Hydrodynamics of topological defects in nematic liquid crystals

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Liquid crystal hydrodynamics with the Beris-Edwards equations based on the tensor order parameter

The simplest equation of motion ignoring hydrodynamics:

$$\frac{\partial \mathbf{Q}}{\partial t} = -\Gamma \frac{dF}{d\mathbf{Q}}$$

Equations of motion including hydrodynamics (Beris-Edwards equations):

$$(\partial_t + u_a \partial_a)\mathbf{Q} = \mathbf{S}(\partial_a u_b, \mathbf{Q}) - \Gamma \frac{dF}{d\mathbf{Q}}$$

Material derivative (time derivative along the flow line)

Coupling between the velocity field and the order parameter

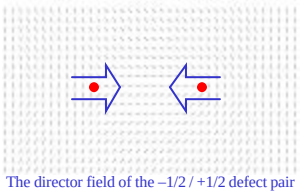
Relaxation to the free energy minimum

+

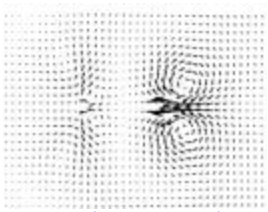
A second equation describes the flow: the Navier-Stokes equation.

Advantage compared to the Ericksen-Leslie model: the change in the magnitude of order and biaxiality can also be described. (These are necessary for the correct description of the defecr core.)

Simulation example I: annihilation of free defects (Phys. Rev. Letters. **88**, 105504, 2002)



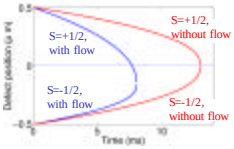
The director field of the –1/2 / +1/2 defect pair



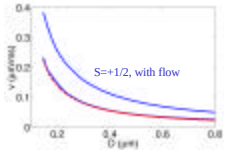
-1/2 defect +1/2 defect

Velocity field of the defects. At the core of the s=+1/2 defect there is a strong vortex. The flow points into the direction of the defect motion. At the s=-1/2 defect the flow is weaker and points into a direction opposite to the defect velocity.

Hydrodynamics (backflow) accelerates the +1/2 defect substantially



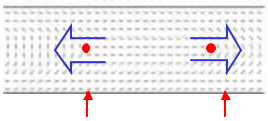
Defect position with and without hydrodynamics, as a function of time. If there is no hydrodynamics then the defects follow symmetrical trajectories.



Defect speed as the function of defect separation, with and without hydrodynamics. The speed anisotropy is small when the defects are close to each other.

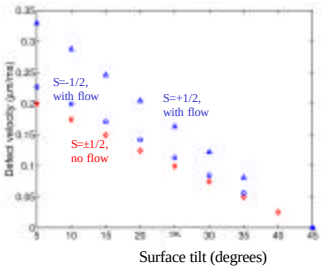
The reason for the speed anisotropy: backflow due to the reorientation of the director.

Simulation example II: 2D domain growth in confined environment



Defect with s= -1/2 Defect with strength s=+1/2

A horizontal domain is growing in a vertical environment due to the almost horizontal surface tilt. Defects are formed at the domain boundaries.

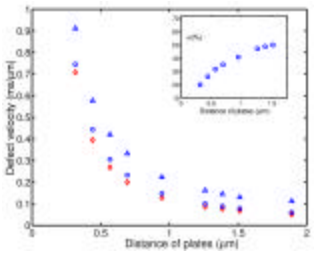


If hydrodynamics is switched off then the two defects move with the same speed (diamonds).

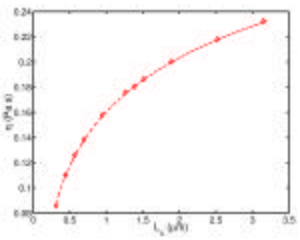
If hydrodynamics is switched on, the s=+1/2 defect goes faster (triangles). The other defect is affected much less by the backflow (circles).

Influence of the sample thickness on the defect speed

The domain wall speed decreases with increasing sample width.



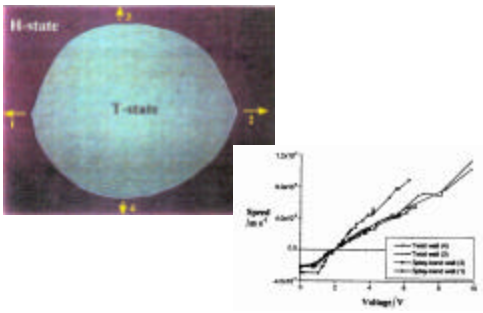
The speed anisotropy increases with the sample width and saturates at around 60%.



Effective viscosity vs. sample width. The effective viscosity increases as log(Lx/const).

Experimental confirmation: domain growth

Acosta, Towler, Walton, Liq. Cryst. **27**, 977.



Twist or vertical domain grows in horizontal environment. The splay-bend wall (1) moves much faster than other three. This is the wall which incorporates a +½ defect.

Conclusions

Hydrodynamics substantially affect the defect movement. Defects with different topological strength move with different speed.