

# Iterative optimization in quantum metrology and entanglement theory using semidefinite programming

Géza Tóth<sup>1,2,3,4,5</sup>,

<sup>1</sup>Theoretical Physics and <sup>2</sup>EHU Quantum Center,

University of the Basque Country (UPV/EHU), Bilbao, Spain

<sup>3</sup>Donostia International Physics Center (DIPC), San Sebastián, Spain

<sup>4</sup>Wigner Research Centre for Physics, Budapest, Hungary

<sup>5</sup>IKERBASQUE, Basque Foundation for Science, Bilbao, Spain

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Wien,

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## 1 Motivation

- What are entangled states useful for?

## 2 Metrological gain and the optimal local Hamiltonian

- Metrological usefulness of a quantum state
- Example for activation in small systems
- Activation in the many-particle case

## 3 Generalization of the ideas

- Computational details for bipartite systems
- Alternative method for finding the optimal Hamiltonian
- Upper bounds instead of lower bounds
- Wigner-Yanase skew information
- The Computable Cross Norm-Realignment (CCNR) criterion

# What are entangled states useful for?

- Entanglement is needed for beating the shot-noise limit in quantum metrology.
- However, not all entangled states are more useful than separable states.
- Intriguing questions:
  - Can we decide which quantum state is more useful than separable states?
  - Can we activate the metrological usefulness of quantum states, if we use several copies?

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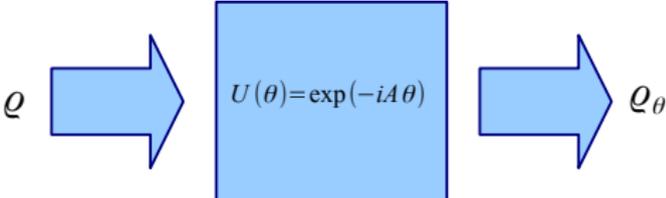
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# The quantum Fisher information

- Cramér-Rao bound on the precision of parameter estimation

$$(\Delta\theta)^2 \geq \frac{1}{mF_Q[\varrho, A]},$$


The diagram illustrates the process of parameter estimation. It starts with an input state  $\varrho$  on the left. A blue arrow points to a central blue box containing the unitary operator  $U(\theta) = \exp(-iA\theta)$ . A second blue arrow points from the box to the output state  $\varrho_\theta$  on the right.

where where  $m$  is the number of independent repetitions and  $F_Q[\varrho, A]$  is the **quantum Fisher information**.

- The quantum Fisher information is

$$F_Q[\varrho, A] = 2 \sum_{k,l} \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} |\langle k|A|l \rangle|^2,$$

where  $\varrho = \sum_k \lambda_k |k\rangle\langle k|$ .

# The quantum Fisher information vs. entanglement

- **Local Hamiltonians** for linear interferometers

$$J_l = \sum_{n=1}^N j_l^{(n)}$$

for  $l = x, y, z$ .

- For separable states of  $N$  **qubits**

$$F_Q[\varrho, J_l] \leq N, \quad l = x, y, z.$$

L. Pezze, A. Smerzi, Phys. Rev. Lett. 102, 100401 (2009);

P. Hyllus, O. Gühne, A. Smerzi, Phys. Rev. A 82, 012337 (2010)

- For states with at most  $k$ -particle entanglement (tight bound if  $k$  is a divisor of  $N$ )

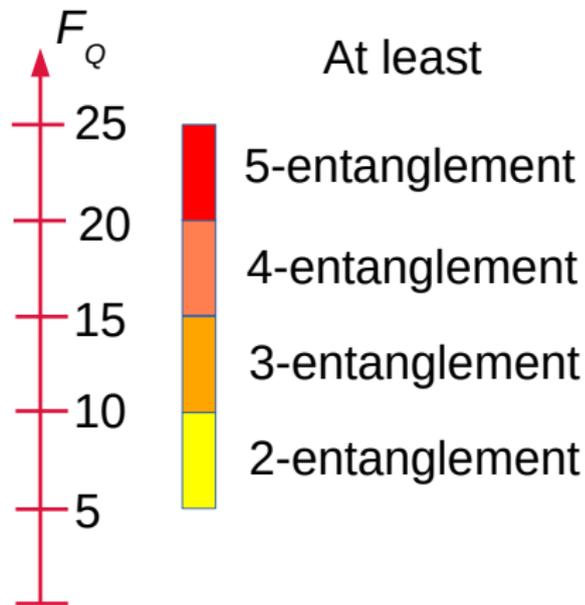
$$F_Q[\varrho, J_l] \leq kN.$$

P. Hyllus *et al.*, Phys. Rev. A 85, 022321 (2012);

GT, Phys. Rev. A 85, 022322 (2012).

# The quantum Fisher information vs. entanglement

5 spin-1/2 particles



(For simplicity, we used  $F_Q[\varrho, J_I] \leq kN$ , which is not tight.)

# The quantum Fisher information vs. entanglement

- Let us consider the fraction of the QFI and the maximum QFI for separable states for  $N$  qubits

$$\frac{F_Q[\varrho, H]}{N}$$

- The maximum for separable states is  $N$  for any Hamiltonian of the form

$$H = \vec{c}^{(1)}\vec{\sigma}^{(1)} + \vec{c}^{(2)}\vec{\sigma}^{(2)} + \vec{c}^{(3)}\vec{\sigma}^{(3)} + \dots,$$

where  $|\vec{c}^{(n)}| = 1$ , and  $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ .

P. Hyllus, O. Gühne, A. Smerzi, Phys. Rev. A 82, 012337 (2010)

# Metrological usefulness

- **Qudits** are more complicated!
- Metrological gain for a given Hamiltonian

$$g_{\mathcal{H}}(\varrho) = \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})},$$

where  $\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})$  is the maximum of the QFI for separable states.

- Metrological gain

$$g(\varrho) = \max_{\text{local } \mathcal{H}} g_{\mathcal{H}}(\varrho) = \max_{\text{local } \mathcal{H}} \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})}.$$

optimized over all **local** Hamiltonians

$$\mathcal{H} = H_1 + H_2 + \dots + H_N.$$

- A state  $\varrho$  is **entangled and metrologically useful** if  $g(\varrho) > 1$ .
- The metrological gain is convex in the state.

G. Toth, T. Vertesi, P. Horodecki, R. Horodecki, PRL 2020.

# Metrologically useful $k$ -entanglement

- If  $g > k - 1$  then we have metrologically useful  $k$ -particle entanglement.
- We have  $k$ -particle entanglement, and the state is more useful than any state with at most  $(k - 1)$  particle entanglement.

R. Trényi *et al.*, New J. Phys. 26, 023034 (2024).

# The quantum metrological gain is an useful quantity!

- The metrological gain is connected to multipartite entanglement in a meaningful way.
- One could also calculate

$$\mathcal{F}_Q[\varrho, \mathcal{H}] - \mathcal{F}_Q^{(\text{sep})}(\mathcal{H}),$$

which could also be used for entanglement detection. However, the relation for multipartite entanglement is less direct.

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# Method for maximizing $g$

- Metrological gain optimized over all local Hamiltonians

$$g(\varrho) = \max_{\text{local } \mathcal{H}} \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})}$$

$\leftarrow$  metrological performance of  $\varrho$   
 $\leftarrow$  best metrological performance of separable states

- It is a fundamental quantity in metrology!
- **Difficult to compute, since  $\mathcal{H}$  is in both the numerator and the denominator!**
- We reduce the problem to maximize  $\mathcal{F}_Q$  over a set of local Hamiltonians.

# Method for finding the optimal local Hamiltonian I

- Direct maximization of  $\mathcal{F}_Q[\varrho, \mathcal{H}]$  over  $\mathcal{H}$  is difficult: it is convex in  $\mathcal{H}$ .
- Let us consider the error propagation formula

$$(\Delta\theta)^2_M = \frac{(\Delta M)^2}{\langle i[M, \mathcal{H}] \rangle^2},$$

which provides a bound on the quantum Fisher information

$$\mathcal{F}_Q[\varrho, \mathcal{H}] \geq 1/(\Delta\theta)^2_M.$$

M. Hotta and M. Ozawa, Phys. Rev. A 2004; B. M. Escher, arXiv:1212.2533;  
K. Macieszczak, arXiv:1312.1356; F. Fröwis, R. Schmied, and N. Gisin,  
Phys. Rev. A 2015.

# Method for finding the optimal Hamiltonian II

The maximum over local Hamiltonians can be obtained as

$$\max_{\text{local } \mathcal{H}} \mathcal{F}_Q[\varrho, \mathcal{H}] = \max_{\text{local } \mathcal{H}} \max_M \frac{\langle i[M, \mathcal{H}] \rangle_{\varrho}^2}{(\Delta M)^2}.$$

G. Toth, T. Vertesi, P. Horodecki, R. Horodecki, PRL 2020.

# Iterative see saw methods (ISS)

- **Iterative See Saw (ISS)** has been used for optimizing over the state, rather than over  $\mathcal{H}$ :

K. Macieszczak, arXiv:1312.1356; K. Macieszczak, M. Fraas, and R. Demkowicz-Dobrzański, New J. Phys. 16, 113002 (2014);

GT, Vértesi, Phys. Rev. Lett. (2018): PPT states are metrologically useful.

- ISS-based method for an optimization over a general dynamics using its Choi- Jamiołkowski representation

Y. L. Len, T. Gefen, A. Retzker, and J. Kołodyński, Quantum metrology with imperfect measurements, Nat. Commun. 13, 6971 (2022).

- An ISS-based method has also been used for optimizing over adaptive strategies, when coherently probing several independent quantum channels

S. Kurdziątek, P. Dulian, J. Majsak, S. Chakraborty, and R.

Demkowicz-Dobrzański, Quantum metrology using quantum combs and tensor network formalism, New J. Phys. 27, 013019 (2025).

## Example: Maximally entangled state

- We consider the  $d \times d$  maximally entangled state

$$|\Psi^{(\text{me})}\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^d |k\rangle|k\rangle.$$

- The optimal Hamiltonian is

$$\mathcal{H}^{(\text{me})} = D \otimes \mathbb{1} + \mathbb{1} \otimes D,$$

where

$$D = \text{diag}(+1, -1, +1, -1, \dots).$$

- We add white noise.

# Numerical results

- The  $3 \times 3$  isotropic state is useful if for the noise

$$p < \frac{25 - \sqrt{177}}{32} \approx 0.3655.$$

- Then, we have the following results for two copies.

	Analytic example	Numerics
Second copy	0.4164	0.4170

- In the case of two copies,  
the metrological usefulness has been activated in the spirit of  
P. Horodecki, M. Horodecki and R. Horodecki, PRL 1989!

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# Multicopy metrology without interaction

- $M$  copies of a quantum state, all undergoing a dynamics governed by the Hamiltonian  $\mathcal{H}$ .
- For the quantum Fisher information we obtain

$$\mathcal{F}_Q[\varrho^{\otimes M}, \mathcal{H}^{\otimes M}] = M\mathcal{F}_Q[\varrho, \mathcal{H}],$$

while the maximum for separable states also increases

$$\mathcal{F}_Q^{(\text{sep})}(\mathcal{H}^{\otimes M}) = M\mathcal{F}_Q^{(\text{sep})}(\mathcal{H}).$$

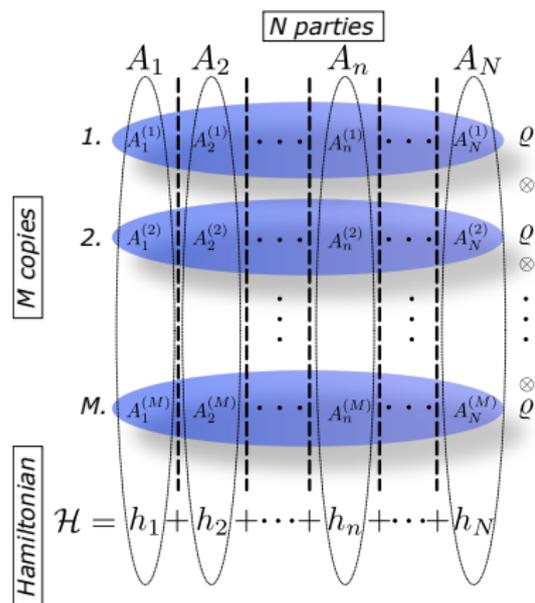
- The metrological gain does not change

$$g_{\mathcal{H}^{\otimes M}}(\varrho^{\otimes M}) = g_{\mathcal{H}}(\varrho).$$

- (Unbiased estimators.)

# Multicopy metrology with interaction

- We need **interaction** between the copies.
- Weakly entangled states can reach maximal metrological usefulness in the many-copy case.



- Metrology with  $M$  copies of an  $N$ -partite quantum state  $\rho$ .
- There is no interaction between particles corresponding to different parties.

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# Local Hamiltonian

- A local Hamiltonian is given as

$$\mathcal{H} = H_1 \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes H_2,$$

- For the quantum Fisher information

$$\mathcal{F}_Q[\rho, c\mathcal{H}] = |c|^2 \mathcal{F}_Q[\rho, \mathcal{H}]$$

holds.

- Thus, we need to normalize, before we maximize  $\mathcal{F}_Q$ .
- We could normalize it with some norm of  $\mathcal{H}$ . We suggest to **normalize it with**

$$\mathcal{F}_Q^{(\text{sep})}(\mathcal{H}) = \sum_{n=1,2} [\sigma_{\max}(H_n) - \sigma_{\min}(H_n)]^2,$$

where  $\sigma_{\max}(X)$  and  $\sigma_{\min}(X)$  denote the maximal and minimal eigenvalues, respectively.

# Local Hamiltonian II

The expression with the square root of the maximum for separable states

$$\sqrt{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})}$$

is a seminorm for  $\mathcal{H} \in \mathcal{L}$ .

- $\sqrt{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})} = 0$  if  $\mathcal{H} = c\mathbb{1}$ , where  $c$  is a real constant.
- $\sqrt{\mathcal{F}_Q^{(\text{sep})}(\mathcal{H})} = 0$  does not imply that  $\mathcal{H}$  is a zero matrix, thus it is not a norm, only a seminorm.

# Method for finding the optimal Hamiltonian

- We define

$$g_{\mathcal{H}}(\varrho) = \frac{\mathcal{F}_{\mathcal{Q}}[\varrho, \mathcal{H}]}{\mathcal{F}_{\mathcal{Q}}^{(\text{sep})}(\mathcal{H})}.$$

- We want to compute

$$g(\varrho) = \max_{\mathcal{H} \in \mathcal{L}} g_{\mathcal{H}}(\varrho).$$

- How can we avoid optimizing the numerator and the denominator?

# Method for finding the optimal Hamiltonian

- Local Hamiltonians must fulfill

$$\sigma_{\min}(H_n) = -c_n, \quad \sigma_{\max}(H_n) = +c_n$$

for  $n = 1, 2$ . Then, for separable states we have

$$\mathcal{F}_Q^{(\text{sep})}(\mathcal{H}) = 4(c_1^2 + c_2^2).$$

- Metrological gain for such local Hamiltonians

$$g_{c_1, c_2}(\varrho) = \max_{\mathcal{H} \in \mathcal{L}_{c_1, c_2}} \frac{\mathcal{F}_Q[\varrho, \mathcal{H}]}{4(c_1^2 + c_2^2)},$$

where we call  $\mathcal{L}_{c_1, c_2}$  the set of local Hamiltonians satisfying the condition with  $c_1$  and  $c_2$ .

- Finally, the maximal gain over all possible local Hamiltonians is given as

$$g(\varrho) = \max_{c_1, c_2} g_{c_1, c_2}(\varrho).$$

# Method for finding the optimal Hamiltonian

- Change the constraints to inequalities

$$c_n \mathbb{1} \pm H_n \geq 0,$$

where  $n = 1, 2$  and  $c_n > 0$  is some constant.

- This way we make sure that

$$\sigma_{\min}(H_n) \geq -c_n, \quad \sigma_{\max}(H_n) \leq +c_n,$$

for  $n = 1, 2$ .

- We maximize a convex function over a convex set.
- the optimum is taken on the boundary of the set where the eigenvalues of  $H_n$  are  $\pm c_n$ . In this case,

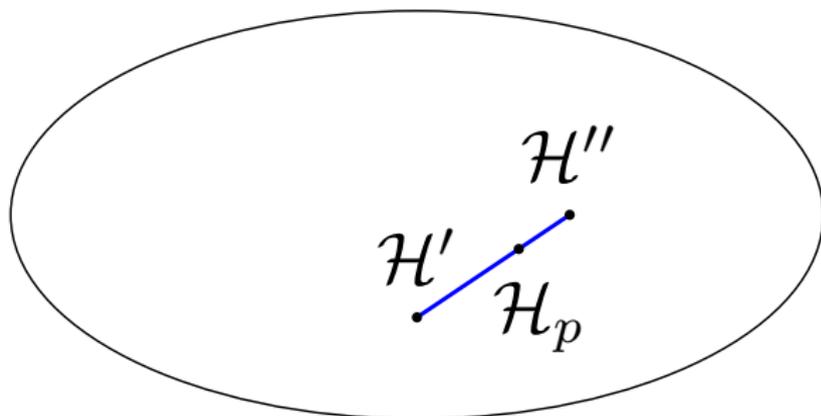
$$H_n^2 = c_n^2 \mathbb{1}.$$

# Method for finding the optimal Hamiltonian

- If two Hamiltonians,  $\mathcal{H}'$  and  $\mathcal{H}''$ , are of the required form then their convex combination, i. e.,

$$\mathcal{H}_p = p\mathcal{H}' + (1 - p)\mathcal{H}''$$

with  $0 \leq p \leq 1$  is also of that form.



**Figure:** The convex set of local Hamiltonians.  $\mathcal{H}_p$  is a "mixture" of  $\mathcal{H}'$  and  $\mathcal{H}''$ .

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# See saw methods in science

- General idea of the see-saw iteration used in physics and engineering

$$\max_{\vec{v}} \vec{v}^T R \vec{v} = \max_{\vec{v}, \vec{w}} \vec{v}^T R \vec{w}.$$

R is a positive semidefinite symmetric matrix with real values.

- Let us see some further applications in quantum metrology and entanglement theory.

# Alternative method for finding the optimal Hamiltonian

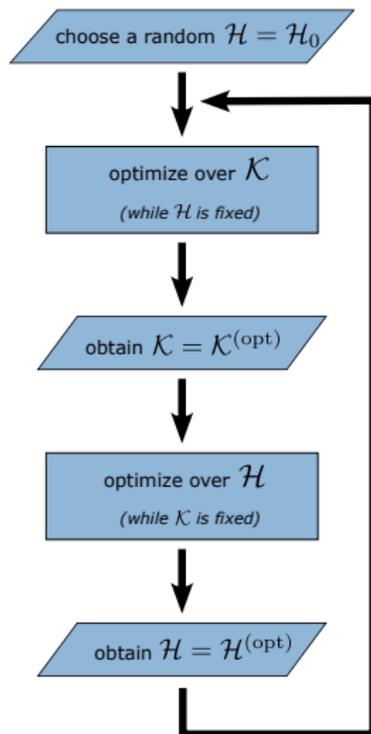
Maximize the QFI using the quantum Fisher matrix elements as

$$\max_{\mathcal{H} \in \mathcal{L}_{c_1, c_2}} \mathcal{F}_Q[\varrho, \mathcal{H}] = \max_{\mathcal{H}, \mathcal{K} \in \mathcal{L}_{c_1, c_2}} \mathcal{F}_Q[\varrho, \mathcal{H}, \mathcal{K}].$$

Here, we define

$$\mathcal{F}_Q[\varrho, \mathcal{H}, \mathcal{K}] = \sum_{kl} 2 \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} \left[ \mathcal{H}_{kl}^r \mathcal{K}_{kl}^r + \mathcal{H}_{kl}^i \mathcal{K}_{kl}^i \right].$$

# Method for finding the optimal Hamiltonian



**Figure:** We can optimize the quantum Fisher information over local Hamiltonians for a fixed probe state with an iterative see-saw (ISS) method.

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# Upper bound with SDP

- The maximization

$$\max_{\vec{v}} \vec{v}^T R \vec{v}$$

is replaced by the Shor relaxation maximizing

$$\text{Tr}(RX),$$

where the Hermitian  $X$  is constrained as

$$X \geq \vec{v}\vec{v}^T,$$

and it does not have to be rank-1.

- We can also add further and further conditions on  $X$  that lead to lower and lower values for the maximum.

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# Wigner-Yanase skew information

- The Wigner-Yanase skew information is defined as

$$I_{\varrho}(\mathcal{H}) = \text{Tr}(\mathcal{H}^2 \varrho) - \text{Tr}(\mathcal{H} \sqrt{\varrho} \mathcal{H} \sqrt{\varrho}).$$

- We maximize over  $\mathcal{H}$ . It is clear how to maximize it with a see-saw method:

$$\max_{\mathcal{H}, \mathcal{K} \in \mathcal{L}_{c_1, c_2}} I_{\varrho}(\mathcal{H}) = \text{Tr} \left[ \frac{1}{2} (\mathcal{H}\mathcal{K} + \mathcal{K}\mathcal{H}) \varrho \right] - \text{Tr}(\mathcal{H} \sqrt{\varrho} \mathcal{K} \sqrt{\varrho}).$$

- Here, instead of  $\mathcal{H}\mathcal{K}$  we use  $\frac{1}{2}(\mathcal{H}\mathcal{K} + \mathcal{K}\mathcal{H})$ , since the latter is Hermitian.
- Note that

$$\mathcal{F}_Q[\varrho, \mathcal{H}] \geq 4I_{\varrho}(\mathcal{H}).$$

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# The Computable Cross Norm-Realignment (CCNR) criterion

- The trace norm is defined as

$$\|X\|_{\text{tr}} = \text{Tr} \left( \sqrt{XX^\dagger} \right).$$

- Dual relation

$$\|X\|_{\text{tr}} = \max_{Y \in \mathbb{R}^{m \times m}: YY^T \leq \mathbb{1}} \text{Tr}(X^T Y).$$

- The maximization of  $\|X\|_{\text{tr}}$  over a set  $S$  can then be written as

$$\max_{X \in S} \|X\|_{\text{tr}} = \max_{X \in S} \max_{Y \in \mathbb{R}^{m \times m}: YY^T \leq \mathbb{1}} \text{Tr}(X^T Y),$$

which can be calculated by a see-saw.

- We have to maximize  $\text{Tr}(X^T Y)$  alternatingly by  $X$  and  $Y$ .

# The Computable Cross Norm-Realignment (CCNR) criterion

The CCNR criterion says that for every bipartite separable state  $\rho$  we have

$$\|R(\rho)\|_{\text{tr}} \leq 1,$$

where  $R(\rho)$  is the realigned matrix obtained by a certain permutation of the elements of  $\rho$ .

- If the inequality is violated then the state is entangled.
- The criterion can detect **PPT bound entangled states** not detected by the Peres-Horodecki criterion.
- Clearly, the larger the left-hand side, the larger the violation.

# CCNR criterion

Dimension $d_1 \times d_2$	Maximum of $\ R(\rho)\ _{\text{tr}}$
$2 \times 2$	1
$2 \times 4$	1
$3 \times 3$	1.1891
$3 \times 4$	1.2239
$4 \times 4$	1.5
$5 \times 5$	1.5
$6 \times 6$	1.5881

**Table:** The largest values for  $\|R(\rho)\|_{\text{tr}}$  for  $d \times d$  PPT states for various  $d$ . The results were obtained by numerical maximization.

## CCNR criterion

We present the  $4 \times 4$  bound entangled state for which the violation of the CCNR criterion is maximal. The state is

$$\varrho_{\text{CCNR}} = \sum_{i=1}^4 p_i |\Psi_i\rangle\langle\Psi_i|_{AB} \otimes \varrho_{A'B'}^{(i)},$$

where the probabilities are

$$p_1 = p_2 = p_3 = 1/6, \quad p_4 = 1/2,$$

where the four Bell states are defined as

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle),$$

and the components for  $A'$  and  $B'$  are given as

$$\begin{aligned} \varrho_{A'B'}^{(1)} &= |\Psi^+\rangle\langle\Psi^+|, & \varrho_{A'B'}^{(2)} &= |\Psi^-\rangle\langle\Psi^-|, \\ \varrho_{A'B'}^{(3)} &= |\Phi^+\rangle\langle\Phi^+|, & \varrho_{A'B'}^{(4)} &= (\mathbb{1} - |\Phi^-\rangle\langle\Phi^-|) / 3. \end{aligned}$$

For the state we have

$$\|R(\varrho_{\text{CCNR}})\|_{\text{tr}} = 1.5.$$

# Summary

- We discussed methods for finding the optimal Hamiltonian for a quantum state using see-saw iterations.
- We discussed some other problems where similar see-saws could be used.

Á. Lukács, R. Trényi, T. Vértesi, and G. Tóth, Iterative optimization in quantum metrology and entanglement theory using semidefinite programming, *Quantum Sci. Technol.* 11, 015042 (2026)

R. Trényi, Á. Lukács, P. Horodecki, R. Horodecki, T. Vértesi, and G. Tóth, Activation of metrologically useful genuine multipartite entanglement, *New J. Phys.* 26, 023034 (2024).

THANK YOU FOR YOUR ATTENTION!