Mapping the spatial distribution of entanglement in optical lattices

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Outline

1. **Motivation**
   - Why cluster states are important?
2. **Experiments so far**
   - Former experiments
   - New technics
3. **Entanglement witnesses**
4. **Our proposal to create a 2D cluster state**

Why cluster states are important?

- Cluster states are a resource for measurement-based quantum computing
[R. Raussendorf, D. E. Browne, and H. J. Briegel, Phys. Rev. A 68, 022312 (2003).]

- They are states with genuine multipartite entanglement that are robust against decoherence.
[H.-J. Briegel and R. Raussendorf, Phys. Rev. Lett. 86, 910 (2001).]

- They arise naturally in Ising spin chains, which is a spin model appearing often in physics.

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Experiments for creating cluster states

- 4-qubit cluster states with photonic systems
  - N. Kiesel *et al.,* Phys. Rev. Lett. 2005, Weinfurter group.
  - P.Walther *et al.,* Nature 2005, Zeilinger group.
- Other graph states, apart from GHZ states with photons
  - 6 qubits, C. Lu *et al.,* Nature Phys. 2007, Pan group.
- Optical lattices of cold atoms
  - O. Mandel *et al.,* Nature 2003, Bloch group.

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New experimental technics for optical lattices

- In the 2003 experiment, the atoms were not accessible individually.

- The atomic ensemble had to expand and the the presence of the cluster state was concluded from an interference pattern. The fidelity could not be measured.

- Now there are new technics such that the state of the atoms can be measured individually.
  [K.-A. B. Soderberg, N. Gemelke, and C. Chin, New J. Phys. 11, 055022 (2009).]
  [A. Klinger, S. Degenkolb, N. Gemelke, K. Brickman Soderberg, and C. Chin, Rev. Sci. Instrum. 81, 013109 (2010).]

Entanglement witnesses

- An entanglement witness W is an operator such that

1. $\langle W \rangle \geq 0$ for all states without genuine multipartite entanglement.
2. $\langle W \rangle < 0$ for some states with genuine multipartite entanglement.

- Typically such witnesses are based on a projector to the state.

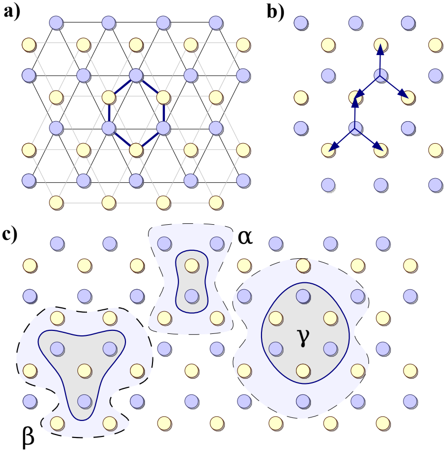
- A local decomposition of the witness is needed to measure it

$$W = \sum_k c_k A_k \otimes B_k \otimes C_k \otimes ...$$

It is often difficult since too many measurements are needed.

The experimental setup for a 2D cluster state

- Bipartite lattice, trapping two different atomic species, A and B. (e.g., Li and Cs atoms).



- By moving the lattices with respect to each other, atoms at neighboring site can interact, realizing an Ising dynamics.

Entanglement detection

- The stabilizing operators, g_i, for the graph states are

$$g_i|G_{\hexagon}\rangle = +1|G_{\hexagon}\rangle, \quad \forall i \in A \cup B.$$

with the stabilizing operators

$$g_i = \sigma_i^x \prod_{j \in Neigh(i)} \sigma_j^z.$$

- For a set of lattice sites, Ω, the projector onto a stabilizer state is

$$P_\Omega = \prod_{i \in \Omega} \frac{1}{2}(1 + g_i).$$

- The fidelity can be computed with this operator as

$$F_{A \cup B} = \mathrm{tr}(P_{A \cup B}\varrho),$$

where the region under study now encloses the *A* and *B* sublattices. Difficult to measure locally.

Witness with a simple decomposition

- A witness for multipartite entanglement in that region Ω,

  $$W_\Omega = \frac{1}{2}1 - P_\Omega.$$

  Also difficult to measure locally.

- There is an alternative:

  $$\tilde{P}_\Omega = P_{\Omega\cap A} + P_{\Omega\cap B} - 1$$

  This observable provides a lower bound for the fidelity

  $$F_\Omega \geq \langle \tilde{P}_\Omega \rangle,$$

  and is also helps to bild an entanglement witness

  $$\tilde{W}_\Omega = \frac{1}{2}1 - \tilde{P}_\Omega.$$

G. Tóth and O. Gühne, Phys. Rev. Lett. (2005).

Witness with a simple decomposition II

- $\langle P_{\Omega\cap A}\rangle$ and $\langle P_{\Omega\cap B}\rangle$ can be extracted from just two settings of measurements.
- In particular, for the graph state one such expectation value

$$\langle P_{\Omega\cap A}\rangle = \left\langle \prod_{i\in\Omega\cap A} \frac{1}{2}\left(1+\sigma_i^x \prod_{j\in N(i)} \sigma_j^z\right)\right\rangle,$$

  is obtained measuring
  - σ^x in all Cs atoms,
  - σ^z in the Li atoms.

- The other expectation value is obtained measuring
  - σ^z in all Cs atoms,
  - σ^x in the Li atoms.

Localizable fidelity I

- It does not make sense to ask, what is the fidelity of a 100 atom graph states to the ideal one.
- We have to ask instead questions about 3-atom, 4-atom, 5-atom units.
- However, all reduced few-atom states are mixed.

Localizable fidelity II

- Given a simply connected set of sites, Ω, and a perfect graph state, $|G_{\hexagon}\rangle$, we can extract another perfect graph state in that region.

- There are two ways to do that:

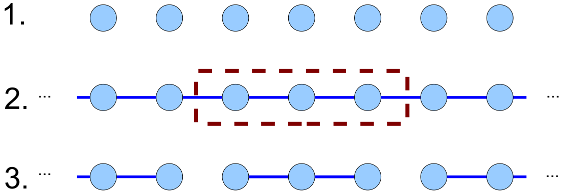
1. Local method:
    1. Measure the boundary qubits, $\partial\Omega$,
    2. Apply phase gates depending on the outcome.
2. Disentangling gates.

Localizable fidelity III

- Localizable fideility: the fidelity that we could get in a small region α, after extracting a local graph state
- $$P_\alpha = \prod_{i\in\alpha} \frac{1}{2}(1 + g_i).$$

  Postprocessing the **same** set of measurement results we can compute all fidelity bounds for all regions.

- **Example** on a linear cluster state, we are interested in the 3-qubit sunit:

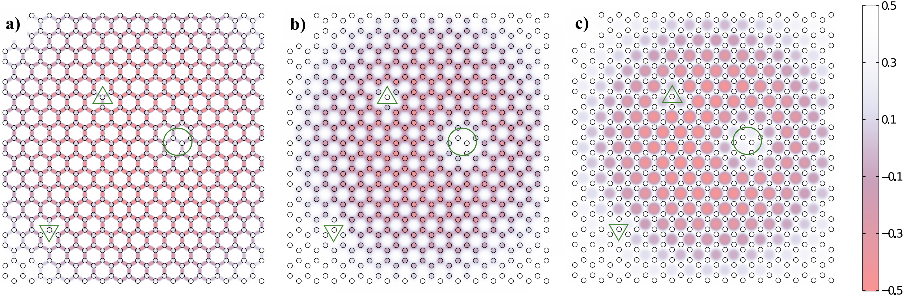


Calculation for the noisy cluster state

- Not only the entanglement detection is easier with the operators bounding the fidelity.

- The dynamics of their expectation values under decoherence is also easier to compute.

Calculation for the noisy cluster state II



Two-dimensional distribution of the entanglement witnesses for

(a) two-,
(b) four-,
(c) six-particle arrangements

Summary

- We discussed how the 2D cluster states can be created in optical lattices of cold atoms, and how their entanglement can be detected.
- We obtain an entanglement map, detecting the two-,three-,etc. partite entanglement that can be obtained in a region after local operations on the boundary of the region.

See: E. Alba, G. Tóth, and J.J. García-Ripoll,
"Mapping the spatial distribution of entanglement in optical lattices",
Phys. Rev. A 82, 062321 (2010).

THANK YOU FOR YOUR ATTENTION!

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    - Mitchell group, Barcelona, (cold gases)
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