Spin-squeezing inequalities for entanglement detection in cold gases Phys. Rev. Lett. 107, 240502 (2011)

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QIPC 2013, Firenze,
1 July 2013


F.Y.R.O.M. - The Former Yugoslav Republic of Macedonia
Greenland (DENMARK)
Jan Mayen (NORWAY)
Greenland Sea
Denmark Strait
Barents Sea
Norwegian Sea
Arctic Circle
ICELAND
Reykjavík
NORWAY
SWEDEN
FINLAND
RUSSIA
Tórshavn
Faroe Islands (DENMARK)
SHETLAND ISLANDS
ORKNEY ISLANDS
HEBRIDES
Rockall (U.K.)
North Atlantic Ocean
North Sea
UNITED KINGDOM
IRELAND
Dublin
London
Celtic Sea
English Channel
Bay of Biscay
Bilbao
FRANCE
Paris
SPAIN
Madrid
PORTUGAL
Lisbon
GERMANY
Berlin
POLAND
Warsaw
DENMARK
Copenhagen
Oslo
Stockholm
Helsinki
ESTONIA
Tallinn
LATVIA
Riga
LITHUANIA
Vilnius
BELARUS
Minsk
UKRAINE
Kiev
CZECH REPUBLIC
Prague
SLOVAKIA
AUSTRIA
Vienna
HUNGARY
Budapest
ROMANIA
Bucharest
MOLDOVA
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BULGARIA
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LUX.
BEL.
Brussels
NETH.
Amsterdam
ANDORRA
MONACO
Corsica
Sardinia
Sicily
BALEARIC ISLANDS
Mediterranean Sea
Adriatic Sea
Tyrrhenian Sea
Ionian Sea
Aegean Sea
Baltic Sea
MOROCCO
Rabat
ALGERIA
Algiers
TUNISIA
Tunis
Valletta
MALTA
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles

Outline

1. **Motivation**
   - Why spin squeezing inequalities are important?
2. **Physical systems**
   - Cold gases
3. **Multipartite entanglement**
   - Definition of entanglement
4. **Spin squeezing entanglement criteria for $j = 1/2$**
   - Collective measurements
   - The original criterion
   - Generalized criteria for $j = \frac{1}{2}$
5. **Spin squeezing inequality for an ensemble of spin-j atoms**
   - Conditions with the angular momentum coordinates for $j > \frac{1}{2}$
   - The usual spin squeezing inequality for $j > \frac{1}{2}$
   - Conditions with the SU(d) generators
   - Detection of singlets

Why spin squeezing inequalities for $j > \frac{1}{2}$ is important?

- Many experiments are aiming to create entangled states with many atoms.
- Only collective quantities can be measured.
- Most experiments use atoms with $j > \frac{1}{2}$.

Outline

Physical systems

State-of-the-art in experiments

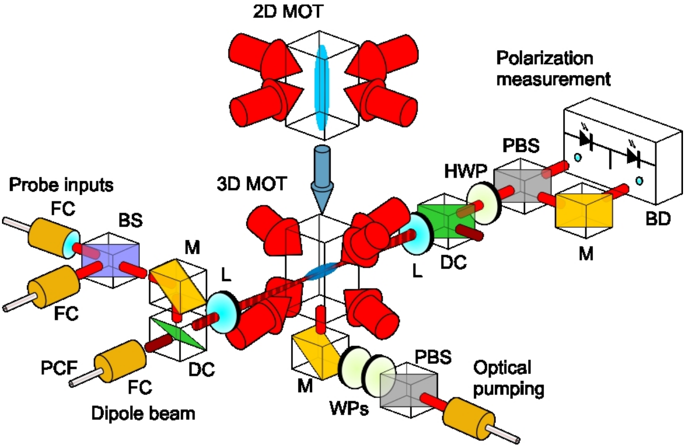
- 100,000 atoms realizing an array of 1D Ising spin chains (Nature, 2003)
- Spin squeezing with 10^6 - 10^{12} atoms (Nature, 2001)

Main challenge

- The particles cannot be addressed individually.
- Only collective quantities can be measured.
- New type of entangled states and entanglement criteria are needed.

Physical systems II

For example: Spin squeezing in a cold atomic ensemble



Picture from M.W. Mitchell, ICFO, Barcelona.

Outline

Entanglement

Definition

A multiparticle state is (fully) separable if it can be written as

$$\sum_k p_k \varrho_1^{(k)} \otimes \varrho_2^{(k)} \otimes ... \otimes \varrho_N^{(k)}.$$

If a state is not fully separable, then it is called entangled.

Outline

Many-particle systems for j=1/2

- For spin-$\frac{1}{2}$ particles, we can measure the collective angular momentum operators:

$$J_l := \frac{1}{2}\sum_{k=1}^{N}\sigma_l^{(k)},$$

where $l = x, y, z$ and $\sigma_l^{(k)}$ a Pauli spin matrices.

- We can also measure the variances

$$(\Delta J_l)^2 := \langle J_l^2\rangle - \langle J_l\rangle^2.$$

Outline

The standard spin-squeezing criterion

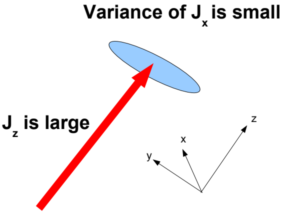
- The spin squeezing criteria for entanglement detection is

$$\frac{(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq \frac{1}{N}.$$

- If it is violated then the state is entangled.

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature 409, 63 (2001).]

- States violating it are like this:



Outline

Generalized spin squeezing criteria for $j = \frac{1}{2}$

- Let us assume that for a system we know only

$$\vec{J} := (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle),$$
$$\vec{K} := (\langle J_x^2 \rangle, \langle J_y^2 \rangle, \langle J_z^2 \rangle).$$

- Then any state violating the following inequalities is entangled.

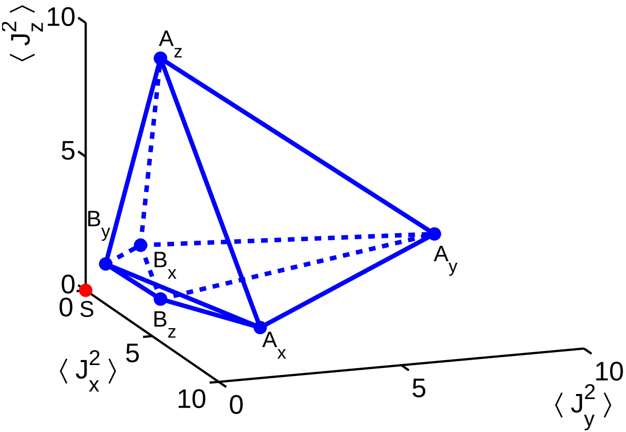
$$\begin{aligned}
\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle &\leq \frac{N(N+2)}{4}, \\
(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 &\geq \frac{N}{2}, \\
\langle J_k^2 \rangle + \langle J_l^2 \rangle &\leq (N-1)(\Delta J_m)^2 + \frac{N}{2}, \\
(N-1)\left[(\Delta J_k)^2 + (\Delta J_l)^2\right] &\geq \langle J_m^2 \rangle + \frac{N(N-2)}{4},
\end{aligned}$$

where k, l, m take all the possible permutations of x, y, z.

[GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL 99, 250405 (2007)]

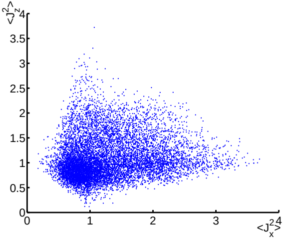
Generalized spin squeezing criteria for $j = \frac{1}{2}$

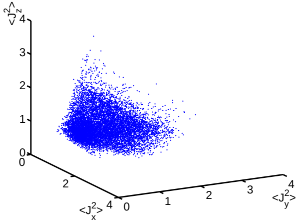
- The previous inequalities, *for fixed* $\langle J_{x/y/z} \rangle$, describe a polytope in the $\langle J_{x/y/z}^2 \rangle$ space.
- For $\langle \vec{J} \rangle = 0$ and $N = 6$ the polytope is the following:



Completeness

- Random separable states:

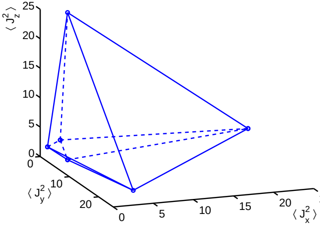




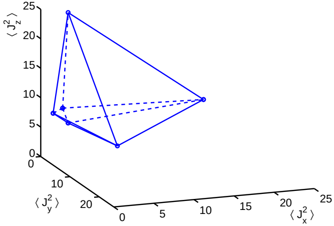
- The completeness can be proved for large *N*.

The polytope for $N = 10$ and

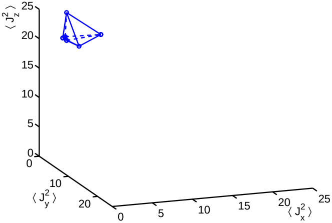
$J = (0, 0, 0)$,



$J = (0, 0, 2.5)$,



and $J = (0, 0, 4.5)$.



Outline

"Modified" quantities for $j > \frac{1}{2}$

- For the $j = \frac{1}{2}$ case, the SSIs were developed based on the first and second moments and variances of the such collective operators.

- For the $j > \frac{1}{2}$ case, we define the modified second moment

$$\langle \tilde{J}_k^2 \rangle := \langle J_k^2 \rangle - \langle \sum_n (j_k^{(n)})^2 \rangle = \sum_{m \neq n} \langle j_k^{(n)} j_k^{(m)} \rangle$$

  and the modified variance

$$(\tilde{\Delta} J_k)^2 := (\Delta J_k)^2 - \langle \sum_n (j_k^{(n)})^2 \rangle.$$

- These are essential to get tight equations for $j > \frac{1}{2}$.

The inequalities for $j > \frac{1}{2}$ with the angular momentum coordinates

- For fully separable states of spin-j particles, all the following inequalities are fulfilled

$$
\begin{aligned}
\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle &\leq Nj(Nj+1), \\
(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 &\geq Nj, \\
\langle \tilde{J}_k^2 \rangle + \langle \tilde{J}_l^2 \rangle - N(N-1)j^2 &\leq (N-1)(\tilde{\Delta} J_m)^2, \\
(N-1)\left[(\tilde{\Delta} J_k)^2 + (\tilde{\Delta} J_l)^2\right] &\geq \langle \tilde{J}_m^2 \rangle - N(N-1)j^2,
\end{aligned}
$$

  where k, l, m take all possible permutations of x, y, z.

- Violation of any of the inequalities implies entanglement.

Completeness

- In the large N limit, the inequalities with the angular momentum are complete.
- It is not possible to find new entanglement conditions based on $\langle J_k \rangle$ and $\langle \tilde{J}_k^2 \rangle$ that detect more states.

Outline

The usual spin squeezing inequality for $j > \frac{1}{2}$

- The standard spin-squeezing inequality becomes

$$\frac{(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} + \frac{\sum_n (j^2 - \langle (j_x^{(n)})^2\rangle)}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq \frac{1}{N}.$$

  Violated only if there is entanglement between the spin-j particles.

- The second term on the LHS is nonnegative.

Outline

The inequalities for $j > \frac{1}{2}$ with the G_k's

- Collective operators:

$$G_l := \sum_{k=1}^{N} g_l^{(k)},$$

where $l = 1, 2, ..., d^2 - 1$ and $g_l^{(k)}$ are the SU(d) generators.

- We can also measure the

$$(\Delta G_l)^2 := \langle G_l^2 \rangle - \langle G_l \rangle^2$$

variances.

The inequalities for $j > \frac{1}{2}$ with the G_k's

- The SSIs for G_k have the general form

$$(N-1)\sum_{k\in I}(\tilde{\Delta} G_k)^2 - \sum_{k\notin I}\langle(\tilde{G}_k)^2\rangle \geq -2N(N-1)\frac{(d-1)}{d}.$$

- For instance, for the $d = 3$ case, the SU(d) generators can be the eight Gell-Mann matrices.
- I is a subset of indices between 1 and M. We have 2^M equations!

Outline

An example: The criterion for SU(d) singlets

A condition for two-producibility (i.e., higher form of entanglement) for *N* qudits of dimension *d* is

$$\sum_k (\Delta G_k)^2 \geq 2N(d-2).$$

A condition for separability is

$$\sum_k (\Delta G_k)^2 \geq 2N(d-1).$$

[G. Vitagliano, P. Hyllus, I.L. Egusquiza, and G. Tóth,
Spin squeezing inequalities for arbitrary spin, PRL 2011.]

Group

Philipp Hyllus	Research Fellow (2011-2012)
Zoltán Zimborás	Research Fellow (2012-)
Iñigo Urizar-Lanz	Ph.D. Student
Giuseppe Vitagliano	Ph.D. Student
Iagoba Apellaniz	Ph.D. Student

- Topics
  - Multipartite entanglement and its detection
  - Metrology, cold gases
  - Collaborating on experiments:
    - Weinfurter group, Munich, (photons)
    - Mitchell group, Barcelona, (cold gases)
- Funding:
  - European Research Council starting grant 2011-2016, 1.3 million euros
  - CHIST-ERA QUASAR collaborative EU project
  - Grants of the Spanish Government and the Basque Government

Links to QIPC Posters

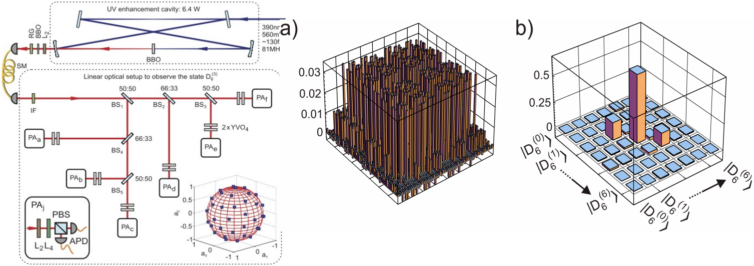
- G. Vitagliano, Spin squeezing and entanglement for arbitrary spin (more details)
- I. Urizar-Lanz, Differential magnetometry using singlets
- I. Apellaniz, Accuracy Bounds for Gradient Metrology in Atomic Ensembles

Links to a QIPC Talk

H. Weinfurter, Tuesday 11:30-12:15, Analyzing multi-qubit quantum states – Permutationally Invariant Tomography.

Permutationally invariant tomography

- Full state tomography is not feasible even for modest size systems.
- PI tomography is a scaleable alternative (PRL 2010).
- We developed a scaleable method for fitting a physical density matrix on the measured data (before: bootle neck of state reconstruction)



Ongoing experiment at the Max Planck Institute for Quantum Optics, München.

G. Tóth *et al.*, Phys. Rev. Lett. **105**, 250403 (2010);
T. Moroder *et al.*, New J. Phys **14**, 105001 (2012).

Summary

- Full set of generalized spin squeezing inequalities with J_l with $l = x, y, z$ for $j > \frac{1}{2}$.
- Large set of inequalities with the other collective operators.
- These might make possible new experiments and make existing experiments simpler.

See: G. Vitagliano, P. Hyllus, I.L. Egusquiza, and G. Tóth, Phys. Rev. Lett. 107, 240502 (2011) + manuscript in preparation.

See `www.gtoth.eu` for the slides

THANK YOU FOR YOUR ATTENTION!

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EUSKO JAURLARITZA GOBIERNO VASCO