Entanglement witnesses in spin models

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Outline

- Entanglement witnesses
- Our proposal: Constructing witnesses for spin models.
- We propose using fundamental quantum operators of spin models for witnessing entanglement. In particular, we use the Hamiltonian.
- We can use our ideas also for spin models in thermal equilibrium.

Entanglement witnesses I

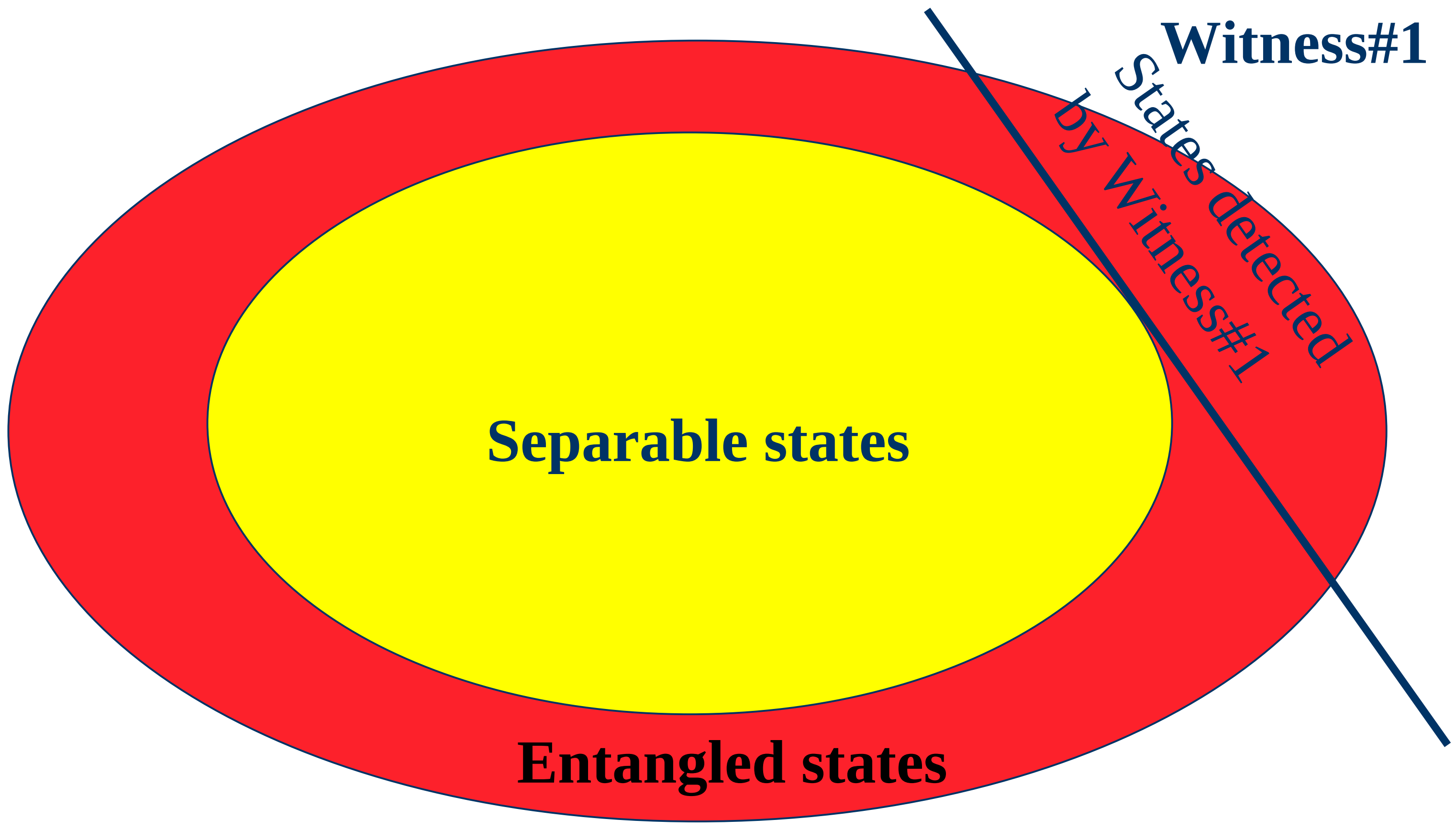
- Entanglement witnesses *W* are observables which have
  - positive or zero expectation value for all separable states

$$\langle W \rangle_{sep} \geq 0$$

  - negative expectation value for *some* entangled states.

$$\langle W \rangle_{ent} < 0$$

Entanglement witnesses II



How to construct entanglement witnesses?

- Usual way: (i) construct a witness operator which detects entangled states close to a given quantum state

  (ii) Decompose it into the sum of locally measurable terms

- Now we do not do that. We use physically interesting quantum operators of spin systems for witnessing entanglement. These have two-particle interaction and certain symmetries.

Simple example: Heisenberg chain without an external magnetic field I

- The Hamiltonian

$$H = J\sum_{k=1}^{N} \sigma_x^{(k)}\sigma_x^{(k+1)} + \sigma_y^{(k)}\sigma_y^{(k+1)} + \sigma_z^{(k)}\sigma_z^{(k+1)}$$

- For product states

$$\left|\left\langle \sigma_x^{(k)}\sigma_x^{(k+1)} + \sigma_y^{(k)}\sigma_y^{(k+1)} + \sigma_z^{(k)}\sigma_z^{(k+1)} \right\rangle\right| =$$

$$\left|\left\langle \sigma_x^{(k)}\right\rangle\left\langle \sigma_x^{(k+1)}\right\rangle + \left\langle \sigma_y^{(k)}\right\rangle\left\langle \sigma_y^{(k+1)}\right\rangle + \left\langle \sigma_z^{(k)}\right\rangle\left\langle \sigma_z^{(k+1)}\right\rangle\right| = \left|v_{Bloch}^{(k)} v_{Bloch}^{(k+1)}\right| \le 1$$

- Also true for mixed separable states.

Heisenberg chain II

- For separable state the energy is bounded as

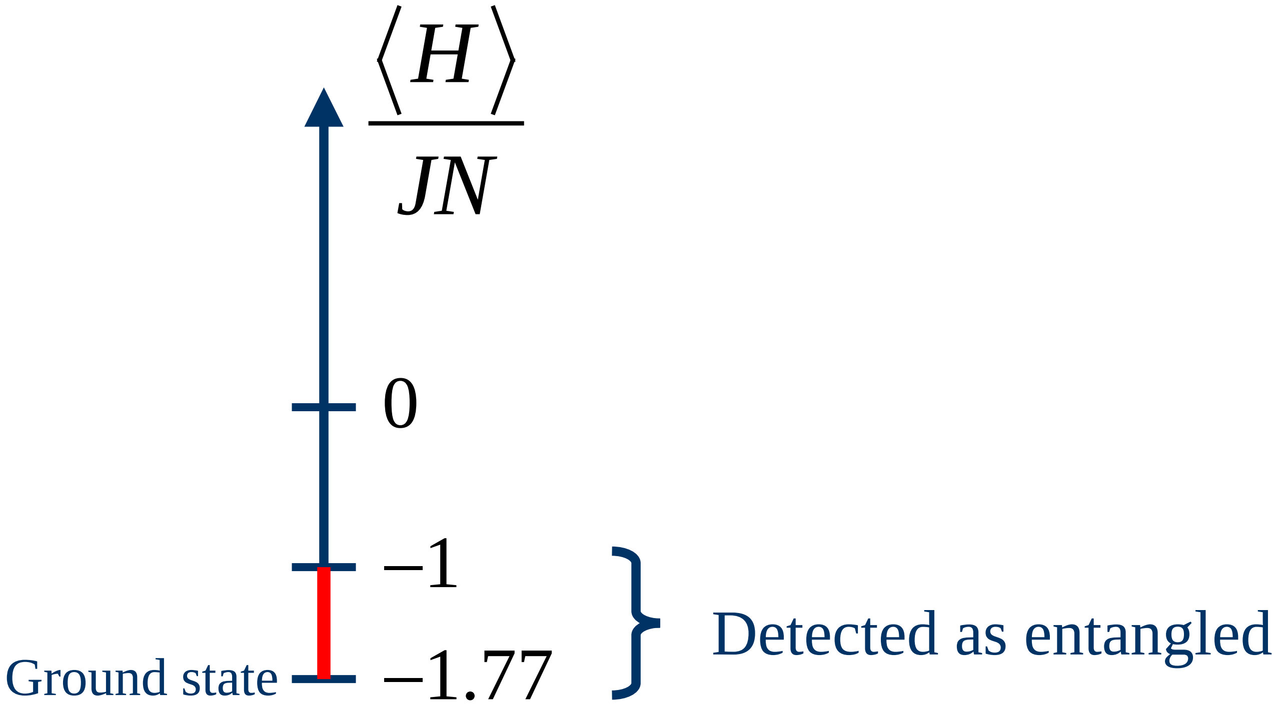
$$\frac{\langle H \rangle}{J} = \left\langle \sum_{k=1}^{N} \sigma_x^{(k)}\sigma_x^{(k+1)} + \sigma_y^{(k)}\sigma_y^{(k+1)} + \sigma_z^{(k)}\sigma_z^{(k+1)} \right\rangle \geq -N$$

- The ground state energy for large N is about

$$\frac{\langle H \rangle}{J} \approx -1.77N$$

The ground state is highly entangled and we detect entanglement in its “proximity.”

Heisenberg chain III



Heisenberg chain and Ising spin chain in an external magnetic field

- Similar ideas work for the Heisenberg spin chain Hamiltonian in an external magnetic field: energy bounds for separable states can be obtained.

- Also, similar bounds can be obtained for the Ising spin Hamiltonian in a transverse field.

Entanglement detection in thermal equilibrium

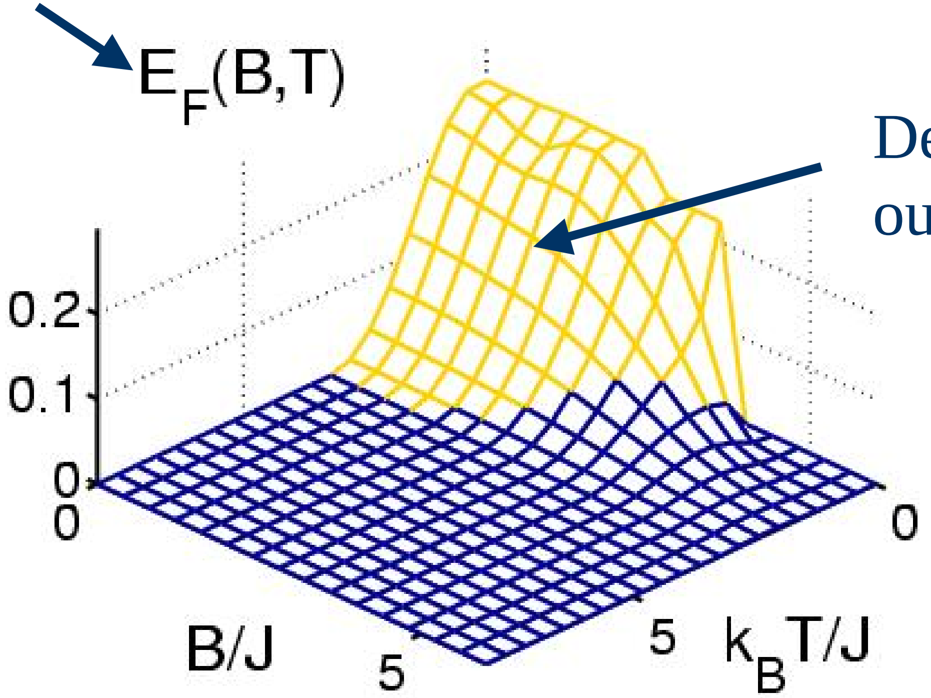
- We can determine a temperature bound corresponding to our energy bound. Below this temperature the quantum state is entangled.
- For the Heisenberg chain this bound is

$$\frac{kT}{J} = 3.18$$

  Here k=1, J=1 and zero field (B=0) was assumed.
- From [Wang, PRA 66, 044305 (2002)] we know that this is the bound for nonzero concurrence.

Heisenberg chain in an external magnetic field

Entanglement of formation of the two-qubit reduced density matrix

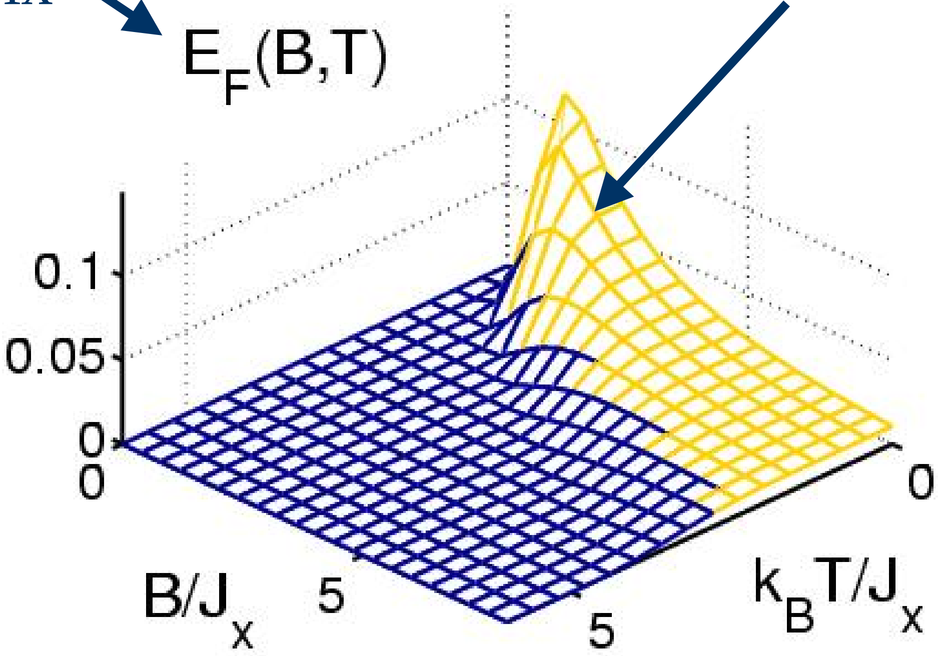


Detected by our bound for energy

Ising spin chain in a transverse magnetic field

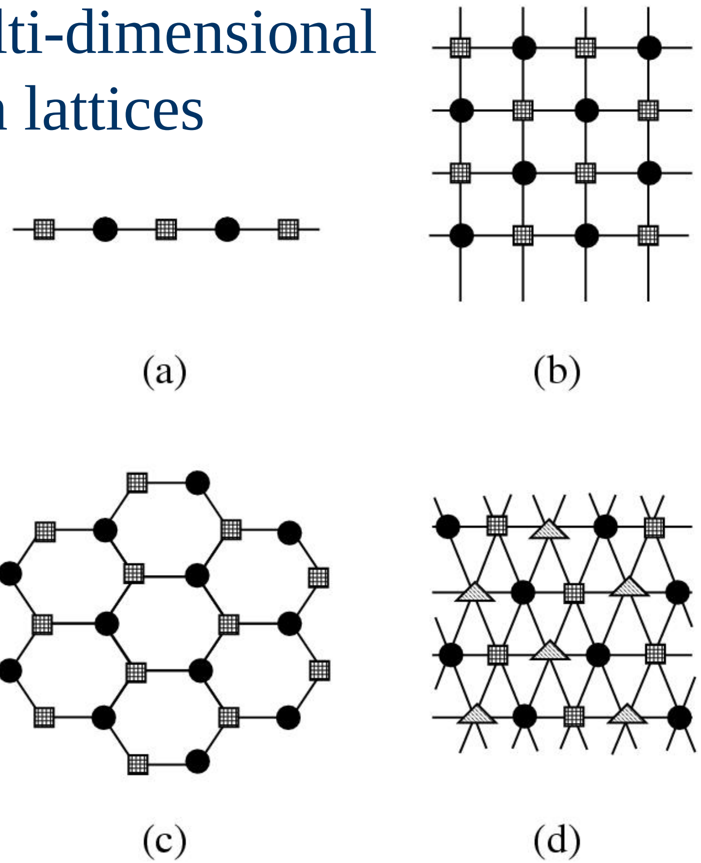
Entanglement of formation of the two-qubit reduced density matrix

Detected by our bound for energy

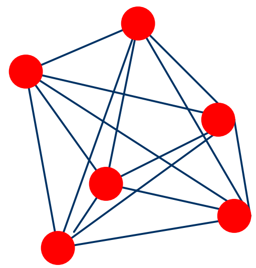


Other models, where energy bound for separable states can be found

Multi-dimensional spin lattices

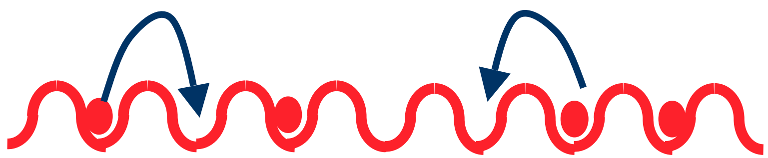


Complete graph with Heisenberg interactions



C=0

Bose-Hubbard model



Practical application

- Detecting entanglement by measuring energy
- Detecting entanglement by measuring temperature

In both cases one has to *trust the physical model*. In the following case this is not needed:

- Detecting entanglement by measuring correlations and computing $\langle H \rangle$.

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Summary

- We constructed entanglement witnesses using the Hamiltonian of spin models
- Home page: http://www.mpq.mpg.de/Theorygroup/CIRAC/people/toth
- *************** SEE YOU IN 15 MINUTES!!! *************
- See also:
  Dowling, Doherty & Bartlett, PRA 70, 062113 (2004) + quant/ph/0408086;
  Brukner & Vedral, quant-ph/0406040;
  Wu, Bandyopadhyay, Sarandy & Lidar, quantph/0412099;
  Gühne, Tóth & Briegel, quant-ph/0502160.