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The Entanglement of the Symmetric Four-photon Dicke State

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Abstract. We present an experimental and theoretical characterization of the symmetric four-qubit entangled Dicke state with two excitations $D_4^{(2)}$. We investigate the state's violation of local realism and study its characteristic properties with respect to quantum information applications. For the experimental observation of the state we used photons obtained from parametric down conversion. This allowed, in a simple experimental set-up, quantum state tomography yielding a fidelity as high as 0.844 ± 0.008.

Keywords. Multi-particle entanglement, quantum information

Introduction

Entanglement in bipartite quantum systems is well understood and can be easily quantified. In contrast, multipartite quantum systems offer a much richer structure and various types of entanglement. Thus, crucial questions are how strongly and, in particular, in which way a quantum state is entangled. Consequently, different classifications of multipartite entanglement have been developed [1,2,3], and quantum states with promising properties have been identified and studied experimentally [4,5,6,7,8,9]. The ongoing effort in this direction leads to a deeper understanding of multipartite entanglement and its applications in quantum communication.

In the following, we present an experimental and theoretical examination of an interesting four-qubit entangled state: $D_4^{(2)}$ – the four-qubit Dicke state with two excita-

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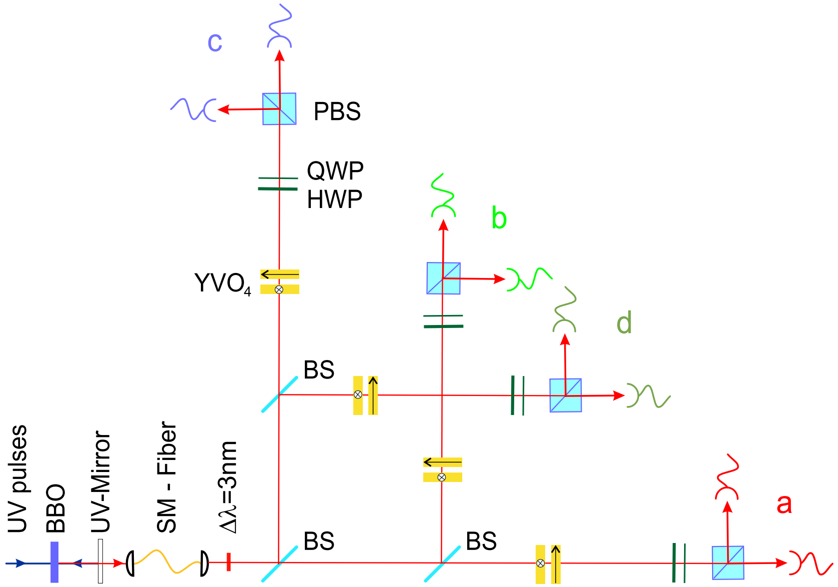

Figure 1. Experimental setup for the analysis of the four-photon polarization-entangled state $D_4^{(2)}$. It is observed after the symmetric distribution of four photons onto the spatial modes a,b,c and d via non-polarizing beam splitters (BS). The photons are obtained from type-II collinear spontaneous parametric down conversion (SPDC) in a 2 mm β-Barium Borate (BBO) crystal pumped by 600 mW UV-pulses. The phases between the four output modes are set via pairs of birefringent Yttrium-Vanadate-crystals (YVO4). Half and quarter wave plates (HWP, QWP) together with polarizing beam splitters (PBS) are used for the polarization analysis.

tions that is symmetric under all permutations of qubits. Generally, a symmetric N-qubit Dicke state [10,11,12] with M excitations, $|D_N^{(M)}\rangle$, is the equally weighted superposition of all permutations of N-qubit product states with M logical 1's and $(N-M)$ logical 0's. The Dicke states naturally appear as the common eigenstates of the total spin-squared and the spin z-component (where z is assumed to be the quantization direction) operators in spin one-half particle systems. Besides the state studied here, another well known example for a Dicke state is the N-qubit W state $|W_N\rangle$ (in the present notation $|D_N^{(1)}\rangle$) [5]. While other symmetric Dicke states have maximum overall spin, $D_4^{(2)}$ is the eigenstate which has minimum spin component along the quantization axis. As we shall see, this fact leads to a set of interesting properties.

1. Experiment

As photons are well suited to emulate spin one-half systems, we use them as the quantum system of choice for the experimental observation and characterization of the Dicke state. Applying polarization encoding, the state $D_4^{(2)}$ has the form:

$$|D_4^{(2)}\rangle = \frac{1}{\sqrt{6}}(|HHVV\rangle + |HVHV\rangle + |VHHV\rangle + |HVVH\rangle + |VHVH\rangle + |VVHH\rangle) \quad (1)$$

with, e.g., $|VVHH\rangle = |V\rangle_a \otimes |V\rangle_b \otimes |H\rangle_c \otimes |H\rangle_d$, where $|H\rangle$ and $|V\rangle$ denote linear horizontal (H) and vertical (V) polarization of a photon in the spatial modes (a, b, c, d) (Fig. 1). It can be seen as the superposition of the six possibilities to distribute two horizontally and two vertically polarized photons into four modes. Accordingly, we create four indistinguishable photons with appropriate polarizations in one spatial mode and distribute them with polarization independent beam splitters (BS) (Fig. 1) [13]. If one photon is detected in each of the four output modes we observe the state $D_4^{(2)}$. For ideal 50:50 BS this occurs with a probability of $p \approx 0.094$ and experimentally we find $p \approx 0.080$ [14].

As source of the four photons we use the second order emission of collinear type II spontaneous parametric down conversion (SPDC). UV pulses with a central wavelength of 390 nm and an average power of about 600 mW from a frequency-doubled mode-locked Ti:sapphire laser (pulse length ≈130 fs) are used to pump a 2 mm thick BBO (β-Barium Borate, type-II) crystal. This results in two horizontally and two vertically polarized photons with the same wavelength. Dichroic UV-mirrors serve to separate the UV-pump beam from the down conversion emission. A half-wave plate together with a 1 mm thick BBO crystal compensates walk-off effects (not shown in Fig. 1). Coupling the four photons into a single mode fiber exactly defines the spatial mode. The spectral selection is achieved with a narrow bandwidth interference filter ($\Delta\lambda = 3$ nm) at the output of the fiber. Birefringence in the non-polarizing beam splitter cubes (BS) is compensated with pairs of perpendicularly oriented 200 μm thick birefringent Yttrium-Vanadate crystals (YVO_4) in each of the four modes. Altogether, the setup is stable over several days. Measurement time is thus mainly limited by misalignment effects in the pump laser system which, however, affects rather the count rate than the quality of the state.

Polarization analysis is performed in all of the four outputs. For each mode we choose the analysis direction with half (HWP) and quarter wave plates (QWP) and detect the photons behind polarizing beam splitters using single photon detectors (Si-APD). The detected signals are fed into a multi-channel coincidence unit which allows to simultaneously register any possible coincidence between the inputs. The rates for each of the 16 characteristic four-fold coincidences were corrected for the different detection efficiencies in each polarization analysis. This experimental scheme allowed the observation of the state with about 60 four-fold coincidences per minute. This count rate was high enough to perform a full quantum state tomography out of which we can determine the experimental state's Fidelity to be $F = 84.4 \pm 0.08\%$ (see Figure 2). The state tomography provides all the necessary data for any further analysis of the state.

2. Analysis of the state

2.1. Correlations and violation of local realism

For a given sate, there exists a single generalized Bell-type inequality which is the sufficient and necessary condition on the local realistic description of the correlation function E [15,16,17]. For four qubits we have

$$\frac{1}{2^4} \sum_{s_1,\ldots,s_4=\pm 1} \left| \sum_{k_1,\ldots,k_4=0,1} s_1^{k_1} \ldots s_4^{k_4} E(\vec{a}_1(k_1), \vec{a}_2(k_2), \vec{a}_3(k_3), \vec{a}_4(k_4)) \right| \leq 1. \quad (2)$$

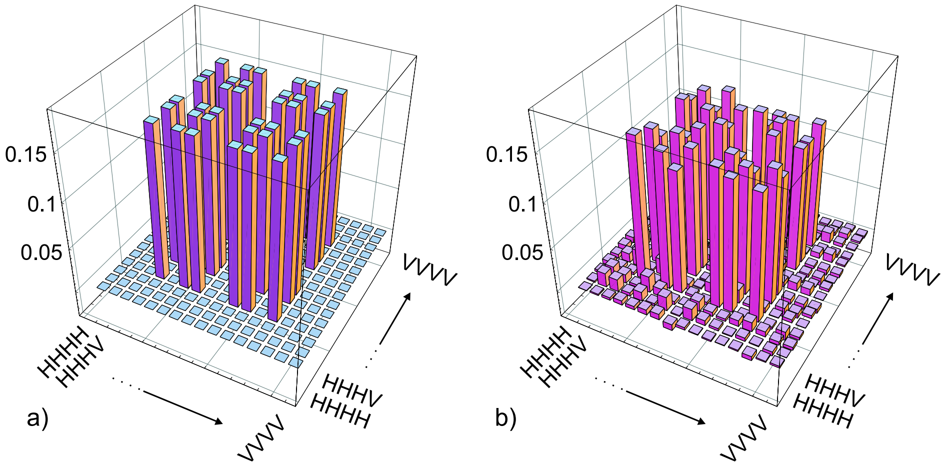

Figure 2. (a) Plot of the density matrix for the ideal state $\rho_{D_4^{(2)}}$ and (b) the Real part of the density matrix derived from the measurement data.

This condition corresponds to the experimental situation, in which the observers can choose between two dichotomic observables with the eigenstates $\vec{a}_i(k_i) = \cos(\theta_i + k_i\frac{\pi}{4})|H\rangle_i + e^{i\phi_i}\sin(\theta_i + k_i\frac{\pi}{4})|V\rangle_i$, $(i = 1, 2, 3, 4)$. The above inequality is satisfied if and only if [17]

$$\max_{\alpha_1,\ldots,\alpha_4}\left\{\sum_{l_1,\ldots,l_4=\{x,y\}}|T_{l_1l_2l_3l_4}|\sin(\alpha_1 + l_1\frac{\pi}{2})\ldots\sin(\alpha_4 + l_4\frac{\pi}{2})\right\} \leq 1, \tag{3}$$

where $\hat{T}$ is the correlation tensor in *any* set of local Cartesian coordinate systems. The components of $\hat{T}$ are given by $T_{l_1l_2l_3l_4} = E(\vec{x}_{l_1}, \vec{x}_{l_2}, \vec{x}_{l_3}, \vec{x}_{l_4})$, where $\vec{x}_{l_k}$, $(l_k = 1, 2, 3)$ represent some set of (local) basis vectors for the kth observer. For the state $D_4^{(2)}$ the correlation tensor has the following non-vanishing coefficients in the standard basis:

$$\begin{aligned}
&T_{xxxx} = T_{yyyy} = T_{zzzz} = 1;\\
&T_{xxyy} = T_{xyxy} = T_{xyyx} = T_{yxxy} = T_{yxyx} = T_{yyxx} = 1/3;\\
&T_{xxzz} = T_{xzxz} = T_{xzzx} = T_{zxxz} = T_{zxzx} = T_{zzxx} = -2/3;\\
&T_{zzyy} = T_{zyzy} = T_{zyyz} = T_{yzzy} = T_{yzyz} = T_{yyzz} = -2/3.
\end{aligned} \tag{4}$$

Ideally, $D_4^{(2)}$ violates inequality (3) by a factor 2.1213. However, the state observed in an experiment is never pure. Therefore it is interesting how robust the state is with respect to admixture of white noise. With the corresponding state, $v|D_4^{(2)}\rangle\langle D_4^{(2)}| + (1 - v)\mathbb{1}/16$, we can derive a measure for that robustness, which is called the critical visibility v^{crit}. That means that for $v > v^{crit}_{D_4^{(2)}}$ no local realistic description of the state exists. For $D_4^{(2)}$ we obtain $v^{crit}_{D_4^{(2)}} = 0.4714$. This result was found using a numerical procedure. In addition we used also the more general method of linear optimization [18] to test the possibility of the quantum probabilities to be describable by a local realistic model. The critical visibility obtained in this method is equal to the 4-digit approximation of $v^{crit}_{D_4^{(2)}}$

obtained from (3). In [19], it was shown that the amount of the violation of the Bell inequality [20] can be used for the detection of N-particle entanglement in a multi-qubit quantum system. In the case of inequality (3) the threshold for four-particle entanglement is equal to 2. If any quantum state violates this inequality by a factor larger than 2, then it contains true four-particle entanglement. In the experiment, for a spectral filtering of the photons to a bandwidth of 3 nm, a violation of inequality (3) was achieved of 1.75 ± 0.08, which is not sufficient to proof four-partite entanglement. By stronger filtering of the photons to a width of 2 nm we reach a violation of 2.03 ± 0.21, which is just at the limit. However, one can use a more sensitive inequality introduced in [21]. This inequality assumes a rotational invariant form of the quantum correlation function. For four qubits it has a form:

$$(E_{HV}, E) \leq 4^4 E_{max}, \tag{5}$$

where $(,)$ represents the scalar product of a real Hilbert space of square integrable functions, E_{HV} is a local realistic correlation function, E is the quantum correlation function and E_{max} is the maximal possible value of the correlation function E for a given state. In the case of local and realistic theories, the correlation function for four qubits must be given by:

$$E_{HV}(\vec{a}_1(\alpha_1), ..., \vec{a}_4(\alpha_4)) = \int \mathrm{d}\lambda \rho(\lambda) \prod_{i=1}^{4} I_i(\alpha_i), \tag{6}$$

where $\rho(\lambda)$ is a certain distribution function of some hidden parameters λ, and $I_i = \pm 1$ is a function, that predetermines the values of the experimental results that can be performed on the given local system. Finally α_i is a certain parametrization of the local setting $\vec{a}_i$. The four qubit quantum correlation function, which has a rotationally invariant form, can be expressed in the following way

$$E(\vec{a}_1, ..., \vec{a}_4) = \hat{T} \circ (\vec{a}_1 \otimes ... \otimes \vec{a}_4), \tag{7}$$

where $\hat{T}$ is the correlation tensor for a quantum state, ρ. By the symbol $\circ$ we represent the scalar product in $\mathbb{R}^{12}$. If we constrain the measurement settings of each observer to one plane the measurement direction vectors can be expressed by $\vec{a}_i(\alpha_i) = \cos\alpha_i \hat{y}_i + \sin\alpha_i \hat{x}_i$, where $\hat{x}_i, \hat{y}_i$ are two basis vectors of $\mathbb{R}^3$ (which can be individually defined by each observer). In such a case, the correlation function is a scalar given by:

$$\begin{aligned} E(\alpha_1, ..., \alpha_N) = \sum_{i_1 ... i_4 = 1,2} & T_{i_1 i_2 i_3 i_4} \sin(\alpha_1 + (i_1 - 1)\frac{\pi}{2}) \\ & \times ... \sin(\alpha_4 + (i_4 - 1)\frac{\pi}{2}). \end{aligned} \tag{8}$$

The inequality (5) can be violated by the quantum value equal to

$$(E, E) = \pi^4 \sum_{i_1, ..., i_4 = \{x,y\}} T^2_{i_1 i_2 i_3 i_4} \tag{9}$$

The violation factor is equal to $v = (E, E)/(E_{HV}, E)$. In [22] it was shown that the threshold, above which four-partite entanglement is confirmed equals to $4(\pi/4)^4 \approx 1.52202$. For the experimental data taken with a 2 nm interference filter we obtain $v = 1.66 \pm 0.13$, which is sufficient to show that the state contains four-qubit entanglement. Consequently if one aims at obtaining information on the violation of local realism and true multi-partite entanglement simultaneously the latter type of Bell inequality is preferable.

2.2. *Genuine four qubit entanglement*

The well-established tool for the test of genuine multi-partite entanglement are usually witness operators, albeit they do not provide any information concerning the local realistic description of a state. One might use the generic form $\mathcal{W}_{\text{g}}$ [23] in which the corresponding expectation value depends directly on the observed fidelity: $Tr(\mathcal{W}_{\text{g}}\rho_{\text{exp}}) = \frac{2}{3} - F_{\text{exp}} = -0.177 \pm 0.008$ and is positive for all biseparable states. For $D_4^{(2)}$, 21 measurement settings, instead of a complete tomography, are sufficient to determine this value. They correspond to the 21 non-vanishing coefficients of $\hat{T}$ in the standard basis (see equation (4)).

Due to the high symmetry of this state, genuine four-partite entanglement can be detected with only two settings via a measurement of the collective spin squared in x- and y- direction ($\langle J_x^2 \rangle$ and $\langle J_y^2 \rangle$). For biseparable states it can be proven that [24,25]

$$\langle \mathcal{W}_4^s \rangle = \langle J_x^2 \rangle + \langle J_y^2 \rangle \leq 7/2 + \sqrt{3} \approx 5.23, \tag{10}$$

where $J_{x/y} = 1/2 \sum_k \sigma_{x/y}^k$ with e.g., $\sigma_x^3 = \mathbb{1} \otimes \mathbb{1} \otimes \sigma_x \otimes \mathbb{1}$. This can be interpreted also by rewriting $\langle J_x^2 \rangle + \langle J_y^2 \rangle = \langle J^2 \rangle - \langle J_z^2 \rangle$ where $J = (J_x, J_y, J_z)$. As for symmetric states $\langle J^2 \rangle = (N/2 + 1)N/2$ our criterion requires $\langle J_z^2 \rangle \geq 5/2 - \sqrt{3}$, i.e. the collective spin squared of biseparable symmetric states in any direction cannot be arbitrarily small [26]. For the state $D_4^{(2)}$, however, $\langle J_z^2 \rangle = 0$ and thus the expectation value of the witness operator in Eq. 10 reaches the maximum of 6. Via measurement of all photons in (± 45)- and (L/R)-basis respectively we find experimentally the value $Tr[\mathcal{W}_4^s \rho_{\text{exp}}] = 5.58 \pm 0.02$ clearly exceeding the bound of biseparable states. Multipartite entanglement is, thus, detected by studying only a certain property of the state. This makes the entanglement witness much more efficient. In principle, the witness works even without individual addressing of qubits. In particular for experiments on multi-photon entanglement, one relies on coincidence detection and therefore usually suffers from low count rates caused by limited detection efficiencies. Thus economic tools, like the two-setting witness operator, which offer a maximum gain of information on a quantum state by a minimal number of measurement settings are important.

2.3. *Residual state after loss or measurement of single qubits*

Let us continue the investigation of properties that make $D_4^{(2)}$ special in comparison with the great variety of other four-qubit entangled states. The various states show great differences in the residual three-qubit state dependent on the measurement basis and/or result: for example, $|GHZ_4\rangle$ [7] can either still render tripartite GHZ like entanglement or become separable, $|W_4\rangle$ as well, but the tripartite entanglement will always be W type. Entanglement in the cluster state $|C_4\rangle$ [6] cannot be easily destroyed and at least bipartite entanglement remains. However, $|\Psi^{(4)}\rangle$ [9,27] and, as described next, $D_4^{(2)}$ yield genuine tripartite entangled states independent of the measurement result and basis, i.e. also under loss of the qubit.

Let us compare the projection of the qubit in mode d onto either $|V\rangle$ or $|-\rangle$ for the state $D_4^{(2)}$:

$$
\begin{aligned}
{}_d\langle V|D_4^{(2)}\rangle &= \frac{1}{\sqrt{3}}(|HHV\rangle + |HVH\rangle + |VHH\rangle),\\
{}_d\langle -|D_4^{(2)}\rangle &= \frac{1}{\sqrt{6}}(|HHV\rangle + |HVH\rangle + |VHH\rangle\\
&\quad -|HVV\rangle - |VHV\rangle - |VVH\rangle).
\end{aligned}
\tag{11}
$$

The first is the state $|W_3\rangle$ [4] and the second one is a so-called G state ($|G_3\rangle$ Ref. [28]). Experimentally we observe these states with fidelities $F_{W_3} = 0.882 \pm 0.015$ and $F_{G_3} = 0.897 \pm 0.019$. Comparable values are observed for measurements of photons in other modes.

The criterion (10) adopted to the three-qubit case, can now be used to detect the tripartite entanglement around $|W_3\rangle$ and $|G_3\rangle$ with the bound $\langle \mathcal{W}_3^s\rangle = \langle J_x^2\rangle + \langle J_y^2\rangle \leq 2 + \sqrt{5}/2 \approx 3.12$. Our measurement results for $|W_3\rangle$ and $|G_3\rangle$ are $Tr\left[\mathcal{W}_3^s \rho_{G_3}\right] = 3.34 \pm 0.03$ and $Tr\left[\mathcal{W}_3^s \rho_{W_3}\right] = 3.33 \pm 0.03$ respectively, which proves both states contain genuine tripartite entanglement.

3. Possible applications

What kind of tripartite entanglement do we observe? The answer to that question takes us directly to a possible application of the Dicke sate. Fascinatingly, the class of tripartite entanglement depends on the measurement basis. While the W state represents the W class, the state $|G_3\rangle$ belongs to the GHZ class. This is remarkable: GHZ and W class states cannot be transformed into one another via SLOCC [1] and not even by entanglement catalysis [29]. $D_4^{(2)}$, however, can be projected into both classes by a local operation, i.e., via a simple von Neumann measurement of one qubit. This also implies that there is no obvious way how to obtain $D_4^{(2)}$ out of either of those three-qubit states via a 2-qubit interaction with an additional photon, as this would directly give a recipe to transform one class of three-qubit entanglement into the other. As the experimentally observed states are not perfect we also have to test whether the observed state $|G_3\rangle$ is GHZ class. To do so, we construct an entanglement witness from the generic one for pure GHZ states, $\mathcal{W}_{GHZ_3} = \frac{3}{4}\mathbb{1} - |GHZ_3\rangle\langle GHZ_3|$, by applying local filtering operations $\widehat{F} = A \otimes B \otimes C$. The resulting witness operator is then $\mathcal{W}' = \widehat{F}^\dagger \mathcal{W}_{GHZ_3} \widehat{F}$ [30,5]. Here A, B and C are 2×2 complex matrices determined through numerical optimization to find an optimal witness for the detected state. Note, that $\mathcal{W}'$ still detects GHZ type entanglement as $\widehat{F}$ is an SLOCC operation. In the measurement GHZ type entanglement is indeed detected with an expectation value of $Tr(\rho_G \mathcal{W}') = -0.029 \pm 0.023$ proving that the observed state is *not* W class.

Entanglement in $D_4^{(2)}$ is not only persistent against projective measurements but also against loss of photons. The state ρ_{abc} after tracing out qubit d is an equally weighted mixture of $|\, W_3 \,\rangle$ and $|\, \overline{W_3} \,\rangle$, which is also tripartite entangled. Applying witness $\mathcal{W}_3^s$ we obtain $Tr\left[\mathcal{W}_3^s \rho_{\text{abc}}\right] = 3.30 \pm 0.01$, proving clearly the genuine tripartite entanglement. The fidelity with respect to the expected state is $F_{\text{abc}} = 0.924 \pm 0.006$, similar values are reached for the loss of the photons in modes a, b and c.

As we have seen, the loss of one photon results in a three-qubit entangled W class state. Thus, the persistency against the loss of a second photon should also be high [27]. It is known that the state $|\, W_4 \,\rangle$ is the symmetric state with the highest persistency against loss of two photons with respect to entanglement measures like the concurrence [1,11]. In contrast, it turns out that for $D_4^{(2)}$ the remaining two photons have the highest possible maximal singlet fraction [31] ($\text{MSF}_{D_4^{(2)}} = 2/3$, experimentally $\text{MSF}_{\text{exp}} = 0.624 \pm 0.005$). This means that the residual state is as close to a Bell state as possible. It was already pointed out in Refs. [27,31] that this is a hint for the applicability of a state in telecloning [32]. Four parties that share the state $D_4^{(2)}$ can use the quantum correlations in each pair of qubits as a quantum channel for a teleportation protocol. Thus, each party can distribute an input qubit to the other parties with a certain fidelity, which depends on the MSF. Using $D_4^{(2)}$ as quantum resource this so-called $1 \rightarrow 3$ telecloning works with the optimal fidelity allowed by the no-cloning theorem. Averaged over arbitrary input states the fidelity is $\text{F}_{1\rightarrow 3}^{\text{clone}} = 0.788$ and the optimal so-called covariant cloning fidelity is $\text{F}_{1\rightarrow 3}^{\text{cov}} = 0.833$ for all input states on the equatorial plane of the Bloch sphere (i.e. all states $\frac{1}{\sqrt{2}}(|\, H \,\rangle + \exp(i\phi)|\, V \,\rangle)$). Assuming perfect cloning machines, the fidelity obtained by using the experimentally observed state for the protocol would be $\text{F}_{1\rightarrow 3}^{\text{expclone}} \approx 0.75$ and $\text{F}_{1\rightarrow 3}^{\text{expcov}} \approx 0.79$ in the covariant case. Note that for the experimentally observed state also the latter value is not independent of ϕ and varies between 0.81 and 0.77 for $\phi \in [0, 2\pi]$.

What if the receiving parties decide that one of them should get a perfect version of the input state? Probabilistically this is still possible, if the other two parties abandon their part of the information by a measurement of their qubit in the same direction, say (H/V). In case they find orthogonal measurement outcomes the sender and the only remaining receiver share a Bell state $_{cd}\langle\, HV \,|D_4^{(2)}\rangle = \frac{1}{\sqrt{3}}(|\, HV \,\rangle + |\, VH \,\rangle) = \sqrt{\frac{2}{3}}|\, \psi^+ \,\rangle_{ab}$. This enables perfect teleportation in 2/3 of the cases and therefore, as each party could be the receiver, an open destination teleportation (ODT) [8]. The experimentally obtained fidelity of the two photon states in this case was $F_{H_c V_d}^{\psi^+} = 0.883 \pm 0.028$. For other measurement directions different Bell states can be obtained. For example for projections onto the (±45)- and (L/R)-basis we found $F_{+_c -_d}^{\phi^+} = 0.721 \pm 0.043$ and $F_{R_c L_d}^{\phi^-} = 0.712 \pm 0.042$. Note that, in contrast to the deterministic GHZ based ODT protocol, $D_4^{(2)}$ allows to choose between telecloning and ODT.

Finally, as another possible application, we also note that $D_4^{(2)}$ is the symmetric Dicke state which can be used in certain quantum versions of classical games [33]. In these models it might offer new game strategies compared to the commonly used GHZ state.

Conclusion

In conclusion we have presented the experimental observation and analysis of a quantum state $D_4^{(2)}$, obtained with a fidelity of 0.844 ± 0.008 and a count rate as high as 60 counts/minute. The setup and methods used are generic for the observation of symmetric Dicke states with higher photon numbers as well. Our analysis focused on the state's violation of local realism and the particular properties that make the Dicke state suitable for several quantum information applications. As was shown, the two inequivalent classes of genuine tripartite entanglement can be obtained from the Dicke state after projection of one qubit in different bases. The possibility to project two photons into a Bell state makes $D_4^{(2)}$ a resource for an ODT protocol. Further, the state has a high entanglement persistency against loss of two photons. In this case, the singlet fraction of the remaining photons is maximal and from this we inferred applicability of the state for quantum telecloning. We believe that due to the extraordinary properties of the state other applications are likely in the future.

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