General method for obtaining the energy minimum of spin Hamiltonians for separable states arXiv:2605.03022

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Outline

1. **Motivation**
   - Why do we need energy minimum for separable states?
2. **Background**
   - Entanglement detection by energy measurement
   - Entanglement detection by energy measurement using the single-particle reduced state

Motivation

- Hamiltonians can be used as efficient entanglement witnesses if we know the energy minimum for separable states.
- Before, this energy minimum was computed on a case-by-case basis.
- Can we have a general formula at least for a large class of models?
- Can we incorporate the single-particle state in the bound?

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Basic idea

- Energy bound for separable states

$$E_{\mathrm{sep}} = \min_{\varrho_N \in \mathrm{Sep}} \langle H \rangle_{\varrho_N}.$$

- If the inequality

$$\langle H \rangle < E_{\mathrm{sep}}$$

is fulfilled then the state is entangled.

C. Brukner and V. Vedral, Macroscopic Thermodynamical Witnesses of Quantum Entanglement, arXiv: quant-ph/0406040; G. Tóth, Entanglement witnesses in spin models, Phys. Rev. A 71, 010301(R) (2005); M. R. Dowling, A. C. Doherty, and S. D. Bartlett, Energy as an entanglement witness for quantum many-body systems, Phys. Rev. A 70, 062113 (2004).

Basic idea

- Thermal state at temperature T

$$\varrho_T = \frac{e^{-\frac{H}{T}}}{\mathrm{Tr}(e^{-\frac{H}{T}})}.$$

- T_E is the temperature for which

$$\langle H \rangle_{\varrho_{T_E}} = E_{\mathrm{sep}}.$$

- It is easy to calculate T_E.

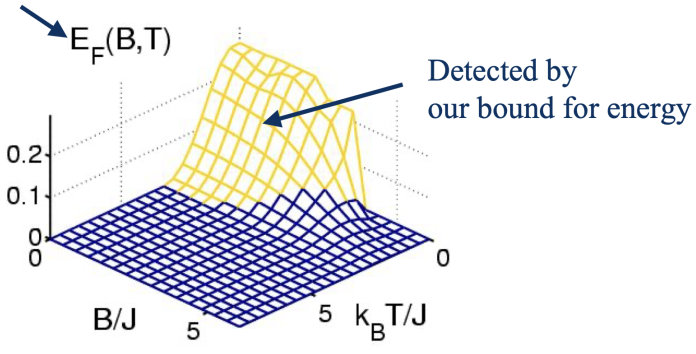
- If $T < T_E$ then the state is entangled.

C. Brukner and V. Vedral, Macroscopic Thermodynamical Witnesses of Quantum Entanglement, arXiv: quant-ph/0406040; G. Tóth, Entanglement witnesses in spin models, Phys. Rev. A 71, 010301(R) (2005); M. R. Dowling, A. C. Doherty, and S. D. Bartlett, Energy as an entanglement witness for quantum many-body systems, Phys. Rev. A 70, 062113 (2004).

Examples

Heisenberg chain in an external magnetic field

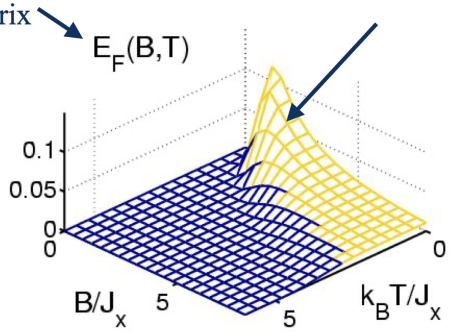
Entanglement of formation of the two-qubit reduced density matrix



Ising spin chain in a transverse magnetic field

Entanglement of formation of the two-qubit reduced density matrix

Detected by our bound for energy



Slides for

G. Tóth, Entanglement witnesses in spin models, Phys. Rev. A 71, 010301(R) (2005).

Formulas

- Let us consider an anti-ferromagnetic Heisenberg Hamiltonian with periodic boundary conditions on a d-dimensional cubic lattice

$$H_{\mathrm{H}} := \sum_{\langle k,l \rangle} \sigma_x^{(k)} \sigma_x^{(l)} + \sigma_y^{(k)} \sigma_y^{(l)} + \sigma_z^{(k)} \sigma_z^{(l)} + B \sum_{k=1}^{N} \sigma_z^{(k)}.$$

  The strength of the exchange interaction is set to be $J = 1$, B is the magnetic field, and $\langle k, l \rangle$ denotes spin pairs connected by an interaction.

- The expectation value of H_{H} for separable states is bounded from below

$$\langle H_{\mathrm{H}} \rangle \geq E_{\mathrm{H,sep}} := \begin{cases} -dN[(B/d)^2/8 + 1] & \text{if } |B/d| \leq 4, \\ -dN(|B/d| - 1) & \text{if } |B/d| > 4, \end{cases}$$

  where N is the total number of spins.

Formulas II

- For the XY Hamiltonian on a d-dimensional cubic lattice with periodic boundary conditions

$$H_{\mathrm{XY}} := \sum_{\langle k,l \rangle} J_x \sigma_x^{(k)} \sigma_x^{(l)} + J_y \sigma_y^{(k)} \sigma_y^{(l)} + B \sum_{k}^{N} \sigma_z^{(k)}$$

  the energy of separable states is bounded from below as

$$\langle H_{\mathrm{XY}} \rangle \geq E_{\mathrm{XY,sep}} := \begin{cases} -dNM\left(1 + b^2/4\right) & \text{if } b \leq 2, \\ -dNMb & \text{if } b > 2. \end{cases}$$

- Here J_x and J_y are the nearest-neighbor coupling along the x and y direction, respectively. B is the magnetic field, $M := \max(|J_x|, |J_y|)$ and $b := |B|/M/d$. This bound is simply the mean-field ground state energy.

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Single-particle density matrix

- In many calculations, apart from the energy, the reduced state is also known.
- In many experiments, we can also assume that the reduced state can be measured.
- Let us assume also in our case that we know the reduced single-particle state ϱ.
- This makes it possible to look at the problem from a different angle.

Hamiltonian

- Hamiltonian with L two-body correlations

$$H = \sum_{\langle n,n' \rangle} \sum_{l=1}^{L} J_l h_l^{(n)} h_l^{(n')} - \vec{B}\vec{G}.$$

- Here, $\vec{B}$ plays the role of the external field and the components of $\vec{G}$ are

$$G_l = \sum_{n=1}^{N} g_l^{(n)},$$

where $\vec{g} = \{g_l\}_{l=1}^{d^2-1}$ are the $SU(d)$ generators.

- For spin-1/2 particles, g_l may be taken as the Pauli matrices σ_x, σ_y, and σ_z.

Hamiltonian given as a sum of two-particle Hamiltonians

- Let us define the two-body Hamiltonian as

$$H_{nn'} = \sum_{l=1}^{L} J_l h_l^{(n)} h_l^{(n')} - \frac{N}{2N_p} \vec{B}(\vec{g}^{(n)} + \vec{g}^{(n')}),$$

- The full Hamiltonian reads for a regular interaction graph as

$$H = \sum_{\langle n,n' \rangle} H_{nn'}.$$

- Here, $H_{nn'}$ acts non-trivially only on spins n and n', and it is acting as the identity operator on the remaining spins, and N_p is the total number of interacting pairs.

Theorem 1

Theorem 1. The minimal energy over separable states with given single-particle marginal ϱ is bounded from below as

$$E_{\mathrm{sep}}(\varrho) := \min_{\varrho_N \in \mathrm{Sep}_N(\varrho)} \langle H \rangle_{\varrho_N} \geq N_p \min_{\varrho_{AB} \in \mathrm{Sep}_2(\varrho)} \langle H_{AB} \rangle_{\varrho_{AB}} ,$$

where $\mathrm{Sep}_N(\varrho)$ denotes the set of N-particle separable states whose single-particle marginals are equal to ϱ.

Similarly, the minimal energy over symmetric separable states satisfies

$$E_{\mathrm{symsep}}(\varrho) := \min_{\varrho_N \in \mathrm{SymSep}_N(\varrho)} \langle H \rangle_{\varrho_N} \geq N_p \min_{\varrho_{AB} \in \mathrm{SymSep}_2(\varrho)} \langle H_{AB} \rangle_{\varrho_{AB}} ,$$

where $\mathrm{SymSep}_N(\varrho)$ is the set of all N-particle symmetric separable states with single-particle marginals ϱ.

Theorem 1

Finally, the constrained energy minimum of the Hamiltonian obeys the lower bound

$$\min_{\varrho_N \in \mathcal{D}_N(\varrho)} \langle H \rangle_{\varrho_N} \geq N_p \min_{\varrho_{AB} \in \mathcal{D}_2(\varrho)} \langle H_{AB} \rangle_{\varrho_{AB}} =: E_L(\varrho),$$

where $\mathcal{D}_N(\varrho)$ denotes the set of all N-particle states with single-particle reduced states ϱ.

Proof. To prove the first inequality we write

$$\begin{aligned} E_{\text{sep}}(\varrho) &= \min_{\varrho_N \in \text{Sep}_N(\varrho)} \left\langle \sum_{\langle n,n' \rangle} H_{nn'} \right\rangle_{\varrho_N} \geq \sum_{\langle n,n' \rangle} \min_{\varrho_{nn'} \in \text{Sep}_2(\varrho)} \langle H_{nn'} \rangle_{\varrho_{nn'}} \\ &= N_p \min_{\varrho_{AB} \in \text{Sep}_2(\varrho)} \langle H_{AB} \rangle_{\varrho_{AB}}, \end{aligned}$$

where the inequality follows because the minimum of a sum is at least as large as the sum of the individual minima. The proofs of other inequalities are analogous. □

Theorem 2

- **Theorem 2.** The bound in Theorem 1 given as

$$E_{\mathrm{sep}}(\varrho) := \min_{\varrho_N \in \mathrm{Sep}_N(\varrho)} \langle H \rangle_{\varrho_N} \geq N_p \min_{\varrho_{AB} \in \mathrm{Sep}_2(\varrho)} \langle H_{AB} \rangle_{\varrho_{AB}} ,$$

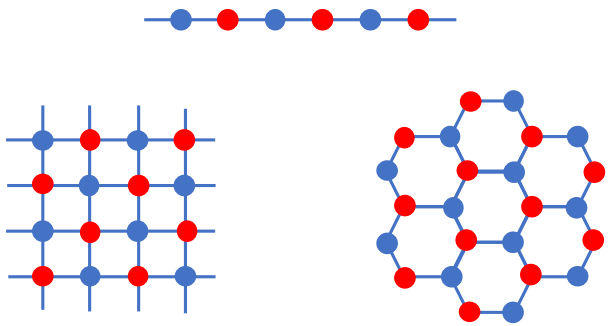
  is saturated with

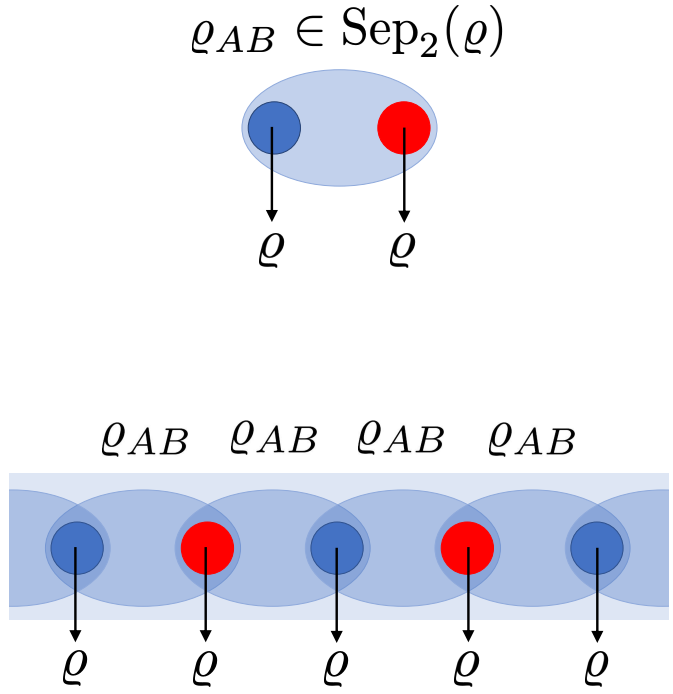
$$N_p = DN = D\Pi_{k=1}^{D} N_k,$$

  for D-dimensional lattices of size $N_1 \times N_2 \times N_3 ... \times N_D$ with nearest-neighbor interaction and periodic boundary conditions, provided all N_k are even.

- A similar statement holds for spin systems on a two-colorable lattice, see **Figs. 1 (a) and (b)** .

Theorem 2





(a) (b)

Figure 1 (a). Spin models on two-colorable graphs. (top) Spin chain. (left) Two-dimensional cubic lattice. (right) Hexagonal lattice.
Figure 1 (b). *N*-particle separable state with two-particle nearest-neighbor marginals ϱ_{AB}. (top) Two-particle separable state ϱ_{AB} with marginals ϱ. (bottom) A portion of an *N*-particle chain.

Gluing separable states

- Let us look for ϱ_{AB} that minimises $\langle H_{AB}\rangle$. We can choose it to be permutationally invariant,

$$\varrho_{AB} = F\varrho_{AB}F.$$

Let us assume that the separable state ϱ_{AB} is decomposed as

$$\varrho_{AB} = \sum_l p_l|\Psi_l\rangle\langle\Psi_l| \otimes |\Phi_l\rangle\langle\Phi_l| = \sum_l p_l|\Phi_l\rangle\langle\Phi_l| \otimes |\Psi_l\rangle\langle\Psi_l|.$$

Gluing separable states II

- Spin chain. It can be partitioned into two disjoint sublattices in such a way that interacting spins correspond to different sublattices, $\mathcal{A}$ and $\mathcal{B}$.

- Then, the Hamiltonian can be given as

$$H = \sum_{l=1}^{N_p} H_{a_l,b_l}.$$

- Then, let us consider the separable state

$$\varrho_{\mathrm{opt}} = \sum_l p_l |\Psi_l\rangle\langle\Psi_l| \otimes |\Phi_l\rangle\langle\Phi_l| \otimes |\Psi_l\rangle\langle\Psi_l| \otimes |\Phi_l\rangle\langle\Phi_l| \otimes \ldots$$

Gluing separable states III

- For the neatest neighbor two-particle reduced states

$$\varrho_{n,n+1} = \varrho_{AB}$$

holds.

- Thus, the energy is the sum of the expectation values of the two-particle Hamiltonians in ϱ_{AB} as

$$\langle H \rangle_{\varrho_{\mathrm{opt}}} = \sum_{k=1}^{N_p} \left\langle H_{a_k,b_k} \right\rangle_{\varrho_{AB}} = N_p \langle H_{AB} \rangle_{\varrho_{AB}} ,$$

and the state saturates the inequality.

- The case of lattices is similar.

Theorem 3

- **Theorem 3.** For a spin system on a complete graph, the bound in Theorem 1 given as

$$E_{\mathrm{symsep}}(\varrho) := \min_{\varrho_N \in \mathrm{SymSep}_N(\varrho)} \langle H \rangle_{\varrho_N} \geq N_p \min_{\varrho_{AB} \in \mathrm{SymSep}_2(\varrho)} \langle H_{AB} \rangle_{\varrho_{AB}} ,$$

  is saturated with

$$N_p = N(N-1)/2,$$

  see **Fig. 1(c)**.

Theorem 3

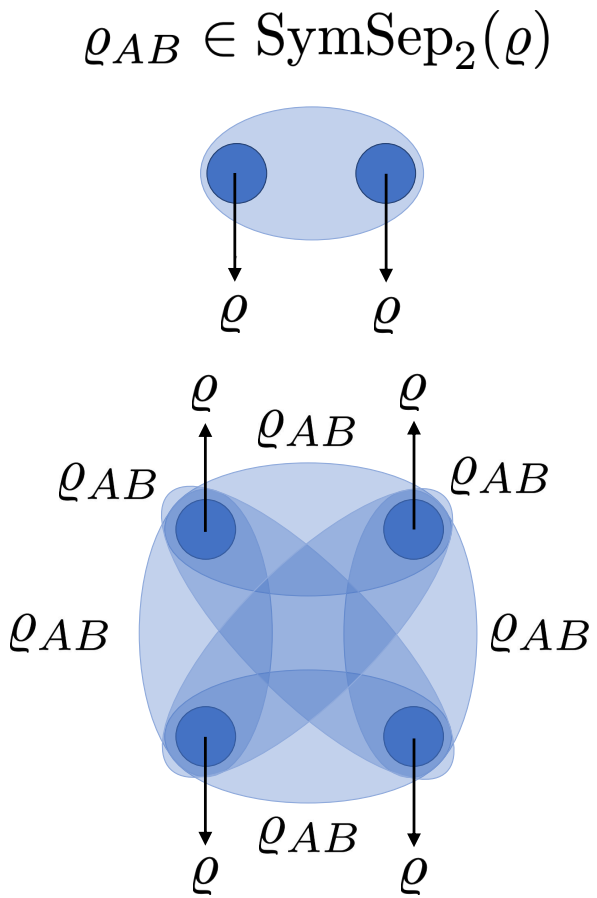


Figure 1(c). *N*-particle symmetric separable state with two-particle nearest-neighbor marginals ϱ_{AB}. (top) Two-particle symmetric separable state ϱ_{AB} with single-particle reduced states ϱ. (bottom) Symmetric separable state for $N = 4$.

Theorem 3

- *Proof.* Let the two-particle reduced state, which is minimizing the energy on the right-hand side of the inequality, be the symmetric separable state

$$\varrho_{AB} = \sum_k p_k |\Psi_k\rangle\langle\Psi_k|^{\otimes 2}.$$

- The N-particle symmetric separable state that is minimizing the energy on the left-hand side of the inequality can be given as

$$\varrho_N = \sum_k p_k |\Psi_k\rangle\langle\Psi_k|^{\otimes N}.$$

It can also be seen that the inequality is saturated. □

Numerics

- A minimization of an operator expectation value over separable states can be carried out numerically for a two-qubit system via semidefinite programming, using the fact that the set of separable states coincides exactly with the set of states with a positive partial transpose.

- For particles with a higher dimension, a similar technique yields a hierarchy of increasingly tight lower bounds.

L. Vandenberghe and S. Boyd, Semidefinite programming, SIAM Review 38, 49 (1996); P. Horodecki, Separability criterion and inseparable mixed states with positive partial transposition, Phys. Lett. A 232, 333 (1997); A. Peres, Separability criterion for density matrices, Phys. Rev. Lett. 77, 1413 (1996); A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, Distinguishing separable and entangled states, Phys. Rev. Lett. 88, 187904 (2002); A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, Complete family of separability criteria, Phys. Rev. A 69, 022308 (2004).

Analytical formulas

Line	L	Set of states	$\max \sum_{l=1}^{L} \langle h_l \otimes h_l \rangle_{\varrho_{AB}}$
(a)	1	$\mathrm{Sep}_2, \mathrm{SymSep}_2$	$\left\langle h_1^2 \right\rangle_{\varrho} - \mathcal{F}_Q[\varrho, h_1]/4$
(b)	≥ 2	$\mathrm{Sep}_2, \mathrm{SymSep}_2$	$\leq \sum_l \left\langle h_l^2 \right\rangle_{\varrho} - \mathcal{F}_Q[\varrho, h_l]/4$
(c)	1	$\mathcal{D}_2, d = 2$	$\left\langle h_1^2 \right\rangle_{\varrho} - I_{\varrho}^{\mathrm{WY}}(h_1)$
(d)*	3	$\mathrm{Sep}_2(\varrho, \sigma), d = 2$	$F(\varrho, \sigma)/2 - 1/4$
(e)	≥ 1	$\mathrm{Sep}_2, \mathrm{SymSep}_2$, $d = 2$	Exact formula

Table I. Summary of the cases when the maxima of correlation terms or bounds on them can be computed with a closed formula.
We assume that both single-particle reduced states of ϱ_{AB} are ϱ.
Here, Sep_N and SymSep_N denote the set of N-particle separable states and symmetric separable states, respectively.
$\mathcal{D}_N$ is the set of N-particle quantum states.
* indicates that $h_1 = j_x, h_2 = j_y, h_3 = j_z$, and the reduced states are ϱ and σ.

Analytical formulas

Line	L	Set of states	$\min \sum_{l=1}^{L} \langle h_l \otimes h_l \rangle_{\varrho_{AB}}$
(a)	≥ 1	Sep_2	$\leq \sum_l \langle h_l \rangle_\varrho^2$
(b)	$1, 2$	SymSep_2	$= \sum_l \langle h_l \rangle_\varrho^2$
(c)	≥ 3	SymSep_2	$\geq \sum_l \langle h_l \rangle_\varrho^2$
(d)	≥ 1	SymSep_2, $d = 2$	Exact formula

Table II. Summary of the cases when the minima of correlation terms or bounds on them can be computed with a closed formula.

Analytical formulas

- For the Tables, the quantum Fisher information reads

$$\mathcal{F}_Q[\varrho, h] = 2 \sum_{k,l} \frac{(\lambda_k - \lambda_l)^2}{\lambda_k + \lambda_l} |\langle k|h|l\rangle|^2,$$

where $\varrho = \sum_k \lambda_k |k\rangle\langle k|$ is the eigendecomposition of the density matrix.

- The Wigner-Yanase skew information is defined as

$$I_\varrho^{\mathrm{WY}}(h) = \mathrm{Tr}(h^2\varrho) - \mathrm{Tr}(h\sqrt{\varrho}h\sqrt{\varrho}),$$

and $F(\varrho, \sigma)$ is the Uhlmann-Jozsa quantum fidelity

- Finally, j_x, j_y and j_z refer to angular momentum components.

Quantum Fisher information

- The maximum correlations for separable states with marginals ϱ are given as

$$\max_{\varrho_{AB}\in \mathrm{Sep}_2(\varrho)} \langle h\otimes h\rangle_{\varrho_{AB}} = \left\langle h^2\right\rangle_{\varrho} - \frac{1}{4}\mathcal{F}_Q[\varrho, h].$$

- The maximum correlations for symmetric separable states with marginals ϱ are given as

$$\max_{\varrho_{AB}\in \mathrm{SymSep}_2(\varrho)} \langle h\otimes h\rangle_{\varrho_{AB}} = \left\langle h^2\right\rangle_{\varrho} - \frac{1}{4}\mathcal{F}_Q[\varrho, h].$$

- h can be of any operator of any dimension!

- Sanity check: $\varrho = |\Psi\rangle\langle\Psi|$.
  $\langle h\otimes h\rangle_{\varrho_{AB}} = \langle h\rangle_{\varrho}^2$, $\quad \left\langle h^2\right\rangle_{\varrho} - \frac{1}{4}\mathcal{F}_Q[\varrho, h] = \left\langle h^2\right\rangle_{\varrho} - (\Delta h)_{\varrho}^2 = \langle h\rangle_{\varrho}^2$.

Quantum Fisher information II

- The QFI is the convex roof of the variance times four:

$$F_Q[\varrho, h] = 4 \min_{\{p_k, \Psi_k\}} \sum_k p_k (\Delta h)^2_\Psi$$

where

$$\varrho = \sum_k p_k |\Psi_k\rangle\langle\Psi_k|.$$

GT, D. Petz, PRA 2013; S. Yu, arxiv 2013

Quantum Fisher information III

- Transforming it into a two-body problem:

$$F_Q[\varrho, h] = 4 \min_{\{p_k, \Psi_k\}} \sum_k p_k (\Delta h)^2_{\Psi_k}$$
$$= 4 \min_{\omega_{12}} \left\langle h^2 \otimes \mathbb{1} - h \otimes h \right\rangle$$

  where ω_{12} is a symmetric separable state with marginals ϱ

$$\omega_{12} = \sum_k p_k |\Psi_k\rangle\langle\Psi_k| \otimes |\Psi_k\rangle\langle\Psi_k|.$$

- Optimization over general separable states leads to the same result.

  GT, D. Petz, PRA 2013; S. Yu, arxiv 2013;
  GT, T. Moroder, O. Gühne, PRL 2015;
  GT, J. Pitrik, Quantum 2023.

Wigner-Yanase skew information

- The maximum correlations for two-qubit quantum states with marginals ϱ are given as

$$\max_{\varrho_{AB}\in\mathcal{D}_2(\varrho)} \langle h\otimes h\rangle_{\varrho_{AB}} = \left\langle h^2\right\rangle_{\varrho} - I_{\varrho}^{\mathrm{WY}}(h).$$

Ising spin chain I

- **Theorem 4.** The minimal energy over separable states of a ferromagnetic spin lattice of dimension D with even N_k considered in Theorem 2 with $L = 1$ and $J_1 = -J < 0$ is given by

$$E_{\mathrm{sep}}(\varrho) = -DNJ\left(\left\langle h_1^2\right\rangle_\varrho - \frac{1}{4}\mathcal{F}_Q[\varrho, h_1]\right) - N\vec{B}\left\langle\vec{g}\right\rangle_\varrho .$$

  The expression holds for spins of arbitrary local dimension, not restricted to qubits.

- The corresponding lower bound valid for general states reads

$$E_{\mathrm{LB}}(\varrho) = -DNJC_{\max}(\varrho) - N\vec{B}\left\langle\vec{g}\right\rangle_\varrho ,$$

  where $C_{\max}(\varrho) = \max_{\varrho_{AB}\in\mathcal{D}_2(\varrho)} \langle h\otimes h\rangle_{\varrho_{AB}}$ is the maximal two-body correlation attainable by any (possibly entangled) state with marginals ϱ.

Ising spin chain II

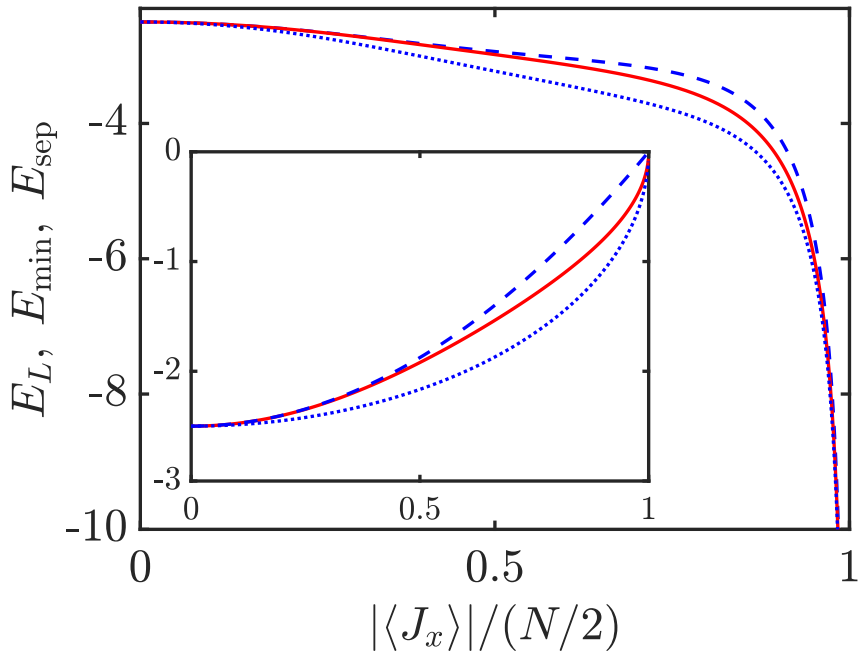


Figure 2(a). Ground state energy and bounds for the ferromagnetic Ising spin chain plotted as a function of $\langle J_x \rangle$. (solid) Energy minimum, i.e., ground state energy. (dashed) Lower bound for separable states based on the quantum Fisher information. It is also an upper bound on the energy minimum. (dotted) Lower bound on the ground state energy based on the Wigner-Yanase skew information. Inset: The same quantities for the expectation value of the correlation terms, without the single-particle terms.

Quantum Fisher information - Entanglement witness

- Energy bound for separable states given with marginal ϱ

$$E_{\mathrm{sep}}(\varrho) = \min_{\varrho_N \in \mathrm{Sep}(\varrho)} \langle H \rangle_{\varrho_N}.$$

- If the inequality

$$\langle H \rangle < E_{\mathrm{sep}}(\varrho)$$

  is fulfilled then the state is entangled.

- $T_E(\varrho)$ also depends on ϱ.

Quantum Fisher information - Entanglement witness II

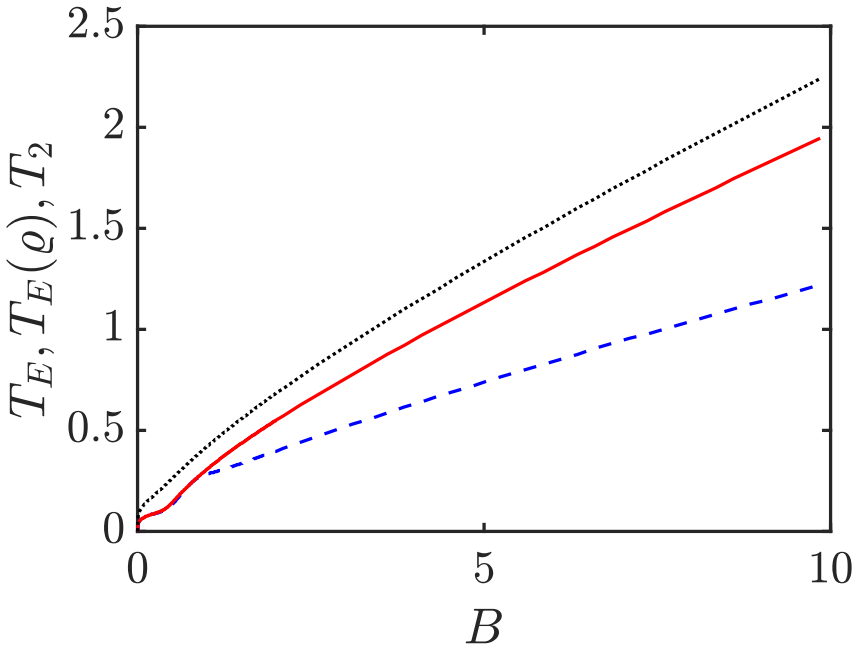


Figure S1. Entanglement temperature as a function of external field. Ferromagnetic Ising spin chain of $N = 8$ spins in a transverse field. (solid) Using Theorem 4 to determine the temperature bound. (dashed) Bound without the knowledge of the single-particle states. (dotted) Entanglement temperature based on two-body nearest-neighbor entanglement.

QFI from correlation measurements

- **Theorem 5.** Let us consider a ferromagnetic system of spins with a Hamiltonian given as before on a completely connected graph and $L = 1$, where $J_1 = -J < 0$.

- The quantum Fisher information can be bounded from below as

$$\mathcal{F}_Q[\varrho, h_1] \geq \mathcal{F}_Q^{(N)}[\varrho, h_1] = \frac{8}{N(N-1)J} \langle H \rangle_{\omega_N} + \frac{8}{N(N-1)J} N\vec{B} \left\langle \vec{g} \right\rangle_{\varrho} + 4 \left\langle h_1^2 \right\rangle_{\varrho},$$

  where ω_N is the zero-temperature thermal state of H, and ϱ is its single-particle reduced state.

- The single-particle density matrix ϱ can be tuned by varying $\vec{B}$.

QFI from correlation measurements II

- Let us define Δ as the difference of the left-hand side and right-hand side of the previous inequality.

- We find the remarkable relation for the N-particle approximation

$$\frac{N-1}{N}\mathcal{F}_Q[\varrho, h_1] \leq \mathcal{F}_Q^{(N)}[\varrho, h_1] \leq \mathcal{F}_Q[\varrho, h_1].$$

- It is bounded from above as

$$\Delta = \mathcal{F}_Q[\varrho, h_1] - \mathcal{F}_Q^{(N)}[\varrho, h_1] \leq \frac{1}{N}[\lambda_{\max}(h_1) - \lambda_{\min}(h_1)]^2 = O\left(\frac{1}{N}\right).$$

QFI from correlation measurements III

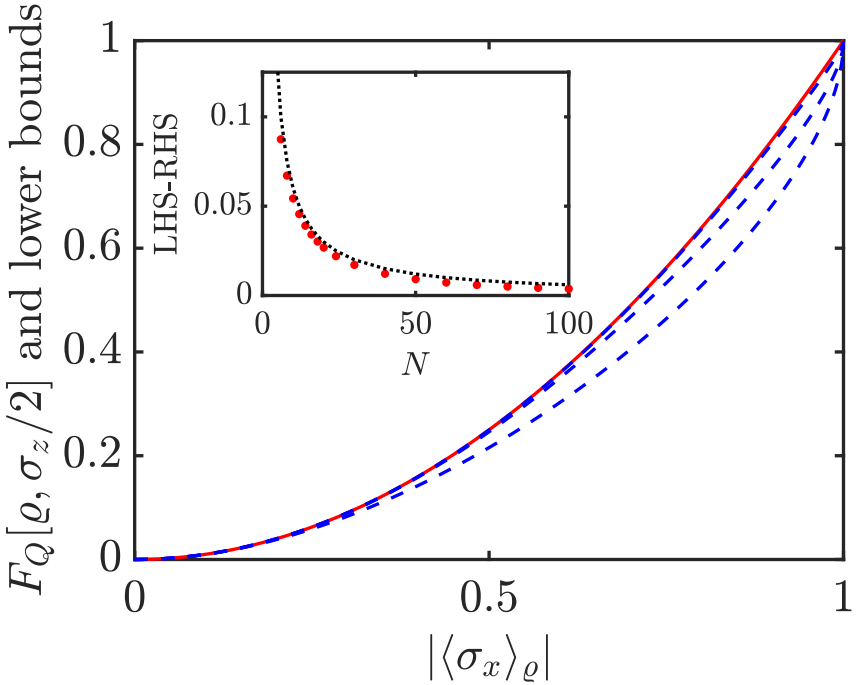


Figure 2. Quantum Fisher information from correlation measurements.
(solid) $\mathcal{F}_Q[\varrho, \sigma_z/2]$ as a function of $|\langle\sigma_x\rangle_\varrho|$.
(dashed) Lower bounds from bottom to top for $N = 4, 10$, and 60 particles, respectively.
Inset: (disks) Error of the estimation compared to the quantum Fisher information as a function of N.
(dotted) Upper bound, $0.6/N$.

Uhlmann-Jozsa Fidelity

- The maximum of the correlations for separable states is given by the fidelity

$$\max_{\varrho_{AB}\in \mathrm{Sep}_2(\varrho,\sigma)} \sum_{l=x,y,z} \langle j_l \otimes j_l \rangle_{\varrho_{AB}} = \frac{1}{2} F(\varrho,\sigma) - \frac{1}{4}.$$

- This can be used to obtain the Fidelity from correlation measurements in spin systems.

Multipartite entanglement

Theorem 6. The energy of any k-producible state for a ferromagnetic spin chain with $L = 1$ and $J_1 = -J < 0$ satisfies

$$E_{k-\mathrm{prod}}(\varrho) \geq \frac{N}{k} \min_{\varrho_{k-\mathrm{part}} \in D_k(\varrho)} \left\langle H_{k-\mathrm{part}} \right\rangle_{\varrho_{k-\mathrm{part}}} - \frac{N}{k} J \left(\left\langle h^2 \right\rangle_{\varrho} - \frac{1}{4} \mathcal{F}_Q[\varrho, h] \right) - \frac{N}{k} \vec{B} \left\langle \vec{g} \right\rangle_{\varrho} .$$

Any quantum state that violates the above condition has $(k+1)$-particle entanglement.

Summary

- We determined the energy minimum for separable states for spin models with nearest-neighbor interaction.
- We consider a formulation that includes the knowledge of the single-particle reduced state.
- We show how to obtain the QFI and the fidelity from correlation measurements.
- We present bounds for multiparticle entanglement.

G. Tóth and J. Pitrik, arXiv:2605.03022.

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