Multipartite entanglement in quantum optical systems

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Denmark Strait
Barents Sea
Norwegian Sea
Arctic Circle
ICELAND
Reykjavík
NORWAY
SWEDEN
FINLAND
RUSSIA
Hammerfest
Tromsø
Murmansk
Kiruna
White Sea
Arkhangel'sk
Luleå
Oulu
Umeå
Lake Onega
Trondheim
Gulf of Bothnia
Tampere
Lake Ladoga
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Mediterranean Sea
Palermo
Gibraltar (U.K.)
Málaga
Sicily
Strait of Gibraltar
Ceuta (SPAIN)
Alborán Sea
Melilla (SPAIN)
Oran
Algiers
Tunis
Crete
Rabat
Casablanca
MOROCCO
ALGERIA
TUNISIA
Valletta
MALTA
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles

Outline

1. **Motivation**
   - Why many-body entanglement is important?
2. **Different types of multipartite entanglement**
   - Two and three qubits
   - Multipartite entanglement
3. **Systems with few particles**
   - Physical systems
   - Designing entanglement witnesses
   - Experiments
4. **Systems with very many particles**
   - Physical systems
   - Spin squeezing and generalized spin squeezing
   - An experiment
5. **Metrology and multipartite entanglement**
   - Quantum Fisher information
   - Quantum Fisher information and entanglement

Why is multipartite entanglement interesting?

- Many experiments aim to create many-body entangled states.
- Quantum Information Science can help to find good targets for such experiments.
- Multipartite entangled states are needed in applications such as metrology.

Outline

Two qubits

Fact

Remember: There is only a single type of two-qubit entanglement.

- From a single copy of any pure entangled two-qubit state, we can get to any other entangled two-qubit state through Stochastic Local Operations and Classical Communication (SLOCC).

  That is, for any entangled $|\Psi\rangle$ and $|\Phi\rangle$, there are invertible A and B such that
  $$|\Psi\rangle = A \otimes B|\Phi\rangle.$$

  Note that A and B do not have to be Hermitian.

Bipartite systems

- For the mixed case, the definition of a separable state is (Werner 1989)

$$\rho = \sum_k p_k \rho_k^{(1)} \otimes \rho_k^{(2)}.$$

Definition

Local Operation and Classical Communications (LOCC):

- Single-party unitaries,
- Single party von Neumann measurements,
- Single party POVM measurements,
- We are even allowed to carry out measurement on party 1 and depending on the result, perform some other operation on party 2 ("Classical Communication").

LOCC and entanglement

It is not possible to create entangled states from separable states, with LOCC.

Distillation

- From many entangled particle pairs we want to create fewer strongly entangled pairs with LOCC.

Two qubits - mixed states

Fact

Remember: There is only a single type of two-qubit entanglement.

- From many copies of mixed entangled states, we can always distill a singlet using Local Operations and Classical Communication (LOCC).

Three-qubit mixed states

Six classes:

Class #1: fully separable states $\sum_k p_k \varrho_1^{(k)} \otimes \varrho_2^{(k)} \otimes \varrho_3^{(k)}$

Class #2: (1)(23) biseparable states $\sum_k p_k \varrho_1^{(k)} \otimes \varrho_{23}^{(k)}$, not in Class #1

Class #3: (12)(3) biseparable states $\sum_k p_k \varrho_{12}^{(k)} \otimes \varrho_3^{(k)}$, not in Class #1

Class #4: (13)(2) biseparable states $\sum_k p_k \varrho_{13}^{(k)} \otimes \varrho_2^{(k)}$, not in Class #1

Class #5: W-class states:
mxtr of pure (W $\cup$ Bisep $\cup$ Sep)-class states, not in Classes #1-4

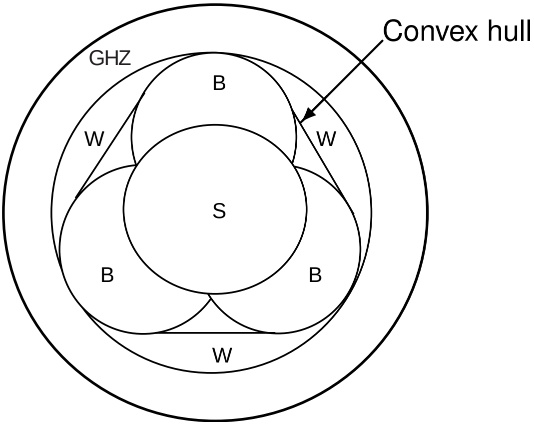
Class #6: GHZ-class states: mxtr of pure (GHZ $\cup$ W $\cup$ Bisep $\cup$ Sep)-class states, not in Classes #1-5

Biseparable states: mixture of states of classes #2, #3 and #4.

Three-qubit mixed states II

- The extension of the classification of pure states to mixed states leads to convex sets:

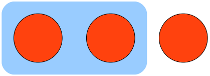
A. Acín, D. Bruss, M. Lewenstein, A. Sanpera, Phys. Rev. Lett. 87, 040401 (2001)



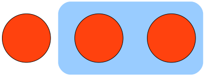
States that are biseparable with respect to all bipartitions

- There are states that are biseparable with respect to all the three bipartitions, but they are *not* fully separable.

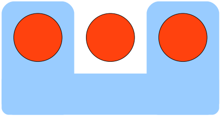
$$\varrho = \sum_k p_k \varrho_{12}^{(k)} \otimes \varrho_3^{(k)}$$



$$\varrho = \sum_k p'_k \varrho_{1}^{(k)} \otimes \varrho_{23}^{(k)}$$



$$\varrho = F_{12} \sum_k p''_k \varrho_{2}^{(k)} \otimes \varrho_{13}^{(k)} F_{12}$$



W. Dür, J.I. Cirac, Phys. Rev. A 61, 042314 (2000)

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More than three qubits

- 4 qubits: There are 9 families and infinite number of SLOCC equivalence classes.
  [F. Verstraete, J. Dehaene, B. De Moor, and H. Verschelde, Phys. Rev. A 65, 052112 (2002)]
- For many qubits, the practically meaningful classification is
  - (Fully) separable
  - Biseparable entangled
  - Genuine multipartite entangled

More than three qubits II

Definition

A state is (fully) separable if it can be written as $\sum_k p_k \varrho_1^{(k)} \otimes \varrho_2^{(k)} \otimes \ldots \otimes \varrho_N^{(k)}$.

Definition

A pure multi-qubit quantum state is called biseparable if it can be written as the tensor product of two multi-qubit states

$$|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle.$$

Here $|\Psi\rangle$ is an N-qubit state. A mixed state is called biseparable, if it can be obtained by mixing pure biseparable states.

Definition

If a state is not biseparable then it is called genuine multi-partite entangled.

k-producibility/*k*-entanglement

Definition

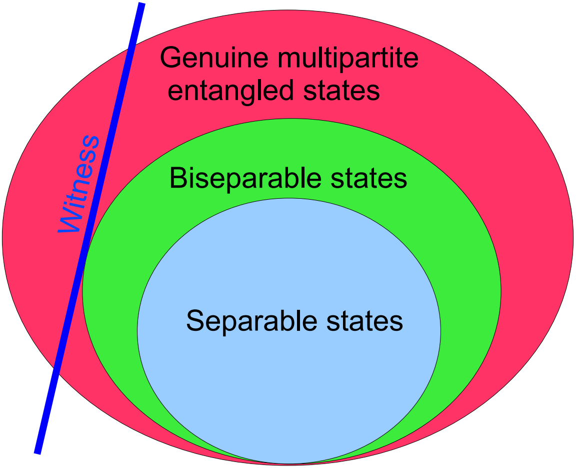
A pure state is *k*-producible or *k*-entangled if it can be written as

$$|\Phi\rangle = |\Phi_1\rangle \otimes |\Phi_2\rangle \otimes |\Phi_3\rangle \otimes |\Phi_4\rangle....$$

where $|\Phi_l\rangle$ are states of at most k qubits. A mixed state is k-producible, if it is a mixture of k-producible pure states.

Convex sets for the multipartite case

- The idea of convex sets also work for the multi-qubit case: A state is biseparable if it can be obtained by mixing pure biseparable states.



Examples

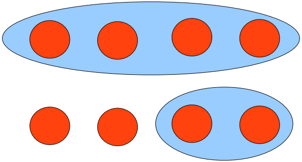
Examples

Two entangled states of four qubits:

$$|GHZ_4\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle),$$

$$|\Psi_B\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |0011\rangle) = \frac{1}{\sqrt{2}}|00\rangle \otimes (|00\rangle + |11\rangle).$$

- The first state is genuine multipartite entangled, the second state is biseparable.

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Outline

Physical systems

State-of-the-art in experiments

- 8 qubits with trapped cold ions (Nature, 2005), 14 qubits (2010)
- 10 qubits with photons (Nature Physics, 2010)

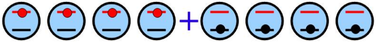
Main Challenges

- How to obtain useful information when only *local* measurements are possible?
- *In principle, the entanglement witness method has the advantage that only one observable, the entanglement witness, needs to be measured. In practice, the measurement of this observable may be done by a series of local measurements. ... At this point the advantage over basic state tomography becomes somewhat questionable.*
  (B. TERHAL, IBM Watson Research Center, 2002)

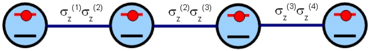
Interesting quantum states

Quantum states in experiments:

- Greenberger-Horn-Zeilinger (GHZ) state or "Schrödinger cat state”



- Cluster state, graph state (obtained in Ising spin chains)



- Symmetric Dicke states

- Singlet states

  $(\Delta J_l)^2 = 0 \qquad \text{for } j = x, y, z.$

Outline

Aims when designing a witness

Definition

An entanglement witness $\mathcal{W}$ is an operator that is positive on all separable (biseparable) states.

Thus, $\mathrm{Tr}(\mathcal{W}\varrho) < 0$ signals entanglement (genuine multipartite entanglement). Horodecki 1996; Terhal 2000; Lewenstein, Kraus, , Cirac, Horodecki 2002

There are two main goals when searching for entanglement witnesses:

Large robustness to noise

Optimization

Few measurements

Robustness to noise

- A state mixed with white noise is given as

$$\varrho(p_{\text{noise}}) = (1 - p_{\text{noise}})\varrho + p_{\text{noise}}\varrho_{\text{noise}}$$

where p_{noise} is the ratio of noise and ϱ_{noise} is the noise. If we consider white noise then $\varrho_{\text{noise}} = \mathbb{1}/2^N$.

Definition

The noise tolerance of a witness $\mathcal{W}$ is characterized by the largest p_{noise} for which we still have

$$\text{Tr}(\mathcal{W}\varrho) < 0.$$

Only local measurements are possible

Definition

A single local measurement setting is the basic unit of experimental effort.

A local setting means measuring operator $A^{(k)}$ at qubit k for all qubits.

$A^{(1)}$	$A^{(2)}$	$A^{(3)}$	...	$A^{(N)}$

- All two-qubit, three-qubit correlations, etc. can be obtained.

$$\langle A^{(1)}A^{(2)}\rangle, \langle A^{(1)}A^{(3)}\rangle, \langle A^{(1)}A^{(2)}A^{(3)}\rangle ...$$

Decomposition of an operator

- All operators must be decomposed into the sum of locally measurable terms and these terms must be measured individually.
- For example,

$$
\begin{aligned}
|GHZ_3\rangle\langle GHZ_3| = & \frac{1}{8}(\mathbb{1} + \sigma_z^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_z^{(3)} + \sigma_z^{(2)}\sigma_z^{(3)}) \\
& + \frac{1}{4}\sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)} \\
& - \frac{1}{16}(\sigma_x^{(1)} + \sigma_y^{(1)})(\sigma_x^{(2)} + \sigma_y^{(2)})(\sigma_x^{(3)} + \sigma_y^{(3)}) \\
& - \frac{1}{16}(\sigma_x^{(1)} - \sigma_y^{(1)})(\sigma_x^{(2)} - \sigma_y^{(2)})(\sigma_x^{(3)} - \sigma_y^{(3)}).
\end{aligned}
$$

O. Gühne and P. Hyllus, Int. J. Theor. Phys. 42, 1001-1013 (2003). M. Bourennane et al., Phys. Rev. Lett. 92 087902 (2004)

- As N increases, the number of terms increases exponentially for projectors to some quantum pure states.

Basic methods for designing witnesses

Three methods for designing witnesses:

- Projector witness, i.e., witness defined with the projector to a highly entangled quantum state
- Witness based on the projector witness
- Witness independent of the projector witness

Projector witness

- A witness detecting genuine multi-qubit entanglement in the vicinity of a pure state $|\Psi\rangle$ is

$$\mathcal{W}_{\Psi}^{(P)} := \lambda_{\Psi}^{2}\mathbb{1} - |\Psi\rangle\langle\Psi|,$$

  where λ is the maximum of the Schmidt coefficients for $|\Psi\rangle$, when all bipartitions are considered.
  M. Bourennane, M. Eibl, C. Kurtsiefer, S. Gaertner, H. Weinfurter, O. Gühne, P. Hyllus, D. Bruß, M. Lewenstein, and A. Sanpera, Phys. Rev. Lett. 2004

- A symmetric witness operator can always be decomposed as

$$P = \sum c_k A_k \otimes A_k \otimes A_k \otimes ... \otimes A_k.$$

- For symmetric operators, the number of settings needed is increasing polynomially with the number of qubits.
  GT, W. Wieczorek, R. Krischek, N. Kiesel, P. Michelberger, and H. Weinfurter, New J. Phys. 2009

Projector witness II

- GHZ states (robustness to noise is $\frac{1}{2}$ for large N!)

$$\mathcal{W}^{(P)}_{\mathrm{GHZ}} := \tfrac{1}{2}\mathbb{1} - |GHZ_N\rangle\langle GHZ_N|.$$

- Cluster states

$$\mathcal{W}^{(P)}_{\mathrm{CL}} := \tfrac{1}{2}\mathbb{1} - |CL_N\rangle\langle CL_N|.$$

- Dicke state

$$\mathcal{W}^{(P)}_{\mathrm{D(N,N/2)}} := \frac{1}{2}\frac{N}{N-1}\mathbb{1} - |D_N^{(N/2)}\rangle\langle D_N^{(N/2)}|.$$

- W-state

$$\mathcal{W}^{(P)}_{\mathrm{W}} := \frac{N-1}{N}\mathbb{1} - |D_N^{(1)}\rangle\langle D_N^{(1)}|.$$

Witnesses based on the projector witness

- We construct witnesses that are easier to measure than the projector witness.

- Idea: If $\mathcal{W}^{(P)}$ is the projector witness and

$$\mathcal{W} - \alpha\mathcal{W}^{(P)} \geq 0$$

is fulfilled for some $\alpha > 0$, then $\mathcal{W}$ is also a witness.
GT and O. Gühne, Phys. Rev. Lett. and Phys. Rev. A 2005

Witnesses based on the projector witness II

Example

Witness requiring only two measurement settings for GHZ states

$$\mathcal{W}_{\mathrm{GHZ}}^{(P)} := \frac{1}{2}\mathbb{1} - |GHZ_N\rangle\langle GHZ_N|$$

$$\leq \mathcal{W}_{\mathrm{GHZ}}^{(P2)} := \mathbb{1} - \frac{1}{2}X_1X_2X_3...X_N - \begin{bmatrix} 1 & & & & \\ & 0 & & & \\ & & ... & & \\ & & & 0 & \\ & & & & 1 \end{bmatrix}.$$

Measurement settings ⇒ [X X X X ...] [Z Z Z Z ...]

- Any state detected by $\mathcal{W}_{\mathrm{GHZ}}^{(P2)}$ is also detected by $\mathcal{W}_{\mathrm{GHZ}}^{(P)}$.
  GT and O. Gühne, Phys. Rev. Lett. and Phys. Rev. A 2005

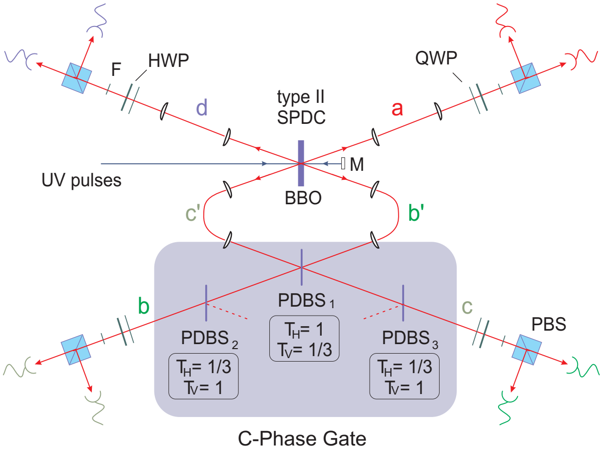
Witnesses independent from the projector witness

- Witnesses without any relation to the projector witness.
- With an easily measurable operator M, we make a witness of the form
$$\mathcal{W} := c\mathbb{1} - M,$$
where c is some constant.
- We have to set c to
$$c = \max_{|\Psi\rangle \in \mathcal{B}} \langle M \rangle_{|\Psi\rangle},$$
where $\mathcal{B}$ is the set of biseparable states. This problem is typically hard to solve.

Outline

An experiment: Cluster state with photons

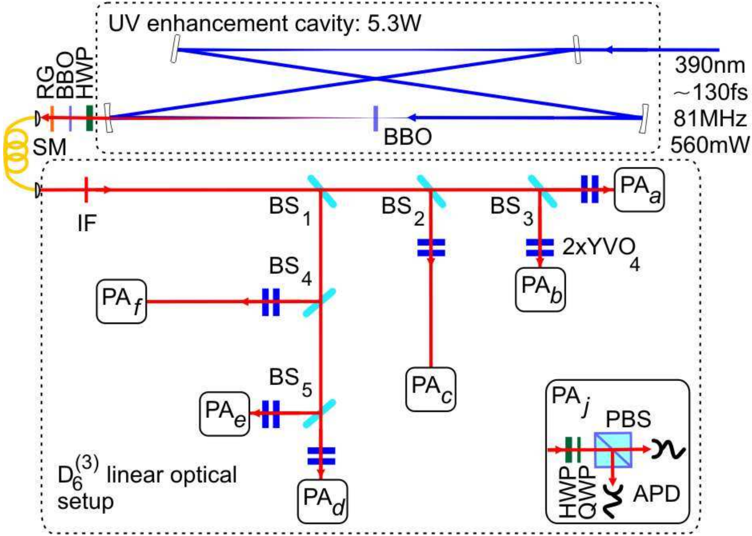
Experiment for creating a four-photon cluster state (Weinfurter group, 2005)



An experiment: Cluster state with photons II

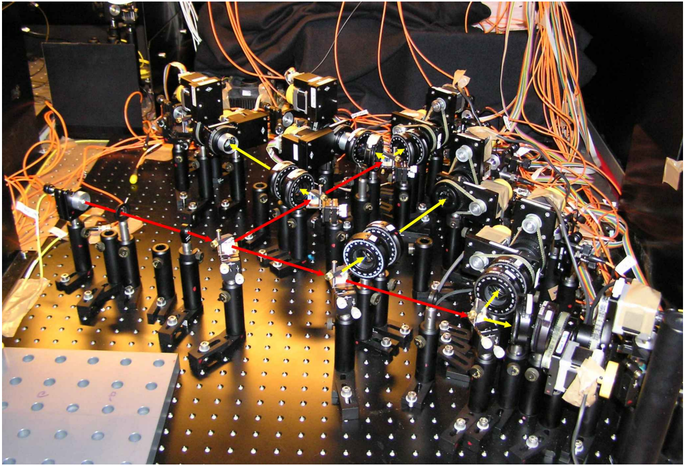
- Note: the experiment works with conditional detection.
- So far the largest experiment is with 6 photons, and with 10 qubits.
- 1 photon can encode more than 1 qubit: hyperentanglement.

An experiment: Dicke state with photons



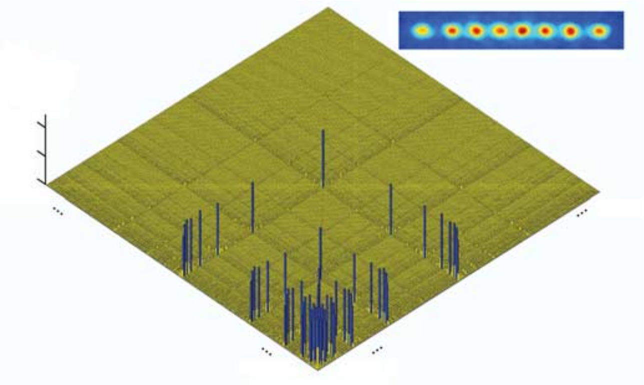
An experiment: Dicke state with photons II

A photo of a real experiment (six-photon Dicke state, Weinfurter group, 2009):



Experiment: W-state with ions

- Experimental observation of an 8-qubit W-state with trapped ions.



H. Haeffner, W. Haensel, C. F. Roos, J. Benhelm, D. Chek-al-kar, M. Chwalla, T. Koerber, U. D. Rapol, M. Riebe, P. O. Schmidt, C. Becher, O. Gühne, W. Dür, R. Blatt, Nature 438, 643-646 (2005).

Quantum state tomography

- The density matrix can be reconstructed from 3^N measurement settings.
- The measurements are

1. XXXX

2. XXXY

3. XXXZ

...

3^4. ZZZZ

- Note again that the number of measurements scales exponentially in N.

Outline

Physical systems

State-of-the-art in experiments

- 100,000 atoms realizing an array of 1D Ising spin chains (Nature, 2003)
- Spin squeezing with 10^6 - 10^{12} atoms (Nature, 2001)

Main challenge

- The particles cannot be addressed individually.
- Only collective quantities can be measured.
- New type of entangled states and entanglement criteria are needed.

Many-particle systems

- For spin-$\frac{1}{2}$ particles, we can measure the collective angular momentum operators:

$$J_l := \frac{1}{2}\sum_{k=1}^{N}\sigma_l^{(k)},$$

where $l = x, y, z$ and $\sigma_l^{(k)}$ a Pauli spin matrices.

- We can also measure the

$$(\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2$$

variances.

Outline

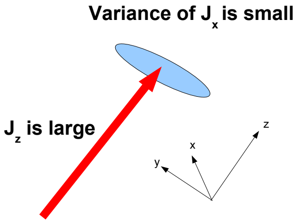
Spin squeezing

Definition

Uncertainty relation for the spin coordinates

$$(\Delta J_x)^2(\Delta J_y)^2 \geq \frac{1}{4}|\langle J_z\rangle|^2.$$

If $(\Delta J_x)^2$ is smaller than the standard quantum limit $\frac{1}{2}|\langle J_z\rangle|$ then the state is called spin squeezed (mean spin in the z direction!).
[M. Kitagawa and M. Ueda, Phys. Rev. A 47, 5138 (1993)]



Spin squeezing II

Definition

Spin squeezing criterion for the detection of quantum entanglement

$$\frac{(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq \frac{1}{N}.$$

If a quantum state violates this criterion then it is entangled.

- Application: Quantum metrology, magnetometry.

[A. Sørensen *et al.*, Nature **409**, 63 (2001)]

Complete set of the generalized spin squeezing criteria

- Let us assume that for a system we know only

$$\vec{J} := (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle),$$

$$\vec{K} := (\langle J_x^2 \rangle, \langle J_y^2 \rangle, \langle J_z^2 \rangle).$$

- Then any state violating the following inequalities is entangled

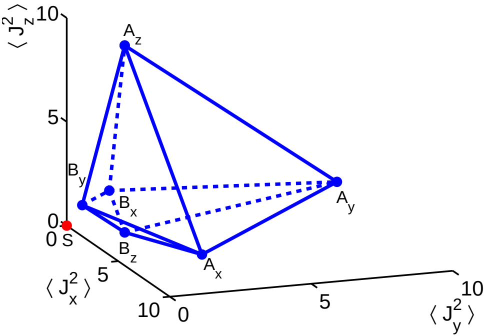
$$\begin{aligned}
\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle &\leq N(N+2)/4, \\
(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 &\geq N/2, \\
\langle J_k^2 \rangle + \langle J_l^2 \rangle - N/2 &\leq (N-1)(\Delta J_m)^2, \\
(N-1)\left[(\Delta J_k)^2 + (\Delta J_l)^2\right] &\geq \langle J_m^2 \rangle + N(N-2)/4.
\end{aligned}$$

where k, l, m takes all the possible permutations of x, y, z.
[GT, C. Knapp, O. Gühne, and H.J. Briegel, Phys. Rev. Lett. 2007]

- Can be used for thermal states of well-known spin chains.

The polytope

- The previous inequalities, *for fixed* $\langle J_{x/y/z}\rangle$, describe a polytope in the $\langle J^2_{x/y/z}\rangle$ space.
- Separable states correspond to points inside the polytope. Note: Convexity comes up again!



The derivation of such criteria

- The derivation of such criteria is partly based on entanglement detection with uncertainty relations.
- For a multi-qubit pure product state $|\Psi_{\mathrm{P}}\rangle = \bigotimes_k |\psi_k\rangle$we have

$$(\Delta J_l)^2 = \sum_k (\Delta j_l^{(k)})^2{}_{\psi_k}.$$

- Hence,

$$\sum_{l=x,y,z} (\Delta J_l)^2_{|\Psi_{\mathrm{P}}\rangle} = \sum_{l=x,y,z} \sum_{k=1}^{N} (\Delta J_l)^2_{|\Psi_k\rangle} =$$

$$\frac{1}{4}\sum_{k=1}^{N}(3 - \langle\sigma_x^{(k)}\rangle^2 - \langle\sigma_y^{(k)}\rangle^2 - \langle\sigma_z^{(k)}\rangle^2) = \frac{N}{2}.$$

- Due to the concavity of the variance, for mixed separable states we have

$$\sum_{l=x,y,z} (\Delta J_l)^2 \geq \frac{N}{2}.$$

[G. Tóth, PRA 2004; O. Gühne, PRL 2004; H.F. Hofmann and S. Takeuchi, PRA 2003.]

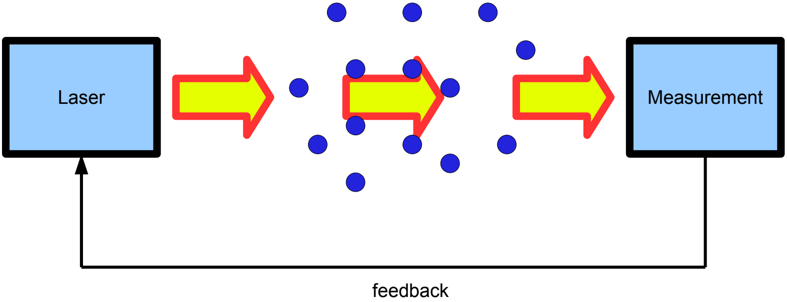
Outline

The physical system

- Bose Eisnetin condensate of atoms: the atoms interact with each other.
- Cold gases: the atoms do not interact with each other. (We consider this case.)

The physical system II

- Cold gases: Rb atoms + light

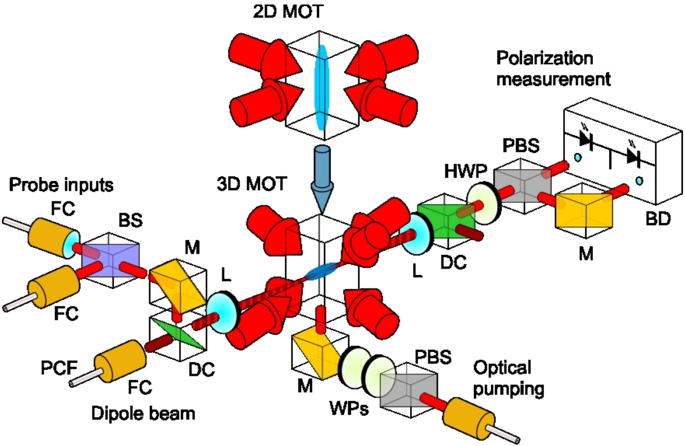


Experimental details

- Atoms interact with light. The light is measured, projecting the atoms into a squeezed state.
- Room temperature experiments: 10^{12} atoms
  [B Julsgaard, A Kozhekin, ES Polzik, Nature 2001].
  - Vapor cells
- Cold atom experiments: 10^6 atoms.
  - Laser cooling, sample in an optical dipole trap.
  - Atoms are transferred from a MOT to a dipole trap.

An experiment

Spin squeezing in a cold atomic ensemble (not BEC!)



Picture from M.W. Mitchell, ICFO, Barcelona.

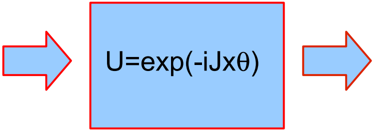
Outline

Quantum Fisher information

- One of the important applications of entangled multipartite quantum states is sub-shotnoise metrology.
  [V. Giovannetti, S. Lloyd, and L. Maccone, Science 306, 1330 (2004).]
- Multipartite entanglement, not simple nonseparability, is needed for extreme spin squeezing, which can be applied in spectroscopy and atomic clocks.
  [A.S. Sørensen and K. Mølmer, Phys. Rev. Lett. 86, 4431 (2001).]
- Not all entangled states are useful for phase estimation, at least in a linear interferometer.
  [P. Hyllus, O. Gühne, and A. Smerzi, arXiv:0912.4349.]

Quantum Fisher information II

- Let us consider the following process:



- The dynamics described above is $\varrho = e^{-i\theta J_{\vec{n}}}\varrho_0 e^{+i\theta J_{\vec{n}}}$.
- We would like to determine the angle θ by measuring ϱ.

Quantum Fisher information III

Quantum Cramér-Rao bound

For such a linear interferometer the phase estimation sensitivity is limited by the Quantum Cramér-Rao bound as

$$\Delta\theta \geq \frac{1}{\sqrt{F_Q[\varrho, J_{\vec{n}}]}},$$

where F_Q is the quantum Fisher information, ϱ is a quantum state and $J_{\vec{n}}$ is a component of the collective angular momentum in the direction $\vec{n}$.

[C.W. Helstrom, *Quantum Detection and Estimation Theory* (Academic Press, New York, 1976);
A. S. Holevo, *Probabilistic and Statistical Aspect of Quantum Theory* (North-Holland, Amsterdam, 1982).]

Outline

Quantum Fisher information and entanglement

Pezzé, Smerzi, PRL 2009

For N-qubit separable states, the values of $F_Q[\varrho, J_l]$ for $l = x, y, z$ are bounded as

$$F_Q[\varrho, J_l] \leq N.$$

Here, $J_l = \frac{1}{2}\sum_{k=1}^{N} \sigma_l^{(k)}$ where $\sigma_l^{(k)}$ are the Pauli spin matrices for qubit (k). The maximum for the left-hand side is N^2.

Thus, for separable states

$$\Delta\theta \geq \frac{1}{\sqrt{N}},$$

while for entangled states

$$\Delta\theta \geq \frac{1}{N}.$$

Quantum Fisher information and multipartite entanglement II

Fact

Genuine multipartite entanglement, not simple nonseparability is needed to achieve maximum sensitivity in a linear interferometer.

[GT, arxiv:1006.4368; P. Hyllus et al., arXiv:1006.4366.]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Summary

- We discussed entanglement detection in multipartite systems.
- We considered
  - systems with few particles in which the particles could be individually addressed.
  - systems with very many particles, without the possibility of individual addressing

Review: O. Gühne and GT, “Entanglement detection”,

Physics Reports 474, 1-75 (2009).

THANK YOU FOR YOUR ATTENTION!