations

$$H_I = i \sum_{jk \in I} (f_j | f'_k) \hat{a}_j^\dagger \hat{a}_k$$

Sensitivity: $F_Q[\rho, H_I] + \langle O \rangle$

$$O = 4 \sum_{kl \in I} \left[(f'_k | f'_l) - \sum_{j \in I} (f'_k | f_j)(f_j | f'_l) \right] \hat{a}_k^\dagger \hat{a}_l$$

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M. Gessner, N. Treps, C. Fabre, arXiv:2201.04050

Origin of Quantum Sensitivity Enhancements

Interpretation

Effective beam splitter moves information about the parameter into derivative modes

Vacuum modes: Noise determined by the vacuum → SQL

Populated modes: Noise can be manipulated with nonclassical states → possibility to achieve sub-SQL fluctuations

Depends on the fluctuations

$$H_I = i \sum_{jk \in I} (f_j | f'_k) \hat{a}_j^\dagger \hat{a}_k$$

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Independent of the fluctuations

M. Gessner, N. Treps, C. Fabre, arXiv:2201.04050

Origin of Quantum Sensitivity Enhancements

Interpretation

Effective beam splitter moves information about the parameter into derivative modes

Vacuum modes: Noise determined by the vacuum → SQL

Populated modes: Noise can be manipulated with nonclassical states → possibility to achieve sub-SQL fluctuations

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ORIGIN OF QUANTUM SENSITIVITY ENHANCEMENTS

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Sensitivity: $$|(f|f')|^2 F_Q[\rho, N] + 4\left[(f'|f') - |(f|f')|^2\right]\langle N\rangle$$

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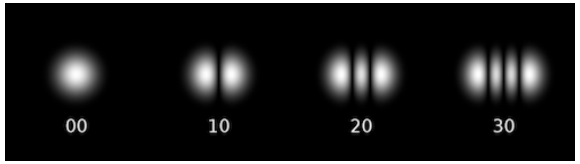

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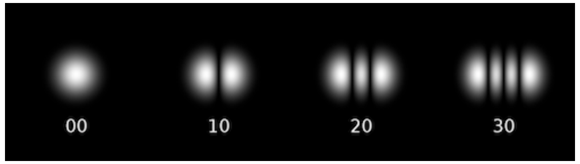

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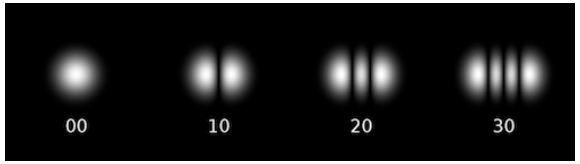

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Example: Phase parameter (LG modes)
Frequency (temporal modes)

$$f(x,\phi) = A(x)e^{-im(\phi+\theta)}$$

$$\Rightarrow |(f|f')|^2 = m^2$$

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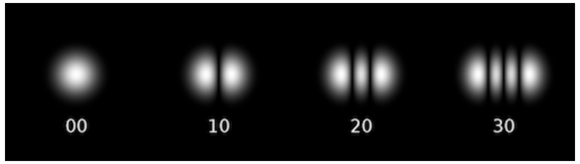

$$|(f|f')|^2 > 0$$

- Nonlinear scaling with *N*
- Quantum enhancements
- States that optimize fluctuations:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |N\rangle)$$

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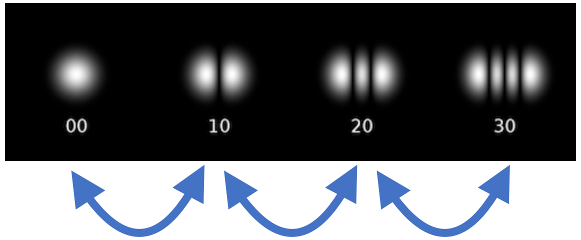
Displacement Sensing

$$w\frac{\partial}{\partial x}\mathrm{HG}_{nm} = \sqrt{n}\mathrm{HG}_{n-1,m} - \sqrt{n+1}\mathrm{HG}_{n+1,w}.$$

Hermite-Gauss modes



Effective Hamiltonian

$$H = \frac{i}{w}\sum_n \sqrt{n+1}(\hat{a}_n^\dagger \hat{a}_{n+1} - \hat{a}_{n+1}^\dagger \hat{a}_n)$$

Mixes neighboring modes with indices ±1

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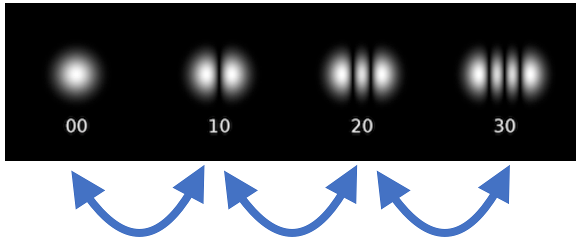

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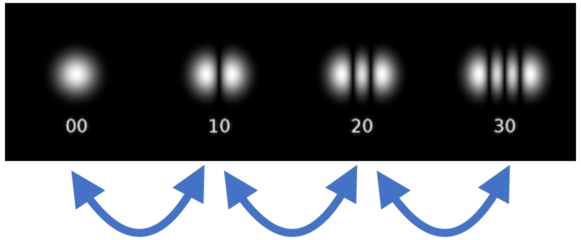

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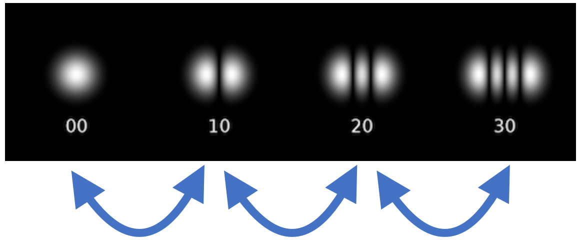

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"Detection mode"

C. Fabre, J. B. Fouet, and A. Maître, Opt. Lett. **25**, 75 (2000)

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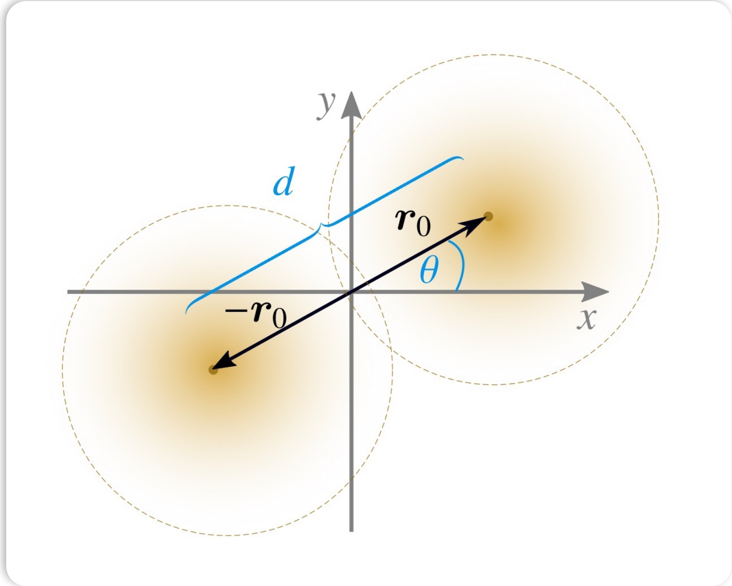

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$$f_{\pm}(x) \simeq \Psi(x + \frac{r_0}{2}) \pm \Psi(x - \frac{r_0}{2})$$

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Real point-spread function
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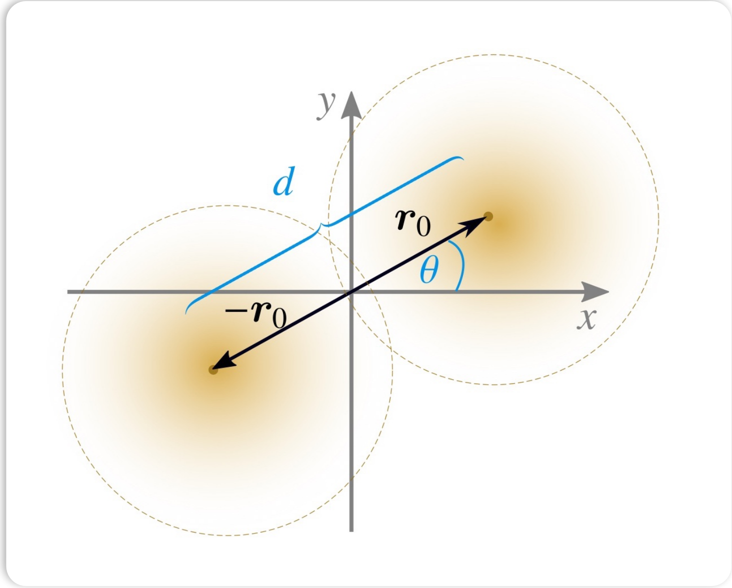

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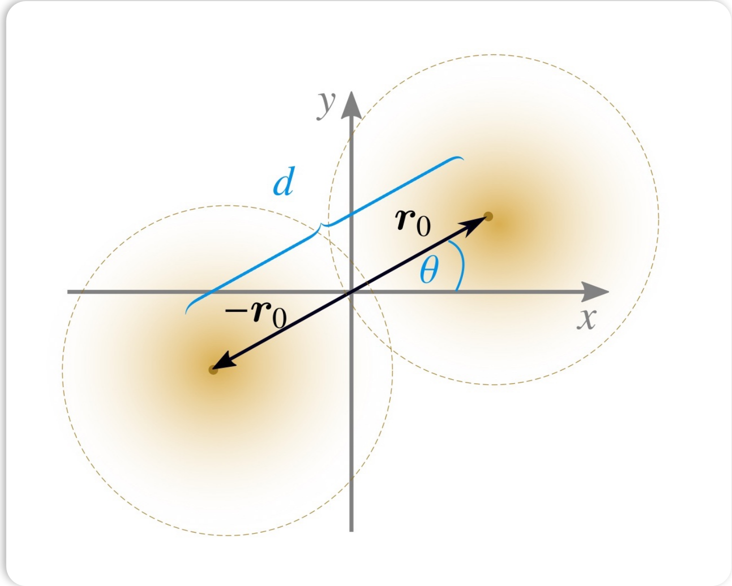

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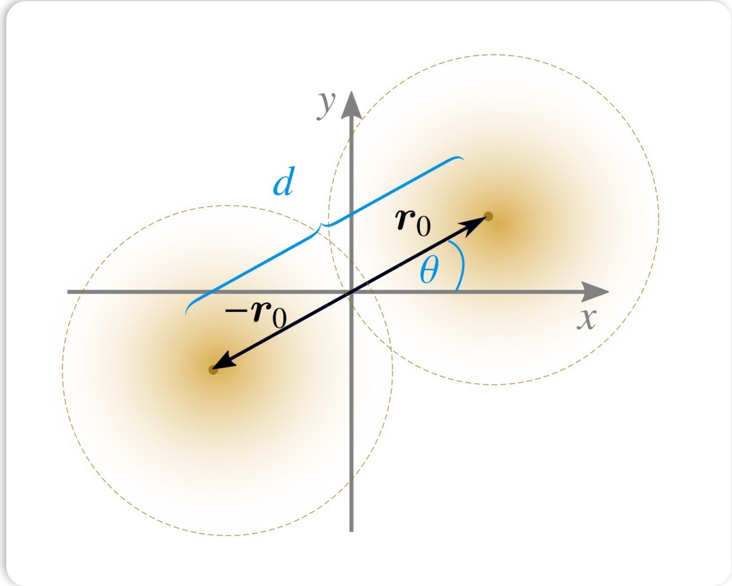

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Population of additional auxiliary modes:
Derivatives of the original modes

Possible in microscopy?

M. Gessner, N. Treps, C. Fabre, arXiv:2201.04050

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Quantum light is defined not only by its quantum state but also by the modes on which it is defined.

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Quantum SPAIN

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