Detecting metrologically useful entanglement in Dicke states

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Introduction

- With the rapid development of quantum control it is now possible to create large scale entanglement in many physical systems, such as cold atoms or trapped ions.
- Entanglement conditions with collective measurements are important since in many quantum experiments the spins cannot be individually addressed.
- We discuss, how to detect multiparticle entanglement in Dicke states prepared in an experiment with few measurements.
- We also show how to verify the metrological usefulness of quantum states based on few measurements, without the need to carry out the metrological procedure itself.

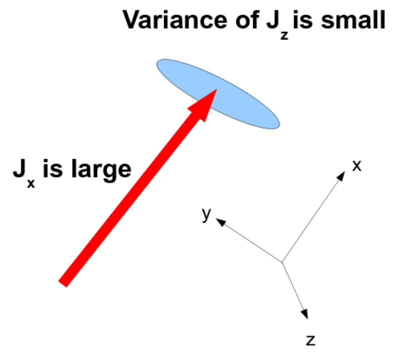
Spin-squeezed states

- Entanglement criterion [4]

$$\xi_s^2 = N\frac{(\Delta J_z)^2}{\langle J_x\rangle^2+\langle J_y\rangle^2}.$$

If $\xi_s^2<1$ then the state is entangled.

- States detected are of the following type:



Dicke states

- Dicke states are defined as

$$|D_N\rangle = \binom{N}{N/2}^{-\frac{1}{2}}\left(|0\rangle^{\otimes\frac{N}{2}}|1\rangle^{\otimes\frac{N}{2}}+\text{permutations}\right).$$

- Dicke states are robust to particle loss.
- Dicke states, in principle, make quantum metrology possible with a Heisenberg scaling.
- States with a high metrological usefulness possess macroscopic entanglement and in a sense, they are close to Schrödinger cats. Hence, Dicke states can be used to study experimentally macroscopic entanglement (F. Fröwis).
- Experiments
  - Photonic systems with four and six qubits [1,5]
  - Bose Einstein condensates, thousands of atoms [2,6]

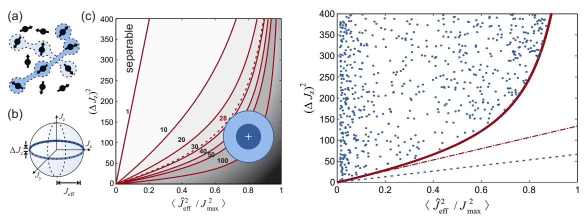
Entanglement depth

- Entanglement criterion for both Dicke states and spin-squeezed states [3,4].
- The inequality

$$(\Delta J_z)^2 \ge NjG_{kj}\left(\frac{\langle J_x^2+J_y^2\rangle - Nj(kj+1)}{N(N-k)j^2}\right)$$

holds for states with an entanglement depth of at most k of an ensemble of N spin-j particles. $G_J(X)$ is a function obtained numerically.

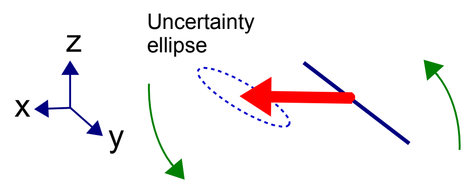
- If a state violates the above criterion then it has at least an entanglement depth $k+1$.



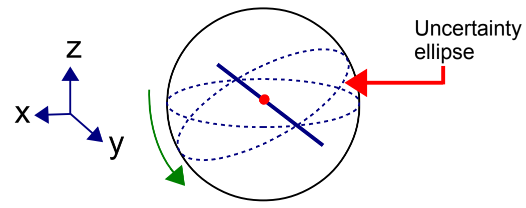
- Used in experiments (Klempt group [2], and [7]).

Quantum metrology

- Spin-squeezed states: Measure $\langle J_z\rangle$ to estimate the angle θ



- Dicke states: Measure $\langle J_z^2\rangle$ to estimate θ. (We cannot measure first moments, since they are zero.)



Quantum Fisher information

- Cramér-Rao bound on the precision of parameter estimation

$$(\Delta\theta)^2 \ge \frac{1}{F_Q[\rho,A]},$$

where $F_Q[\rho,A]$ is the quantum Fisher information (QFI) defined as

$$F_Q[\rho,A] = 2\sum_{k,l}\frac{(\lambda_k-\lambda_l)^2}{\lambda_k+\lambda_l}|\langle k|A|l\rangle|^2,$$

and $\rho=\sum_k\lambda_k|k\rangle\langle k|$.

- For separable states

$$F_Q[\rho,J_l]\le N.$$

- For states with at most k-particle entanglement

$$F_Q[\rho,J_l]\le kN.$$

Estimating the QFI

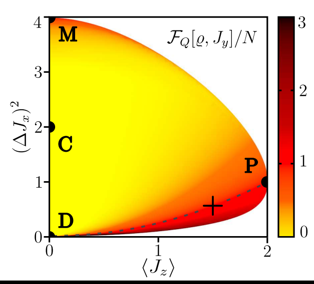
- Bound QFI from below based on $w_k=\langle W_k\rangle$.
- Using the Legendre transform technique, we arrive at the formula [3]

$$\mathcal{F}_Q[\rho,J_l]\ge \sup_{\{r_k\}}\left[\sum_k r_k w_k - \sup_\mu \lambda_{\max}(M)\right],$$

where

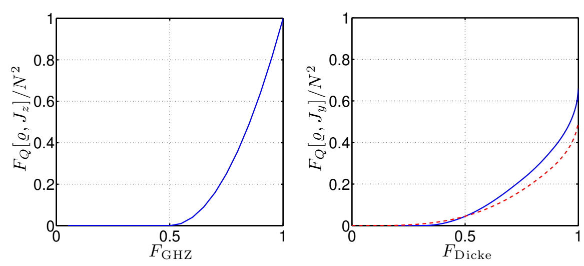
$$M=\sum_k r_k W_k - 4(J_l-\mu)^2.$$

- Our method works for systems with density matrices of size 1000×1000 or even larger.
- Bounding the QFI for spin squeezing:



Estimating the QFI II

- QFI vs. the fidelity



- Bounding the QFI for experiments (for references see [3])

Physical system	Targeted quantum state	Fidelity	$\frac{\mathcal{F}_Q}{N^2}\ge$	Ref.
photons	$\lvert D_4\rangle$	0.844 ± 0.008	0.358 ± 0.011	[31]
		0.78 ± 0.005	0.281 ± 0.059	[34]
		0.8872 ± 0.0055	0.420 ± 0.009	[14]
		0.873 ± 0.005	0.351 ± 0.006	[60]
	$\lvert D_6\rangle$	0.654 ± 0.024	0.141 ± 0.019	[32]
		0.56 ± 0.02	0.0761 ± 0.012	[33]
photons	$\lvert GHZ_4\rangle$	0.840 ± 0.007	0.462 ± 0.019	[25]
	$\lvert GHZ_5\rangle$	0.68	0.130	[61]
	$\lvert GHZ_8\rangle$	0.59 ± 0.02	0.032 ± 0.016	[62]
	$\lvert GHZ_8\rangle$	0.776 ± 0.006	0.3047 ± 0.0134	[27]
	$\lvert GHZ_{10}\rangle$	0.561 ± 0.019	0.015 ± 0.011	[27]
trapped ions	$\lvert GHZ_3\rangle$	0.89 ± 0.03	0.608 ± 0.097	[28]
	$\lvert GHZ_4\rangle$	0.57 ± 0.02	0.020 ± 0.013	[29]
	$\lvert GHZ_6\rangle$	$\ge0.509\pm0.004$	0.0003 ± 0.0003	[63]
	$\lvert GHZ_8\rangle$	0.817 ± 0.004	0.402 ± 0.010	[30]
	$\lvert GHZ_{10}\rangle$	0.626 ± 0.006	0.064 ± 0.006	[30]

Related bibliography

Papers with our contributions

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