Permutationally Invariant Tomography of a Four-Qubit Symmetric Dicke State

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Multi-partite entangled quantum states play an important role in quantum information processing with applications, for example, in quantum enhanced metrology or quantum communication. Therefore, efficient measurement schemes to fully characterize these states are needed. However, conventional quantum state tomography which reveals all properties of a quantum state suffers from an exponentially increasing measurement effort with the number of qubits.

Recently, it was shown that the measurement effort scales only quadratically under the restriction of permutational invariance of the quantum state [1]. This is of great importance since many prominent quantum states like, for example, GHZ states, symmetric Dicke states or W states are permutationally invariant. Here, we present experimental results of the tomographic analysis of a photonic four-qubit symmetric Dicke state [Fig. 1a]. Instead of 3^4=81 basis settings for full tomography only 15 basis settings have to be measured for permutationally invariant tomography. A further advantage of our method is that the single qubits do not have to be individually manipulable since for each measurement the same local setting is applied to all qubits. The best performance of our measurement algorithm is achieved when the basis settings are evenly distributed on the Bloch sphere [Fig. 1b]. In our experiments we showed that the overlap between the density matrix obtained from full tomography and the one obtained from permutationally invariant tomography, and thus the permutationally invariant component of the observed quantum state was 94.7%.

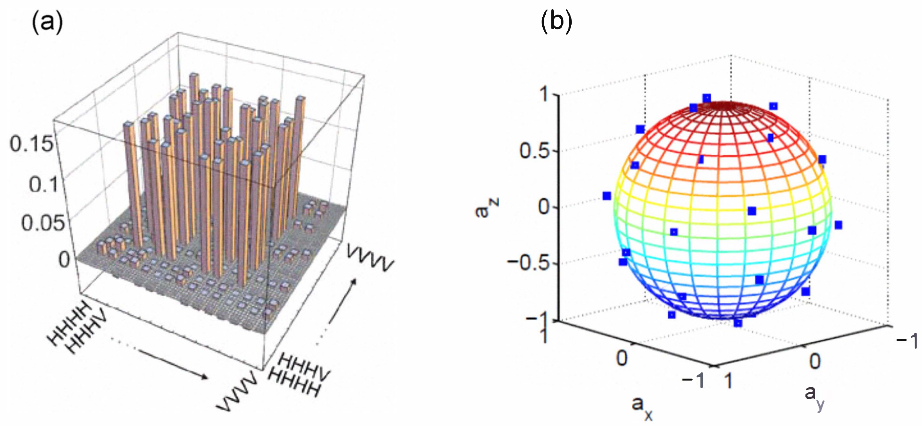

Fig. 1 (a) Real part of the density matrix of a photonic four-qubit symmetric Dicke state reconstructed with permutationally invariant tomography. (b) Optimized measurement settings vizualized on the Bloch sphere; each point on the sphere (a_x, a_y, a_z) corresponds to the projection measurement $\frac{1}{2}(1 \pm a_x\sigma_x + a_y\sigma_y + a_z\sigma_z)$.

Together with a femtosecond enhancement cavity in the ultraviolet regime [2], permutationally invariant tomography should enable us to investigate photonic systems with a larger number of qubits, e.g., the six photon symmetric Dicke state. For such systems full tomography is practically impossible due to the low count rates and the exceedingly high number of 3^6=729 measurements, compared to only 28 measurements for permutationally invariant tomography.

References

[1] G. Tóth, W. Wieczorek, D. Gross, R. Krischek, C. Schwemmer and H. Weinfurter, "Permutationally invariant quantum tomography," Phys. Rev. Lett. **105**, 250403 (2010).

[2] R. Krischek, W. Wieczorek, A. Ozawa, N. Kiesel, P. Michelberger, T. Udem and H. Weinfurter, "Ultraviolet enhancement cavity for ultrafast nonlinear optics and high-rate multiphoton entanglement experiments," Nature Photonics **4**, 170-173 (2010).