Interesting quantum states (Lecture of the Quantum Information class of the Master in Quantum Science and Technology)

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Outline

1. **Interesting quantum states**
   - Motivation
   - A. Single particle states
   - B. Bipartite singlet state
   - C. Werner states
   - D. Isotropic states
   - E. Schrödinger cat states
   - F. Greenberger-Horne-Zeilinger (GHZ) state
   - G. Cluster states
   - H. W state
   - I. Symmetric Dicke states

Which quantum states are interesting?

- We have infinite possibilities to pick a quantum state in a multi-qubit system.
- We would like to find useful ones or states that have interesting symmetries.

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Single particle states

- Pure states. The von Neumann entropy $S = 0$.
- Completely mixed state

$$\varrho_{\mathrm{cm}} = \frac{1}{d} \sum_{k=1}^{d} |k\rangle\langle k|.$$

The von Neumann entropy $S = \log d$, maximal.

Outline

Bipartite singlet state

- The two-qubit singlet state looks like
$$|\Psi_{\text{singlet}}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$
- We get the same form after any basis transformation (if we transform the bases of the two qubits in the same way).This can be seen as follows. Let us choose two vectors as
$$\begin{aligned} |v\rangle &= \alpha|0\rangle + \beta|1\rangle, \\ |v_\perp\rangle &= \beta^*|0\rangle - \alpha^*|1\rangle. \end{aligned}$$
Clearly,
$$\langle v|v_\perp\rangle = 0,$$
Then, simple algebra yields
$$\frac{1}{\sqrt{2}}(|v\rangle \otimes |v_\perp\rangle - |v_\perp\rangle \otimes |v\rangle) = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$
This is true for any $|v\rangle$ and $|v_\perp\rangle$.

Bipartite singlet state II

- Due to the independence from the choice of the local basis, it is invariant under a transformation of the type $U \otimes U$, apart from a global phase ϕ.

  $$U \otimes U |\Psi_{\text{singlet}}\rangle = |\Psi_{\text{singlet}}\rangle \exp(-i\phi).$$

  We can also say that

  $$U \otimes U |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}| (U \otimes U)^{\dagger} = |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}|.$$

  Hence,

  $$U \otimes U |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}| = |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}| U \otimes U.$$

  Thus, the density matrices of such states will commute with all $U \otimes U$:

  $$[U \otimes U, |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}|] = 0$$

  for any U.

Bipartite singlet state III

- Let us consider some operators of the form

  $$\sigma_{\vec{n}} = \sum_{l=x,y,z} n_l \sigma_l$$

  where $|\vec{n}| = 1$. For $\vec{n} = (1,0,0)$, $\sigma_{\vec{n}} = \sigma_x$. For $\vec{n} = (0,1,0)$, $\sigma_{\vec{n}} = \sigma_y$, and in general it is a generalization of the Pauli spin matrices to an arbitrary direction.

- Such operators all have eigenvalues ± 1. If you measure $\sigma_{\vec{n}}$ on party A and get a result, then if you also measure it on party B, you will get the opposite result. This is true for every $\sigma_{\vec{n}}$.

- This can be used in quantum communication to establish a bit sequence that is known only by Alice and Bob and by nobody else.

Bipartite singlet state IV

- For the singlet state

$$|\Psi_{\mathrm{singlet}}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

  we have anticorrelations

$$\begin{array}{rcl} \langle\sigma_x \otimes \sigma_x\rangle & = & -1, \\ \langle\sigma_y \otimes \sigma_y\rangle & = & -1, \\ \langle\sigma_z \otimes \sigma_z\rangle & = & -1. \end{array}$$

- For the collective angular moment, we have

$$[\Delta(\sigma_x^{(1)} + \sigma_x^{(2))})]^2 = [\Delta(\sigma_y^{(1)} + \sigma_y^{(2))})]^2 = [\Delta(\sigma_z^{(1)} + \sigma_z^{(2))})]^2 = 0.$$

Bipartite singlet state V

- Why is it called a singlet? Remember the theory of angular momentum, triplet and singlet subspace.
- Alternatively, in quantum information, the maximally entangled state

  $$|\Psi_{\mathrm{me}}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

  can also be called singlet.
- A generalization for higher dimensions is the maximally entangled state

  $$|\Psi_{\mathrm{me}}\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^{d} |kk\rangle.$$

- For the maximally entangled state, the reduced state is the completely mixed state

  $$\mathrm{Tr}_A(|\Psi_{\mathrm{me}}\rangle\langle\Psi_{\mathrm{me}}|) = \mathrm{Tr}_B(|\Psi_{\mathrm{me}}\rangle\langle\Psi_{\mathrm{me}}|) = \frac{\mathbb{1}}{d}.$$

  Thus, if we have access only to one of the two subsystems, we know nothing.

Bipartite singlet state VI

- For the maximally entangled state

  $$|\Psi_{\mathrm{me}}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

  we have

  $$\begin{aligned} \langle\sigma_x \otimes \sigma_x\rangle &= +1, \\ \langle\sigma_y \otimes \sigma_y\rangle &= -1, \\ \langle\sigma_z \otimes \sigma_z\rangle &= +1. \end{aligned}$$

- For the collective angular moment, we have

  $$[\Delta(\sigma_x^{(1)} - \sigma_x^{(2))})]^2 = [\Delta(\sigma_y^{(1)} + \sigma_y^{(2))})]^2 = [\Delta(\sigma_z^{(1)} - \sigma_z^{(2))})]^2 = 0.$$

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Flip operator

- Flip operator

$$F|\Psi\rangle|\Phi\rangle = |\Phi\rangle|\Psi\rangle.$$

- For two-qubits

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

.

$$F = \frac{1}{2}(\mathbb{1} \otimes \mathbb{1} + \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z).$$

.

- Eigenvalues of F : -1,1,1,1 (-1 for antisymmetric states, +1 for symmetric states).

Flip operator II

- For a comparison, remember that for the projector to the singlet state we have

$$|\Psi_{\mathrm{singlet}}\rangle\langle\Psi_{\mathrm{singlet}}| = \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} - \sigma_x\otimes\sigma_x - \sigma_y\otimes\sigma_y - \sigma_z\otimes\sigma_z).$$

- For the projector to the maximally entangled state we have

$$|\Psi_{\mathrm{me}}\rangle\langle\Psi_{\mathrm{me}}| = \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \sigma_x\otimes\sigma_x - \sigma_y\otimes\sigma_y + \sigma_z\otimes\sigma_z).$$

Werner states

- **Definition.** Werner states are states that are invariant under a transformation of the type $U \otimes U$ (Werner, 1989).
- For qubits, Werner states are noisy singlets for two-qubits

$$\varrho_{\rm Werner}(p) = (1-p)|\Psi_{\rm singlet}\rangle\langle\Psi_{\rm singlet}| + p\frac{\mathbb{1}}{4}.$$

- For two qudits, their density matrix is defined as

$$\alpha\mathbb{1} + \beta F$$

  where F is the flip operator. For systems larger than qubits, we do not have a pure Werner state.
- Twirling

$$\tau(\varrho) = \int M(dU) U \otimes U \varrho (U \otimes U)^\dagger.$$

  Used to transform states into a normal form, for example, before distilling entanglement. Twirling leaves Werner states unchanged. It transforms all quantum states to Werner states.

Werner states II

- Multipartite Werner states are defined as states that are invariant under $U^{\otimes N}$.

- For $d = 3$, there are three-qudit Werner states that are pure. Such a state is the fermionic singlet

$$\Psi_{\mathrm{fs}} = \frac{1}{\sqrt{6}}(|123\rangle - |132\rangle + -...).$$

- There are entangled Werner states that do not violate any Bell inequality. (See Bell inequalities later.)

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Isotropic states

- Isotropic states are invariant under any transformation of the type $U \otimes U^*$.
- Isotropic states are defined as the maximally entangled state

$$|\Psi_{\mathrm{me}}\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^{d} |kk\rangle.$$

mixed with white noise.

- The maximally entangled state above is the pure isotropic state for any dimension.

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Schrödinger cat states

- Think on the usual the Schrödinger's cat experiment. The cat is in a superposition of being dead and alive.
- Questions about the linearity of quantum mechanics, etc. Can superpositions of macroscopically different objects exist?

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Greenberger-Horne-Zeilinger (GHZ) state

- A possible generalization of the maximally entangled state to N qubits is the GHZ state defined as

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|00...00\rangle + |11..11\rangle\right)$$

- The entanglement of the GHZ state is very fragile. Measuring one qubit destroys it, the state becomes separable:

$$\mathrm{Tr}_1(|\mathrm{GHZ}_N\rangle\langle\mathrm{GHZ}_N|) = \frac{1}{2}(|0\rangle\langle 0|^{\otimes(N-1)} + |1\rangle\langle 1|^{\otimes(N-1)}).$$

- Realized experimentally with trapped ions, superconducting circuits for ~ 100 qubits. It has also been realized in photons.

Greenberger-Horne-Zeilinger (GHZ) state II

- Interestingly, the GHZ state is an eigenstate of operators that are the products of single-qubit operators. For example, it is the eigenstate of

  $$\sigma_x \otimes \sigma_x \otimes ... \otimes \sigma_x = \sigma_x^{\otimes N}$$

  with an eigenvalue $+1$. What does this mean? If they flip all the qubits, we get back the original states.

- The GHZ state is also the eigenstate of the operators of the type

  $$\sigma_z^{(m)} \sigma_z^{(n)},$$

  for all $m \neq n$ with an eigenvalue $+1$.

Greenberger-Horne-Zeilinger (GHZ) state III

- If a state is eigenstate of operators O_1 and O_2, then it is also an eigenstate of $O_1 O_2$. Because of that the state is an eigenstate of the products of such operators.
- Note that all these operators commute with each other.
- For all operators, we have $O^2 = \mathbb{1}$.
- These operators form a *group* that is called *stabilizer.* The group has N generators and 2^N elements. Phys. Rev. A. 1996.
- (Definition of a discrete group: If A and B are in the group, so is their product AB.)
- Stabilizer theory is important for quantum error correction.

Greenberger-Horne-Zeilinger (GHZ) state IV

- Just an example: for three-qubit GHZ states, we have the following 8 operators

$$\begin{aligned}
\sigma_x \otimes \sigma_x \otimes \sigma_x,\\
\sigma_z \otimes \sigma_z \otimes \mathbb{1},\\
\mathbb{1} \otimes \sigma_z \otimes \sigma_z,\\
\sigma_z \otimes \mathbb{1} \otimes \sigma_z,\\
-\sigma_y \otimes \sigma_y \otimes \sigma_x,\\
-\sigma_x \otimes \sigma_y \otimes \sigma_y,\\
-\sigma_y \otimes \sigma_x \otimes \sigma_y,\\
\mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}.
\end{aligned}$$

Greenberger-Horne-Zeilinger (GHZ) state V

- The fidelity with respect to GHZ states:

$$\begin{aligned} F(\varrho, |\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|) &= \mathrm{Tr}(\varrho|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|) \\ &= \frac{1}{2}\mathrm{Tr}\left(\varrho\begin{pmatrix} 1 & 0 & ... & 0 & 1 \\ 0 & 0 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & 0 \\ 1 & 0 & ... & 0 & 1 \end{pmatrix}\right) \\ &= \frac{1}{2}(\varrho_{1,1} + \varrho_{2^N,1} + \varrho_{1,2^N} + \varrho_{2^N,2N}). \end{aligned}$$

- It is enough to know four elements of the density matrix.

Greenberger-Horne-Zeilinger (GHZ) state VI

- For any N, we can define the following operators.

$$\begin{aligned} S_1^{(\mathrm{GHZ})} &= \sigma_x^{(1)} \otimes \sigma_x^{(2)} \otimes ... \otimes \sigma_x^{(N)}, \\ S_k^{(\mathrm{GHZ})} &= \sigma_z^{(k)} \sigma_z^{(k+1)}, k = 2, .., N. \end{aligned}$$

They all commute with each other. The GHZ state is uniquely determined by

$$S_k |\mathrm{GHZ}\rangle = |\mathrm{GHZ}\rangle, k = 1, 2, ..., N.$$

- The projector can be written as

$$|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| = \frac{1 + S_1}{2} \frac{1 + S_2}{2} \cdots \frac{1 + S_N}{2}.$$

S_1: xxxx...xx measurement, S_k, $k \geq 2$: zzzz...zz measurement.

Greenberger-Horne-Zeilinger (GHZ) state VII

- Simple lower bound

$$|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| \geq \frac{1+S_1}{2} + \left(\frac{1+S_2}{2}\right)\left(\frac{1+S_3}{2}\right)\left(\frac{1+S_4}{2}\right)... - \mathbb{1}.$$

  It is easier to measure than the projector. Phys. Rev. Lett. 2005.

- The $S_k^{(\mathrm{GHZ})}$ operators commute with each other and their square is the identity $\mathbb{1}$.

- Hence, any element of the group generated by them can be obtained as

$$\left(S_1^{(\mathrm{GHZ})}\right)^{\alpha_1}\left(S_2^{(\mathrm{GHZ})}\right)^{\alpha_2}\left(S_3^{(\mathrm{GHZ})}\right)^{\alpha_3}...\left(S_N^{(\mathrm{GHZ})}\right)^{\alpha_N},$$

  where $\alpha_k = 0, 1$. There are 2^N elements of the group.

Application: Quantum error correction

- Classical error correction: we store 1 bit with odd number of bits (e.g., 3).
- If they are not the same, then majority vote matters. (This is the reason for the odd number. Even number of voters cannot always decide.)
- Quantum error correction: we store 1 qubit on several qubits.
- However, we must be careful. We cannot jut read out and correct. Reading out would destroy the quantum state.

Bit-flip code

- The bit flip code can correct a bit flip, as the name suggests. Thus, it helps to fight the error of the type

$$\epsilon(\varrho) = (1 - p)\varrho + p\sigma_x\varrho\sigma_x.$$

- It is based on using using redundancy:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle.$$

Thus, one qubit is now stored on three qubits.

Bit-flip code II

- The bit flip code can handle the case of 0 or 1 bit flip. Let us see in detail.
- First we need to detect which bit is flipped. This can be done by measuring $\langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle$ and $\langle\sigma_z^{(2)}\sigma_z^{(3)}\rangle$.

$$\begin{aligned}
&\langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle = +1, \quad \langle\sigma_z^{(2)}\sigma_z^{(3)}\rangle = +1 \rightarrow \text{No error,}\\
&\langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle = -1, \quad \langle\sigma_z^{(2)}\sigma_z^{(3)}\rangle = +1 \rightarrow \text{Error on qubit 1,}\\
&\langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle = +1, \quad \langle\sigma_z^{(2)}\sigma_z^{(3)}\rangle = -1 \rightarrow \text{Error on qubit 3,}\\
&\langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle = -1, \quad \langle\sigma_z^{(2)}\sigma_z^{(3)}\rangle = -1 \rightarrow \text{Error on qubit 2.}
\end{aligned}$$

- After detecting the error, we can correct it. We can just flip the qubit on which we found an error.
- We keep repeating these two steps to protect the qubit stored on three qubits.

Bit-flip code III

- Concrete example: let us assume a bit-flip error on the first qubit. Then, our state is
$$\alpha|100\rangle + \beta|011\rangle.$$
- For this state, we have
$$\langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle = -1, \quad \langle\sigma_z^{(2)}\sigma_z^{(3)}\rangle = +1.$$
- We can correct it by flipping the first qubit, i.e.,
$$\varrho \to \sigma_x^{(1)}\varrho\sigma_x^{(1)}.$$
- Then, at the end we will have
$$\alpha|000\rangle + \beta|111\rangle.$$

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Cluster states

- Cluster states are defined with the stabilizing operators

$$\begin{aligned} S_1^{(\mathrm{GHZ})} &= \sigma_x^{(1)} \otimes \sigma_z^{(2)}, \\ S_k^{(\mathrm{GHZ})} &= \sigma_z^{(k-1)} \sigma_x^{(k)} \sigma_z^{(k+1)}, k = 2, .., N-1, \\ S_N^{(\mathrm{GHZ})} &= \sigma_z^{(N-1)} \otimes \sigma_x^{(N)}. \end{aligned}$$

- It can be obtained with an Ising type interaction as

$$|\Psi\rangle = e^{-iH\frac{\pi}{4}} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes N,$$

where an Ising spin-chain Hamiltonian is given as

$$H = \sum_{n=1}^{N-1} (\mathbb{1} - \sigma_z^{(n)})(\mathbb{1} + \sigma_z^{(n+1)}).$$

Cluster states II

- Cluster states are defined with the stabilizing operators The projector can be obtained

$$|\mathrm{C}\rangle\langle\mathrm{C}| = \frac{1+S_1}{2}\frac{1+S_2}{2}\cdots\frac{1+S_N}{2}.$$

S_1, S_3, S_5: zxzxzx... measurement, S_2, S_4, S_6, xzxzxz ... measurement.

- Simple lower bound

$$|\mathrm{C}\rangle\langle\mathrm{C}| \geq \left(\frac{1+S_1}{2}\right)\left(\frac{1+S_3}{2}\right)\left(\frac{1+S_5}{2}\right)... + \left(\frac{1+S_2}{2}\right)\left(\frac{1+S_4}{2}\right)\left(\frac{1+S_6}{2}\right)... - \mathbb{1}.$$

It is easier to measure than the projector, it has been used >10 times in experiments.

Phys. Rev. Lett. 2005, Phys. Rev. Lett. 2005.

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W state

- It is defined as

$$\frac{1}{\sqrt{N}}\left(|100...\rangle + |010...\rangle + |001...\rangle + ... + |000..01\rangle\right).$$

  Interestingly, it maximizes the two-body concurrence among symmetric states.

- I we loose a single particle, we obtain

$$\mathrm{Tr}_1(|\mathrm{W}_N\rangle\langle\mathrm{W}_N|) = \frac{1}{N}|0\rangle\langle 0|^{\otimes(N-1)} + \frac{N-1}{N}|\mathrm{W}_{N-1}\rangle\langle\mathrm{W}_{N-1}|.$$

  The entanglement of the W state is robust. Loosing one qubit does not destroy the entanglement.

- Realized with trapped ions, photons, cold atoms, up to thousands of particles.

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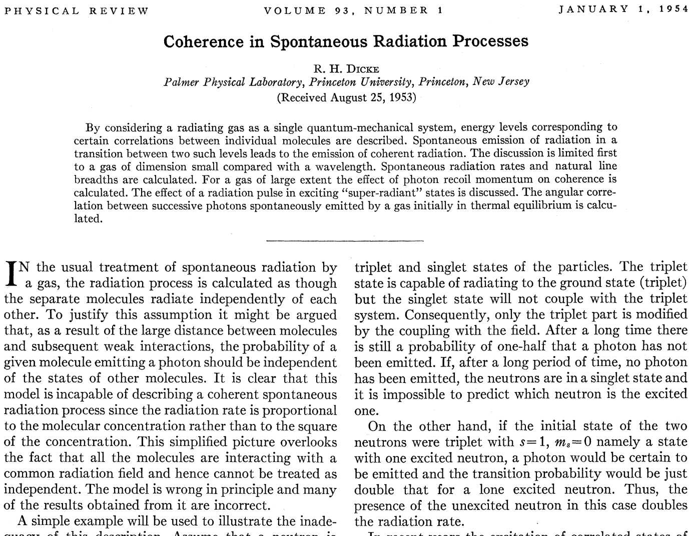
Symmetric Dicke states of qubits



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Coherence in Spontaneous Radiation Processes

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By considering a radiating gas as a single quantum-mechanical system, energy levels corresponding to certain correlations between individual molecules are described. Spontaneous emission of radiation in a transition between two such levels leads to the emission of coherent radiation. The discussion is limited first to a gas of dimension small compared with a wavelength. Spontaneous radiation rates and natural line breadths are calculated. For a gas of large extent the effect of photon recoil momentum on coherence is calculated. The effect of a radiation pulse in exciting "super-radiant" states is discussed. The angular correlation between successive photons spontaneously emitted by a gas initially in thermal equilibrium is calculated.

IN the usual treatment of spontaneous radiation by a gas, the radiation process is calculated as though the separate molecules radiate independently of each other. To justify this assumption it might be argued that, as a result of the large distance between molecules and subsequent weak interactions, the probability of a given molecule emitting a photon should be independent of the states of other molecules. It is clear that this model is incapable of describing a coherent spontaneous radiation process since the radiation rate is proportional to the molecular concentration rather than to the square of the concentration. This simplified picture overlooks the fact that all the molecules are interacting with a common radiation field and hence cannot be treated as independent. The model is wrong in principle and many of the results obtained from it are incorrect.

A simple example will be used to illustrate the inade-

triplet and singlet states of the particles. The triplet state is capable of radiating to the ground state (triplet) but the singlet state will not couple with the triplet system. Consequently, only the triplet part is modified by the coupling with the field. After a long time there is still a probability of one-half that a photon has not been emitted. If, after a long period of time, no photon has been emitted, the neutrons are in a singlet state and it is impossible to predict which neutron is the excited one.

On the other hand, if the initial state of the two neutrons were triplet with $s=1$, $m_s=0$ namely a state with one excited neutron, a photon would be certain to be emitted and the transition probability would be just double that for a lone excited neutron. Thus, the presence of the unexcited neutron in this case doubles the radiation rate.

Symmetric Dicke states of qubits II

- Dicke states are simultaneous eigenstates of $\vec{J}^2 = J_x^2 + J_y^2 + J_z^2$ and J_z

  $$\begin{aligned} \vec{J}^2 |j, j_z, \alpha\rangle &= j(j+1)|j, j_z, \alpha\rangle, \\ J_z |j, j_z, \alpha\rangle &= j_z |j, j_z, \alpha\rangle, \end{aligned}$$

  where α is a label needed for degeneracies.
- For qubits, for $j = N/2$ (maximal) all Dicke states are **symmetric.** $\vec{J}^2$ is also maximal. $j_z = -N/2, -N/2+1, ..., N/2$.
- In this case, the α label is not needed. There is only a single state for a given j_z and $j = N/2$
- Symmetric Dicke states of qubits are the equal superpositions of the permutations of a series of 0's and 1's

  $$|D_N^{(m)}\rangle \propto \sum_k \mathcal{P}_k(|0\rangle^{\otimes(N-m)} \otimes |1\rangle^{\otimes(m)}),$$

  where the sum is over all distinct permutations of 0's and 1's.

Symmetric Dicke states of qubits III

- For $|0\rangle^{\otimes N}$, we have $\langle J_z \rangle = +N/2$.
- For $|1\rangle^{\otimes N}$, we have $\langle J_z \rangle = -N/2$.
- If half of the particles are 1, half of them are 0, we have $m = N/2$. In this case, $\langle J_z \rangle = 0$.
- For example, for $N = 4$ the symmetric Dicke state is

$$|D_4^{(2)}\rangle = \frac{1}{\sqrt{6}}(|0011\rangle + |0101\rangle + |1001\rangle + |0110\rangle + |1010\rangle + |1100\rangle).$$

- Relation to W states: $|D_N^{(1)}\rangle = |W_N\rangle$.
- Realized with trapped ions, photons, cold gases, up to thousands of particles.