Quantum states with a positive partial transpose are useful for metrology: Numerical and analytical examples

G. Tóth and T. Vértesi,
Phys. Rev. Lett. 120, 020506 (2018).

K. F. Pál, G. Tóth, E. Bene, and T. Vértesi,
Phys. Rev. Res. 3, 023101 (2021).

Presented by Géza Tóth

Kwek Leong Chuan's group, Center for Quantum Technologies,
Singapore, 9 November 2021 (online)

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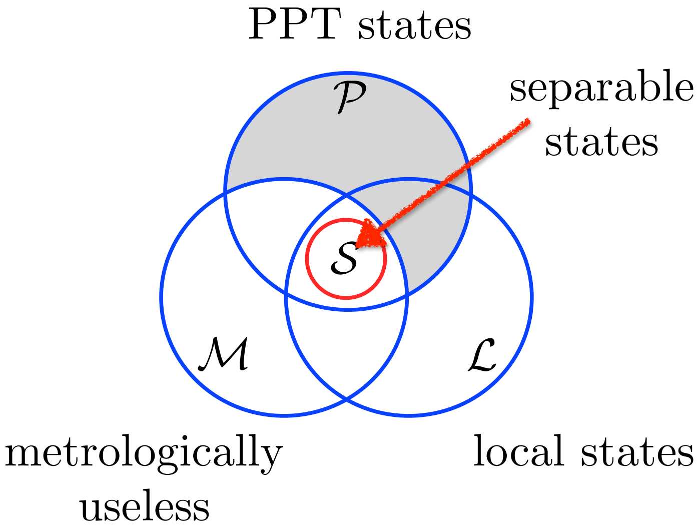
Outline

1. **Motivation**
   - What are entangled states useful for?
2. **Bacground**
   - Quantum Fisher information
   - Recent findings on the quantum Fisher information
3. **Maximizing the QFI for PPT states**
   - Results so far
   - Our results
4. **Analytical examples**
   - Properties of the two families
   - First family of PPT states
   - Second family of PPT states
   - PPT singlet-like states
   - QUBIT4MATLAB programs

What are entangled states useful for?

- Entangled states are useful, but not all of them are useful for some task.
- Entanglement is needed for beating the shot-noise limit in quantum metrology.
- Intriguing question: Are states with a positive partial transpose useful for metrology? Can they also beat the shot-noise limit?

What are entangled states useful for?



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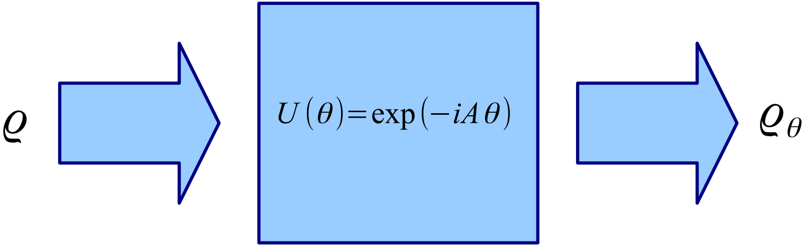
Quantum metrology

- Fundamental task in metrology



- We have to estimate θ in the dynamics

$$U = \exp(-iA\theta).$$

The quantum Fisher information

- Cramér-Rao bound on the precision of parameter estimation

$$(\Delta\theta)^2 \geq \frac{1}{mF_Q[\varrho, A]},$$

where where m is the number of independent repetitions and $F_Q[\varrho, A]$ is the quantum Fisher information.

- The quantum Fisher information is

$$F_Q[\varrho, A] = 2\sum_{k,l} \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} |\langle k|A|l\rangle|^2,$$

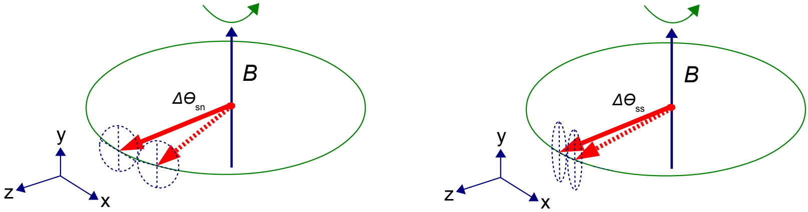
where $\varrho = \sum_k \lambda_k |k\rangle\langle k|$.

Special case $A = J_l$

- The operator A is defined as

$$A = J_l = \sum_{n=1}^{N} j_l^{(n)}, \quad l \in \{x, y, z\}.$$

- Magnetometry with a linear interferometer



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The quantum Fisher information vs. entanglement

- For separable states

$$F_Q[\varrho, J_l] \leq N, \qquad l = x, y, z.$$

Pezze, Smerzi, Phys. Rev. Lett. 102, 100401 (2009); Hyllus, Gühne, Smerzi, Phys. Rev. A 82, 012337 (2010)

- For states with at most k-particle entanglement (k is divisor of N)

$$F_Q[\varrho, J_l] \leq kN.$$

P. Hyllus *et al.*, Phys. Rev. A 85, 022321 (2012); GT, Phys. Rev. A 85, 022322 (2012).

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Results so far concerning metrologically useful PPT states

- Bound entangled states with PPT and some non-PPT partitions.
- Violates an entanglement criterion with three QFI terms.

P. Hyllus, W. Laskowski, R. Krischek, C. Schwemmer, W. Wieczorek, H. Weinfurter, L. Pezze, and A. Smerzi, PRA 85, 022321 (2012).

- Bound entangled states with PPT and some non-PPT partitions.
- Violates the criterion with a single QFI term, better than shot-noise limit.

Ł. Czekaj, A. Przysiężna, M. Horodecki, P. Horodecki, Phys. Rev. A 92, 062303 (2015).

> on nonlocality [43]) to answer would be, Is there any family of quantum states that allows for a general Local Hidden Variables (LHV) model but can be used to obtain sub-shot-noise (i.e., better than classical) quantum metrology? This question is related to another question (especially in the context of both general requirements in quantum metrology [26] and recent results on nonlocality [43]) regarding whether there is any chance for sub-shot-noise metrology for states obeying the PPT condition with respect to *any* cut. While the present result

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Our results

- We look for bipartite PPT entangled states and multipartite states that are PPT with respect to all partitions.

G. Tóth and T, Vértesi, Phys. Rev. Lett. 120, 020506 (2018).

Maximizing the QFI for PPT states: brute force

- Maximize the QFI for PPT states. Remember

$$F_Q[\varrho, A] = 2\sum_{k,l} \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} |\langle k|A|l\rangle|^2,$$

  where $\varrho = \sum_k \lambda_k |k\rangle\langle k|$.

- Difficult to maximize a convex function over a convex set. The maximum is taken on the boundary of the set.

- Not guaranteed to find the global maximum.

- Note: Finding the *minimum* is possible!

Maximizing the QFI for PPT state: our method

- Let us consider the error propagation formula

$$(\Delta\theta)^2{}_M = \frac{(\Delta M)^2}{\langle i[M,\mathcal{H}]\rangle^2},$$

which provides a bound on the quantum Fisher information

$$F_Q[\varrho,\mathcal{H}] \geq 1/(\Delta\theta)^2{}_M.$$

[M. Hotta and M. Ozawa, Phys. Rev. A 2004; B. M. Escher, arXiv:1212.2533; F. Fröwis, R. Schmied, and N. Gisin, Phys. Rev. A 2015. For a summary, see, e.g., the Supplemental Material of Tóth, Vértesi, Horodecki, Horodecki, PRL 2020.]

Maximizing the QFI for PPT state: our method

- The bound is sharp

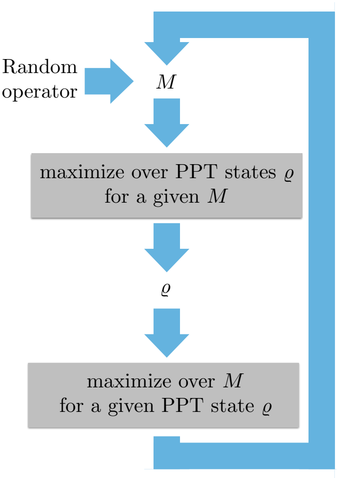
$$F_Q[\varrho, A] = \max_M \frac{\langle i[M, A] \rangle_\varrho^2}{(\Delta M)^2}.$$

M. G. Paris, Int. J. Quantum Inform. 2009. Used, e.g., in F. Fröwis, R. Schmied, and N. Gisin, 2015; I. Appelaniz *et al.*, NJP 2015.

The maximum for PPT states can be obtained as

$$\max_{\varrho \text{ is PPT}} F_Q[\varrho, A] = \max_{\varrho \text{ is PPT}} \max_M \frac{\langle i[M, A] \rangle_\varrho^2}{(\Delta M)^2}.$$

Sew-saw algorithm for maximizing the precision



Similar iterative approach was used for maximzing over ϱ for noisy states: Macieszczak, arXiv:1312.1356v1; Macieszczak, Fraas, Demkowicz- Dobrzanski, NJP 2014.

Maximize over PPT states for a given M

- Best precision for PPT states for a given operator M can be obtained by a semidefinite program.

- *Proof.*—Let us define first

$$\begin{aligned} f_M(X,Y) = \min_{\varrho} \quad & \mathrm{Tr}(M^2\varrho), \\ \text{s.t.} \quad & \varrho \geq 0, \varrho^{\mathrm{T}k} \geq 0 \text{ for all } k, \mathrm{Tr}(\varrho) = 1, \\ & \langle i[M,A]\rangle = X \text{ and } \langle M\rangle = Y. \end{aligned}$$

The best precsion for a given M and for PPT states is

$$(\Delta\theta)^2 = \min_{X,Y} \frac{f_M(X,Y) - Y^2}{X^2}.$$

Maximize over M for a given PPT state

- For a state ϱ, the best precision is obtained with the operator given by the symmetric logarithmic derivative

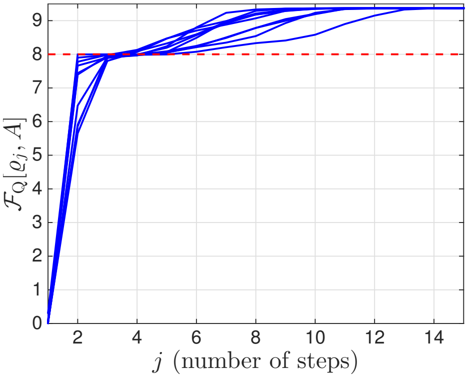
$$M = 2i \sum_{k,l} \frac{\lambda_k - \lambda_l}{\lambda_k + \lambda_l} |k\rangle\langle l| \langle k|A|l\rangle,$$

where $\varrho = \sum_k \lambda_k |k\rangle\langle k|$.

Convergence of the method

- The precision cannot get worse with the iteration!

Convergence of the method II



Generation of the 4×4 bound entangled state.

(blue) 10 attempts. After 15 steps, the algorithm converged.

(red) Maximal quantum Fisher information for separable states.

Robustness of the states

$$\varrho(p) = (1-p)\varrho + p\varrho_{\rm noise}$$

- Robustness of entanglement: the maximal p for which $\varrho(p)$ is entangled for any separable $\varrho_{\rm noise}$.

  Vidal and Tarrach, PRA 59, 141 (1999).

- Robustness of metrological usefulness: the maximal p for which $\varrho(p)$ outperforms separable state for any separable $\varrho_{\rm noise}$.

Robustness of the states II

System	A	$\mathcal{F}_{\mathrm{Q}}[\varrho, A]$	$\mathcal{F}_{\mathrm{Q}}^{(\mathrm{sep})}$	$p_{\mathrm{white\,noise}}$
four qubits	J_z	4.0088	4	0.0011
three qubits	$j_z^{(1)} + j_z^{(2)}$	2.0021	2	0.0005
2×4 (three qubits, only $1:23$ is PPT)	$j_z^{(1)} + j_z^{(2)}$	2.0033	2	0.0008

Multiqubit states

Robustness of the states III

d	$\mathcal{F}_Q[\varrho, A]$	$p_{\text{white noise}}$	$p_{\text{noise}}^{\text{LB}}$
3	8.0085	0.0006	0.0003
4	9.3726	0.0817	0.0382
5	9.3764	0.0960	0.0361
6	10.1436	0.1236	0.0560
7	10.1455	0.1377	0.0086
8	10.6667	0.1504	0.0670
9	10.6675	0.1631	0.0367
10	11.0557	0.1695	0.0747
11	11.0563	0.1807	0.0065
12	11.3616	0.1840	0.0808

- $d \times d$ systems.
- Maximum of the quantum Fisher information for separable states is 8.
- The operator A is not the usual J_z.

Robustness of the states IV

- The QFI is 11.3616 for a 12×12 system.
- Thus, it seems to approach the maximum value, 16, but via numerical calculation we cannot say more.

Robustness of the states V: 4×4 bound entangled PPT state

- Let us define the following six states
  $|\Psi_1\rangle = (|0,1\rangle + |2,3\rangle)/\sqrt{2}$, $|\Psi_2\rangle = (|1,0\rangle + |3,2\rangle)/\sqrt{2}$,
  $|\Psi_3\rangle = (|1,1\rangle + |2,2\rangle)/\sqrt{2}$, $|\Psi_4\rangle = (|0,0\rangle - |3,3\rangle)/\sqrt{2}$,
  $|\Psi_5\rangle = (1/2)(|0,3\rangle + |1,2\rangle) + |2,1\rangle/\sqrt{2}$,
  $|\Psi_6\rangle = (1/2)(-|0,3\rangle + |1,2\rangle) + |3,0\rangle/\sqrt{2}$.

- Our state is a mixture

$$\varrho_{4\times 4} = p\sum_{n=1}^{4} |\Psi_n\rangle\langle\Psi_n| + q\sum_{n=5}^{6} |\Psi_n\rangle\langle\Psi_n|,$$

  where $q = (\sqrt{2}-1)/2$ and $p = (1-2q)/4$.

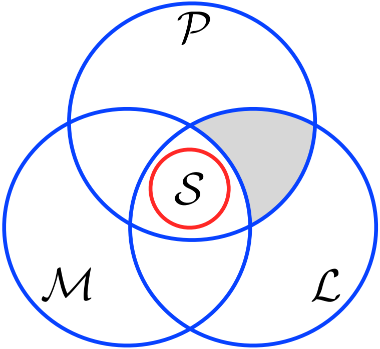
- We consider the operator

$$A = H \otimes \mathbb{1} + \mathbb{1} \otimes H,$$

  where $H = \mathrm{diag}(1, 1, -1, -1)$.

Metrologically useful quantum states with LHV models (PPT)

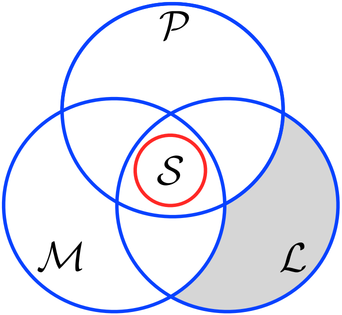
- Consider the 2×4 state listed before. Possible to construct numerically a LHV model for the state.



F. Hirsch, M. T. Quintino, T. Vértesi, M. F. Pusey, and N. Brunner, PRL 2016;
D. Cavalcanti, L. Guerini, R. Rabelo, and P. Skrzypczyk, PRL 2016.

Metrologically useful quantum states with LHV models (non-PPT)

- Two-qubit Werner state $(1-p)|\Psi^-\rangle\langle\Psi^-| + p\mathbb{1}/4$, with $|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$.
- Better for metrology than separable states ($\mathcal{F}_\mathrm{Q} > 2$) for $p > 1 - 0.3596 = 0.6404$.
- They do not violate a Bell inequality for $p < 0.6829$.



F. Hirsch, M. T. Quintino, T. Vértesi, M. Navascués, N. Brunner, Quantum 2017; A. Acín, N. Gisin, B. Toner, PRA 2006.

Cluster states

- Cluster states: resource in measurement-based quantum computing

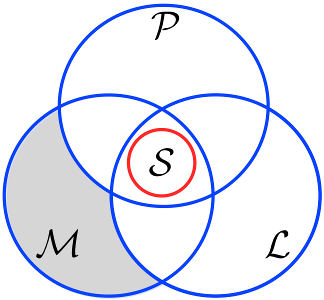
  R. Raussendorf and H. J. Briegel, PRL 2001.

- Fully entangled pure states.
- Violate a Bell inequality

  V. Scarani, A. Acín, E. Schenck, M. Aspelmeyer, PRA 2005; O. Gühne, GT, P. Hyllus, H. J. Briegel, PRL 2005; GT, O. Gühne, and H. J. Briegel, PRA 2006.

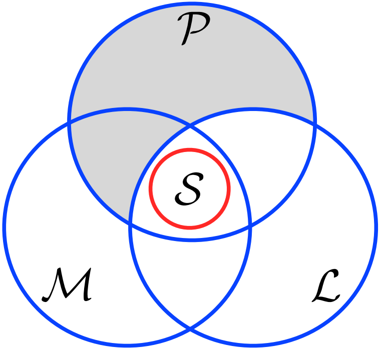
- Ring cluster states for $N \geq 5$ are metrologically not useful

  P. Hyllus, O. Gühne, and A. Smerzi, PRA 2010.



Non-local PPT states

Counterexample for the Peres conjecture



T. Vértesi and N. Brunner, Nature Communications 2015.

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Metrological performance

- **Observation 1.**—We present two families of PPT states. For both families of states,

$$\mathcal{F}_Q[\varrho_{\mathrm{F}n}, H] = \frac{16\sqrt{d}}{1+\sqrt{d}},$$

holds. The Hamiltonian corresponding to the $(AA')(BB')$ partition is

$$H = \sigma_A^z \otimes \mathbb{1}_B \otimes \mathbb{1}_{A'B'} + \mathbb{1}_A \otimes \sigma_B^z \otimes \mathbb{1}_{A'B'},$$

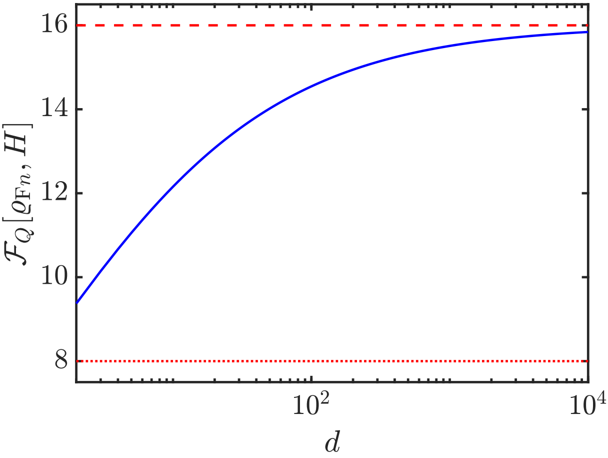
where the dimension of A' and B' is d.

- The quantum Fisher information approaches 16 for large d, which is the maximum achievable value by entangled states.

- Thus, PPT states turn out to be almost as useful as non-PPT entangled states in this metrological task.

K. F. Pál, G. Tóth, E. Bene, and T. Vértesi, Bound entangled singlet-like states for quantum metrology, Phys. Rev. Res. 3, 023101 (2021).

Metrological performance II



- (dashed) Maximum for the QFI for bipartite quantum states.
- (solid) The QFI of the $(2d) \times (2d)$ PPT quantum state.
- (dotted) Maximum for the QFI for separable quantum states.

QFI vs. variance

- **Observation 2.**—For both families of states,

$$\mathcal{F}_Q[\varrho_{\mathrm{F}n}, H] = 4(\Delta H)^2_{\varrho_{\mathrm{F}n}}$$

holds.

- For the expectation value of the Hamiltonian

$$\langle H \rangle_{\varrho_{\mathrm{F}n}} = 0$$

holds.

K. F. Pál, G. Tóth, E. Bene, and T. Vértesi, Bound entangled singlet-like states for quantum metrology, Phys. Rev. Res. 3, 023101 (2021).

The effect of noise on the QFI

- **Observation 3.—**If we mix the quantum state $\varrho_{\mathrm{F}n}$ with white noise,

$$\varrho_{\mathrm{F}n}^{(p)} = p\varrho_{\mathrm{F}n} + (1-p)\frac{\mathbb{1}}{4d^2},$$

  the quantum Fisher information is given as

$$\mathcal{F}_Q[\varrho_{\mathrm{F}n}^{(p)}, H] = \frac{2p_1p^2}{(2p_1-1)p+1}\mathcal{F}_Q[\varrho_{\mathrm{F}n}, H],$$

  where $\mathcal{F}_Q[\varrho_{\mathrm{F}n}, H]$ we discussed before.

- The constant p_1 is given as

$$p_1 = \sqrt{d}/(1+\sqrt{d}).$$

K. F. Pál, G. Tóth, E. Bene, and T. Vértesi, Bound entangled singlet-like states for quantum metrology, Phys. Rev. Res. 3, 023101 (2021).

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First family of PPT states

- **Definition 1.**—In matrix notation, the states $\varrho_{\rm F1}$ can be written as

$$\varrho_{\rm F1} = \frac{1}{2}\begin{bmatrix} p_1\sqrt{XX^\dagger} & 0 & 0 & p_1 X \\ 0 & p_2\sqrt{YY^\dagger} & p_2 Y & 0 \\ 0 & p_2 Y^\dagger & p_2\sqrt{Y^\dagger Y} & 0 \\ p_1 X^\dagger & 0 & 0 & p_1\sqrt{X^\dagger X} \end{bmatrix},$$

  where the two matrices with a unit trace norm acting on $A'B'$ are defined as

$$X = \frac{1}{d\sqrt{d}}\sum_{i,j=0}^{d-1} u_{ij}|ij\rangle\langle ji|,$$

$$Y = \sqrt{d}X^\Gamma = \frac{1}{d}\sum_{i,j=0}^{d-1} u_{ij}|ii\rangle\langle jj|.$$

- Here Γ denotes partial transposition in terms of Bob.

First family of PPT states II

- The density matrix of the ϱ_{F1} state can also be expressed as

$$\begin{aligned}
\varrho_{\mathrm{F1}} = & \frac{p_1}{2d^2} \sum_{i,j=0}^{d-1} \left(|00ij\rangle\langle 00ij| + |11ij\rangle\langle 11ij| \right) \\
& + \frac{p_1}{2d\sqrt{d}} \sum_{i,j=0}^{d-1} \left(u_{ij} |00ij\rangle\langle 11ji| + u_{ij}^{*} |11ji\rangle\langle 00ij| \right) \\
& + \frac{p_2}{2d} \sum_{i=0}^{d-1} \left(|01ii\rangle\langle 01ii| + |10ii\rangle\langle 10ii| \right) \\
& + \frac{p_2}{2d} \sum_{i,j=0}^{d-1} \left(u_{ij} |01ii\rangle\langle 10jj| + u_{ij}^{*} |10jj\rangle\langle 01ii| \right) .
\end{aligned}$$

- The order of subsystems is $ABA'B'$.

First family of PPT states III

- The p_1 probability is

$$p_1 = \sqrt{d}/(1+\sqrt{d}),$$

we define also $p_2 = 1 - p_1$.

- u_{ij} are matrix elements of a unitary operator acting on a d-dimensional space fulfilling

$$|u_{ij}| = 1/\sqrt{d}$$

for all i, j. Such an operator exists for all d, and the one corresponding to the quantum Fourier transform is appropriate

$$u_{ij} = \frac{1}{\sqrt{d}} e^{\mathbf{i}\frac{2\pi}{d}ij}.$$

- Important property

$$\varrho_{\mathrm{F1}} = (\varrho_{\mathrm{F1}})^{\Gamma}, \qquad \mathrm{rank}(\varrho_{\mathrm{F1}}) = d^2 + d.$$

P. Badziag, K. Horodecki, M. Horodecki, J. Jenkinson, and S. J. Szarek, Bound entangled states with extremal properties, Phys. Rev. A 90, 012301 (2014).

First family of PPT states IV

- **Observation 4.**—For the state $\varrho_{\rm F1}$, for the term in the formula of the quantum Fisher information, we have

$$\langle\mu|H|\nu\rangle = \begin{cases} 2, & \text{if } |\mu\rangle = |v_{ij}\rangle \text{ and } |\nu\rangle = |v_{ij}^{-}\rangle \\ & \text{or } |\nu\rangle = |v_{ij}\rangle \text{ and } |\mu\rangle = |v_{ij}^{-}\rangle, \\ 0, & \text{otherwise}, \end{cases}$$

  where $0 \leq i, j \leq d-1$.

- Here $|\mu\rangle$ and $|\nu\rangle$ denote the eigenvectors of $\varrho_{\rm F1}$ listed before. They include all $|v_{ij}\rangle$'s and all $|v_{ij}^{-}\rangle$'s.

- For $|v_{ij}\rangle$ and $|v_{ij}^{-}\rangle$, see

  K. F. Pál, G. Tóth, E. Bene, and T. Vértesi,
  Bound entangled singlet-like states for quantum metrology, Phys. Rev. Res. 3, 023101 (2021).

Relations to our 2018 PRL

- This is the same Hamiltonian operator that appears in [G. Tóth and T. Vértesi, PRL 2018] for two-qudit states.

- The 4×4 analytical state presented in [G. Tóth and T. Vértesi, PRL 2018] can be transformed to $\varrho_{\rm F1}$.

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Second family of PPT states

- **Definition 2.**—The family of states can be written as

$$\varrho_{\mathrm{F2}} = \frac{p_1}{d^2} \sum_{i,j=0}^{d-1} |z_{ij}\rangle\langle z_{ij}| + \frac{p_2}{2d} \sum_{i=0}^{d-1} |s_i\rangle\langle s_i| + \frac{p_2}{2d} \sum_{i=0}^{d-1} |10ii\rangle\langle 10ii|.$$

- The probabilities p_1 and p_2 are the same as before, and

$$|z_{ij}\rangle = \frac{1}{\sqrt{2}} \left(|00ij\rangle + \sum_{k=0}^{d-1} Q^j_{ik} |11jk\rangle \right)$$

for $0 \leq i, j \leq d-1$, where Q^j_{ik} are orthogonal matrices for all values of j, that is,

$$\sum_i Q^j_{ik} Q^j_{ik'} = \delta_{kk'}$$

holds for all j. Q^j_{ik} also have further properties.

Second family of PPT states II

- The states $|s_i\rangle$ are orthonormal vectors in the subspace

$$|01\rangle_{AB} \otimes \mathcal{H}_{A'} \otimes \mathcal{H}_{B'},$$

which will also be specified later in terms of Q_{ik}^j.

- With an appropriate choice of the Q_{ik}^j the partial transpose of ϱ is positive semidefinite.

- Important property

$$\varrho_{\rm F1} \neq (\varrho_{\rm F1})^\Gamma, \quad \text{rank}(\varrho_{\rm F1}) = d2 + 2d.$$

K. F. Pál, G. Tóth, E. Bene, and T. Vértesi,
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Second family of PPT states II

- **Observation 5.—**For the state $\varrho_{\rm F2}$, for the term in the formula of the quantum Fisher information, we have

$$\langle\mu|H|\nu\rangle = \begin{cases} 2, & \text{if } |\mu\rangle = |z_{ij}\rangle \text{ and } |\nu\rangle = |z_{ij}^-\rangle \\ & \text{or } |\nu\rangle = |z_{ij}\rangle \text{ and } |\mu\rangle = |z_{ij}^-\rangle, \\ 0, & \text{otherwise,} \end{cases}$$

  where $0 \leq i, j \leq d-1$.

- Here $|\mu\rangle$ and $|\nu\rangle$ denote the eigenvectors of $\varrho_{\rm F2}$.

Relations to our 2018 PRL

- The numerically found states presented in [G. Tóth and T. Vértesi, PRL 2018] are like ϱ_{F2}.

K. F. Pál, G. Tóth, E. Bene, and T. Vértesi,
Bound entangled singlet-like states for quantum metrology, Phys. Rev. Res. 3, 023101 (2021).

Outline

1. Motivation
   - What are entangled states useful for?
2. Bacground
   - Quantum Fisher information
   - Recent findings on the quantum Fisher information
3. Maximizing the QFI for PPT states
   - Results so far
   - Our results
4. **Analytical examples**
   - Properties of the two families
   - First family of PPT states
   - Second family of PPT states
   - PPT singlet-like states
   - QUBIT4MATLAB programs

Strong evidence that we found the best PPT state for metrology

- Based on extensive numerical maximization, it looks like that our states have the best metrological performance for bipartite states with a given d.
- For large d, the QFI equals the maximum, corresponding to a two-qubit singlet.

PPT singlet-like states

- Starting from a PPT state, LOCC will lead to PPT states only.
- If we have only PPT states, we can still try to distill the PPT state best for metrology.
- We could find concrete examples where using F as a local filter

$$\varrho' = \frac{(F \otimes F)\varrho_{\mathrm{noisy}}(F \otimes F)^\dagger}{\mathrm{Tr}[(F \otimes F)\varrho(F \otimes F)^\dagger]},$$

we could increase the QFI.

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QUBIT4MATLAB programs

- The routine `BES_private.m` defines the states of the first family For the u_{ij} unitaries, the quantum Fourier transform is used.

- The routine `BES_metro4x4.m` defines the state presented in PRL 2018.

- The routine `BES_metro.m` defines the states of the second family.

- We also included other routines that show their usage. They are called `example_BES_private.m`, `example_BES_metro4x4.m`, and `example_BES_metro.m`.

- The programs `BES_private.m` and `BES_metro.m` can give the states corresponding to the order of the subsystems given as $ABA'B'$, as in this paper.

- The programs can also give the states corresponding to the order of the subsystems given as $AA'BB'$, which is more appropriate for studying bipartite entanglement between AA' and BB'.

Summary

- We presented quantum states with a positive partial transpose with respect to all bipartitions that are useful for metrology.

G. Tóth and T. Vértesi,

Quantum states with a positive partial transpose
are useful for metrology,

Phys. Rev. Lett. 120, 020506 (2018).

K. F. Pál, G. Tóth, E. Bene, and T. Vértesi,

Bound entangled singlet-like states for quantum metrology,

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