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Theory of robust quantum many-body scars in long-range interacting systems

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$$$$$

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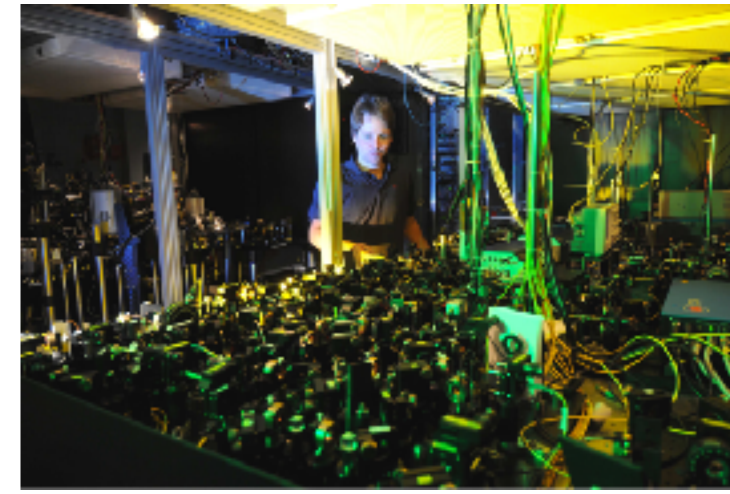
arxiv 2309.xxxxx (to appear soon)

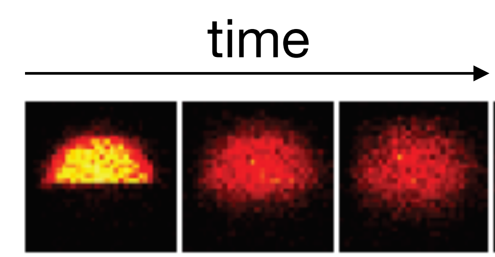
Synthetic matter

- Ultracold atoms
- Rydberg atom arrays
- Trapped ions
- Superconducting qubits
- …



Beyond “traditional” quantum many-body physics:

- Isolated (no phonon/heat bath) → **Coherent quantum dynamics**

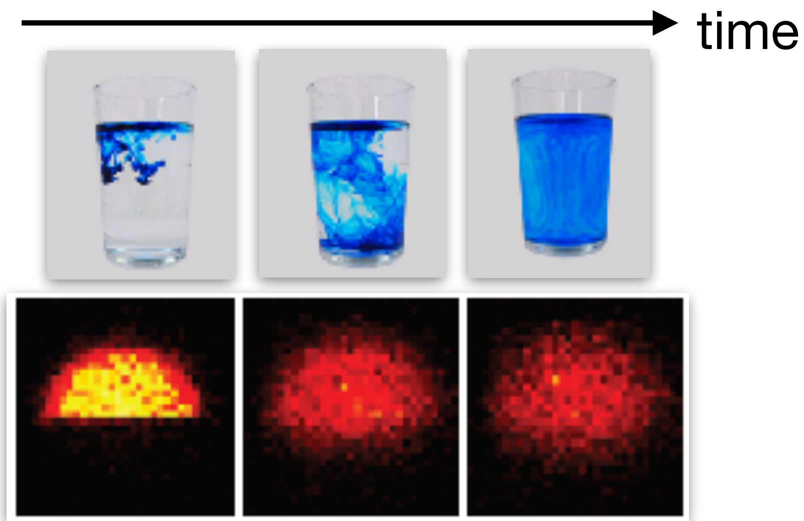


- Long timescales: $\sim 10^{-3}\ s$ vs $10^{-12}\ s$ → **Single-site, real-time resolution**

- Design dimensionality, lattice, interactions,… → **Tunability**

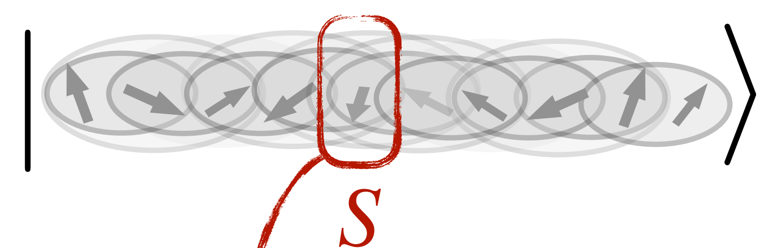
pictures: I. Bloch group @ Munich

Thermalization of isolated systems?

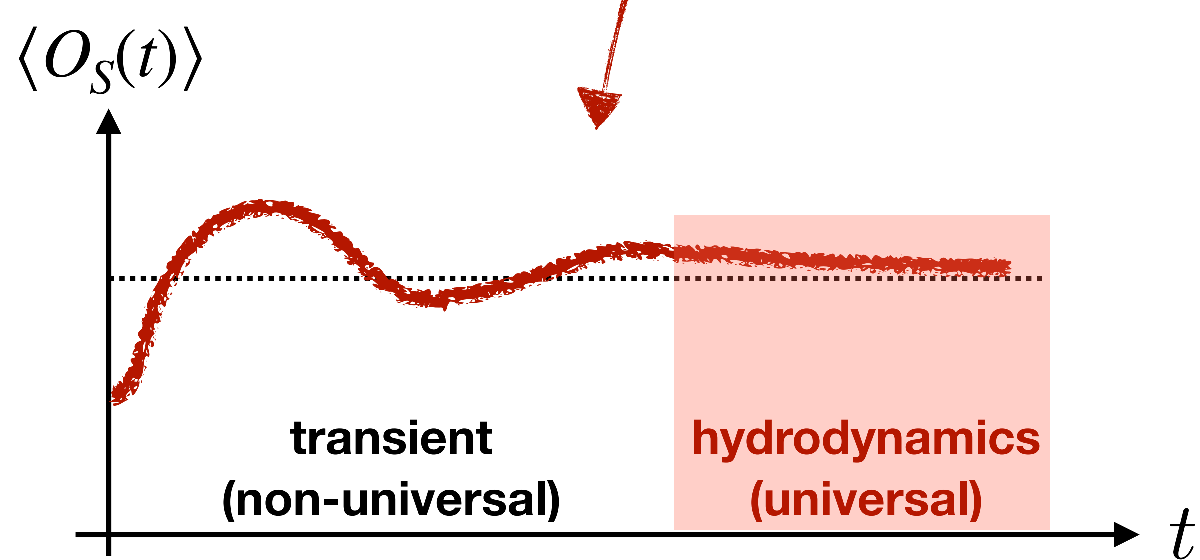


experiments:
Kinoshita et al. - Nature (2006)
Trotzky et al. - Nat. Phys. (2012)
Langen et al. - Nat. Phys. (2013)
Kaufman et al. - Science (2016)
…

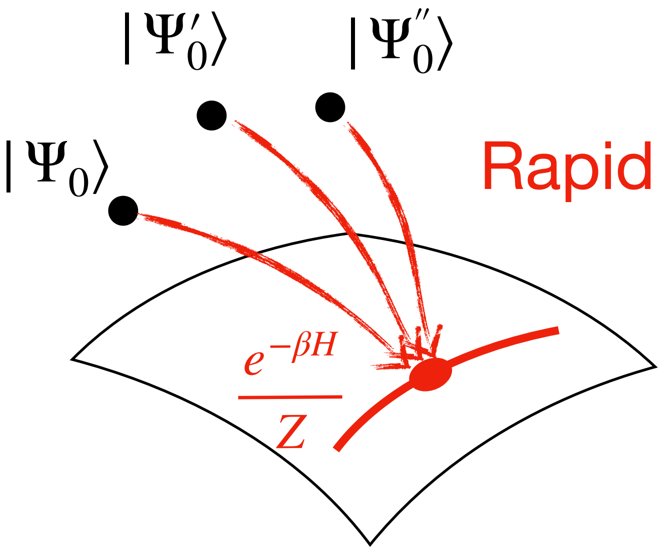
$$|\psi(t)\rangle = e^{-iHt}\,|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle \quad =$$



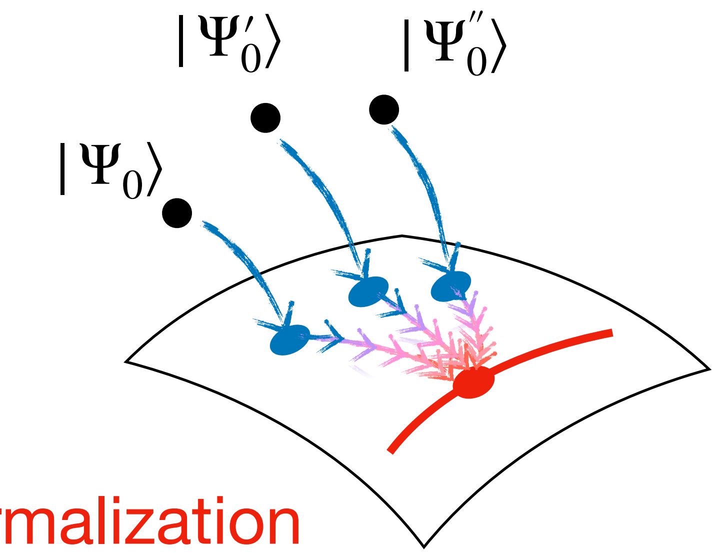
$$\langle\psi(t)\,|\,O_S\,|\,\psi(t)\rangle \;\underset{t\to\infty}{\overset{?}{\longrightarrow}}\; \mathrm{Tr}\left(O_S\frac{e^{-\beta H}}{Z}\right)$$



Nonequilibrium states/phases of matter



Rapid thermalization



Suppressed/Slow thermalization

Many-body localization

Quantum glasses

review:
Abanin et al. - RMP (2020)

Prethermalization

Metastability

Disorder-free localization

Quantum many-body scars

…

Quantum many-body chaos

review:
D'Alessio et al. - Adv. Phys. (2016)

THIS TALK

Experimental discovery of quantum many-body scars

Two states per atom:

ground $|\circ\rangle$

Rydberg $|\bullet\rangle$

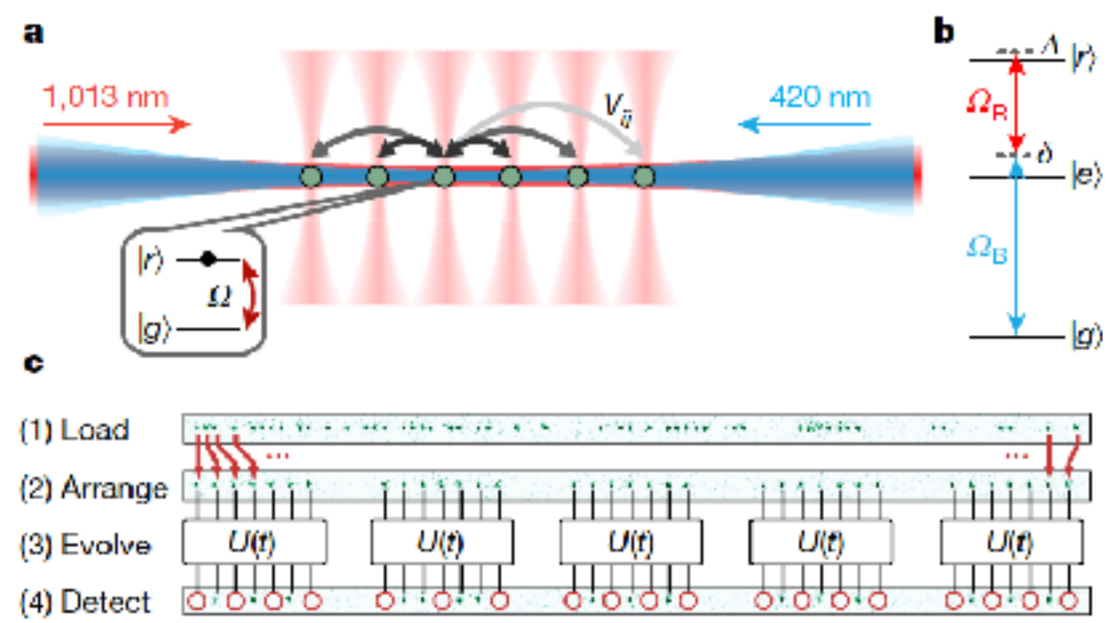
Initial state:

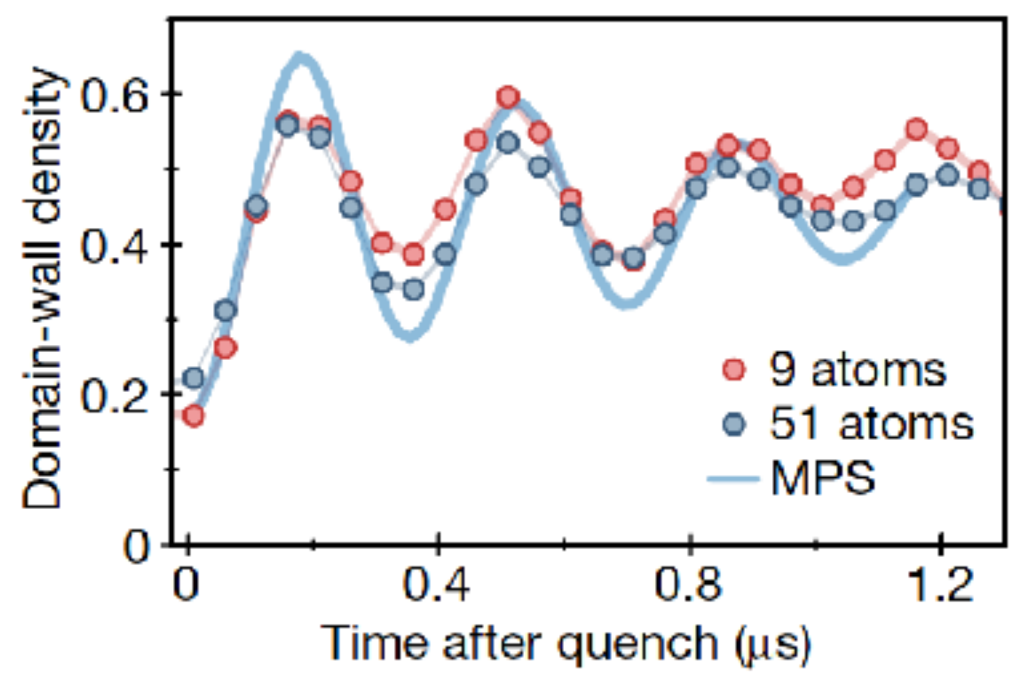
○ ● ○ ● ○ ● ○ ● ○ ● ○ ●

1 L

Surprising long-lived revivals

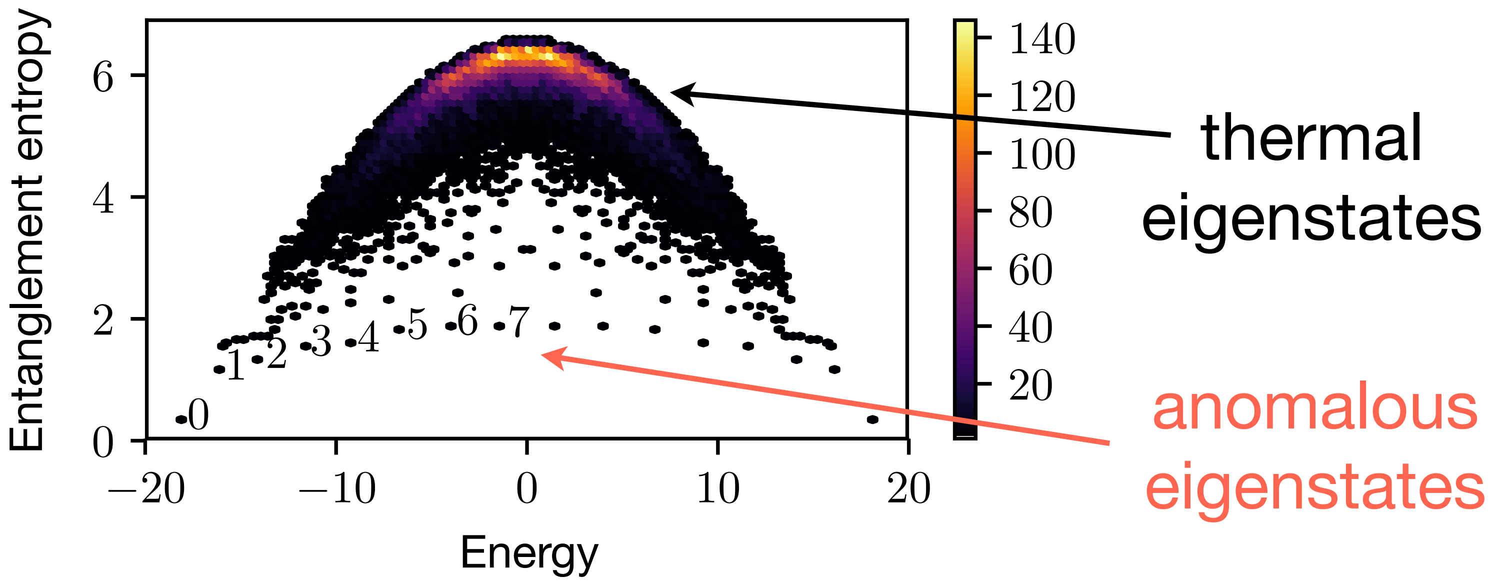
Bernien et al. - Nature (2017)

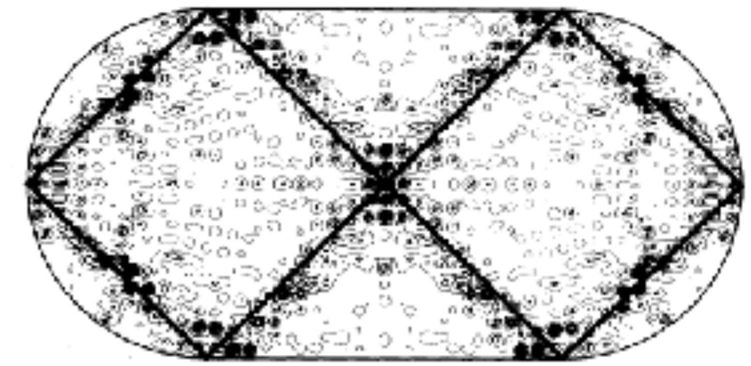




Theory of quantum many-body scars

Turner et al. - Nat. Phys. (2018)





cf quantum scars in billiards

Heller - PRL (1984)

Construction of local H with exact scars

Shiraishi & Mori - PRL (2017)
Motrunich et al., Bernevig et al., Iadecola et al., Surace et al. …

Instability to generic perturbations

Lin et al. - PRR (2020), Surace et al. - PRB (2021)

***Q1:* Is there a class of systems with *robust* scars?**

Long-range interactions in synthetic matter

- Two-level systems (“spins-1/2”/“qubits”)
- Interactions mediated by spatially delocalized degrees of freedom

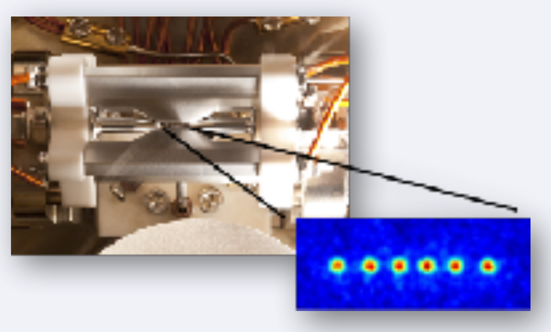
Variable-range quantum Ising chain

$$H = -\sum_{i<j}^{L} J_{ij}\sigma_i^x\sigma_j^x - h\sum_{i}^{L}\sigma_i^z \qquad J_{ij} \sim \frac{J}{|i-j|^{\alpha}}$$

Trapped ions

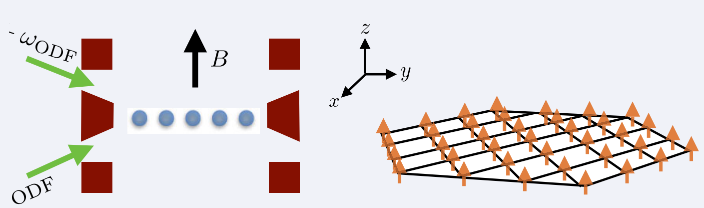
Paul trap $0 < \alpha < 3$

Maryland (C. Monroe)
Innsbruck (R. Blatt)



Penning trap $0.02 < \alpha < 0.2$

Boulder (J. Bollinger)



Dipolar atoms $\alpha = 3 \text{ or } 6$

Polar molecules $\alpha = 3$

Spinor condensates $\alpha = 0$

Atoms in cavity $\alpha \approx 0$

Nuclear spins $\alpha = 3$

Diamond NV-centers $\alpha = 3$

...

Mean-field dynamics $\alpha = 0$

$$H = -J \sum_{i,j} \sigma_i^x \sigma_j^x - g \sum_i \sigma_i^z$$

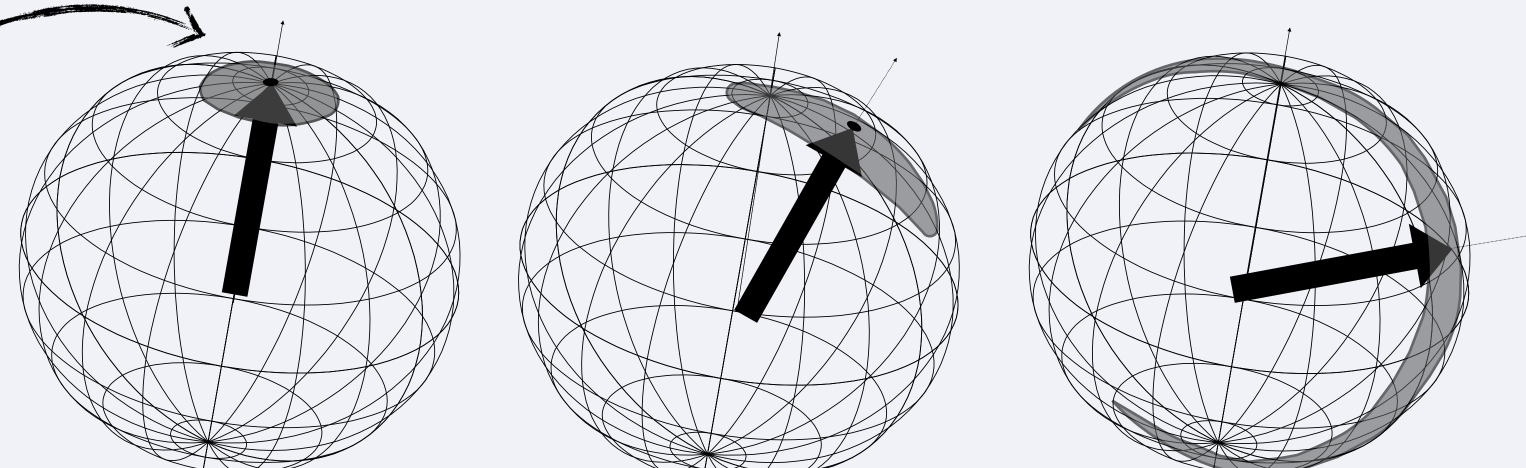
$\sum_{i,j} \sigma_i^x \sigma_j^x \rightarrow (S^x)^2$

$\sum_i \sigma_i^z \rightarrow S^z$

Single-body dynamics $\Longrightarrow$ Solvable

Semiclassical picture of **collective spin squeezing**

$\hbar_{\text{eff}} \sim \hbar / L$

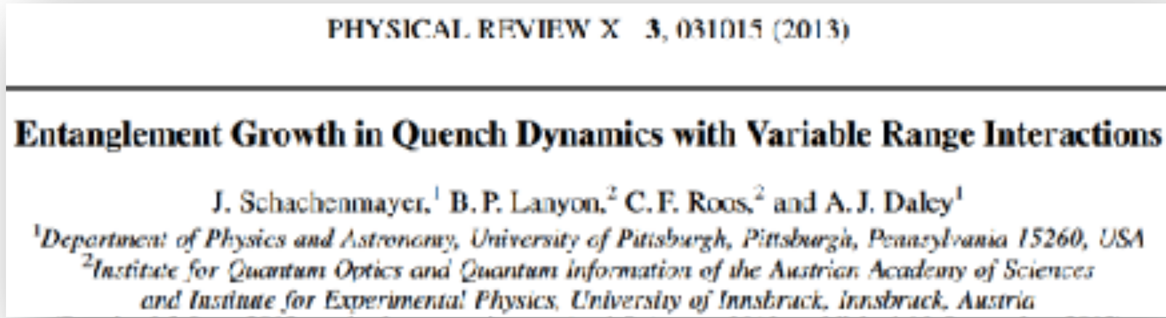


$\vec{S}(t) \propto L$

$\Longrightarrow$ **Thermalization impossible** (spin size conserved)

Thermalization with finite-range interactions $\alpha \neq 0$?

2013:



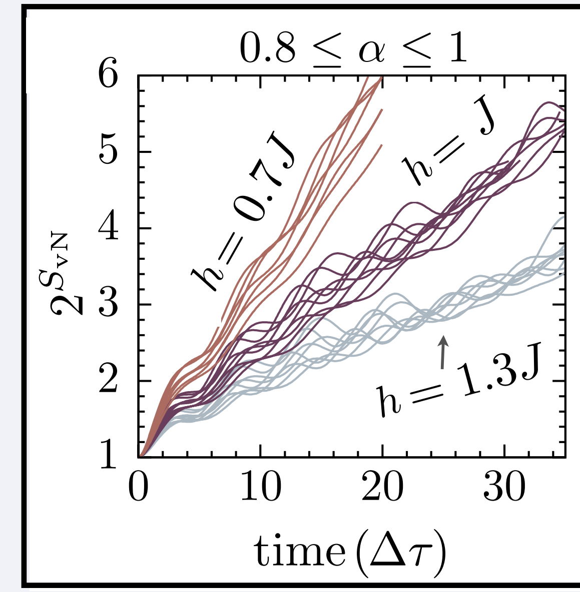
PHYSICAL REVIEW X 3, 031015 (2013)

Entanglement Growth in Quench Dynamics with Variable Range Interactions

J. Schachenmayer,[1] B. P. Lanyon,[2] C. F. Roos,[2] and A. J. Daley[1]

[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences and Institute for Experimental Physics, University of Innsbruck, Innsbruck, Austria*

different behavior. Counterintuitively, quenches above the critical point for these long-range interactions lead only to a logarithmic increase of bipartite entanglement in time, so that in this regime, long-range interactions produce a slower growth of entanglement than short-range interactions.



$N = 30, 40, 50$ $D_{\text{MPS}} = 120$

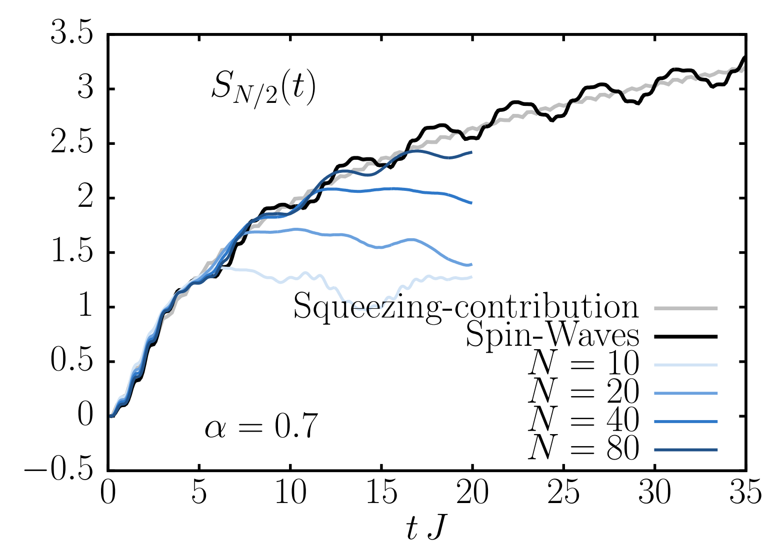
...

2020: **Theory of entanglement entropy growth for $0 \leq \alpha \leq d$**

AL & Pappalardi - PRR (2020)

$\Longrightarrow$ Slow (logarithmic) growth for $0 < t \ll N^{\beta}$ in absence of semiclassical chaos

$\Longrightarrow$ Fast (linear) growth for $0 < t \ll \log N$ in presence of semiclassical chaos



Q2: Do long-range systems ultimately thermalize?

This talk:

Long-range interacting quantum spin systems

$\updownarrow$

Robust quantum many-body scars

➡ No thermalization for strongly polarized initial states

➡ Robust non-equilibrium states with useful entanglement properties

I. Numerical analysis of spectrum

II. Analytical theory of eigenstate localization

III. Verification of new theory predictions

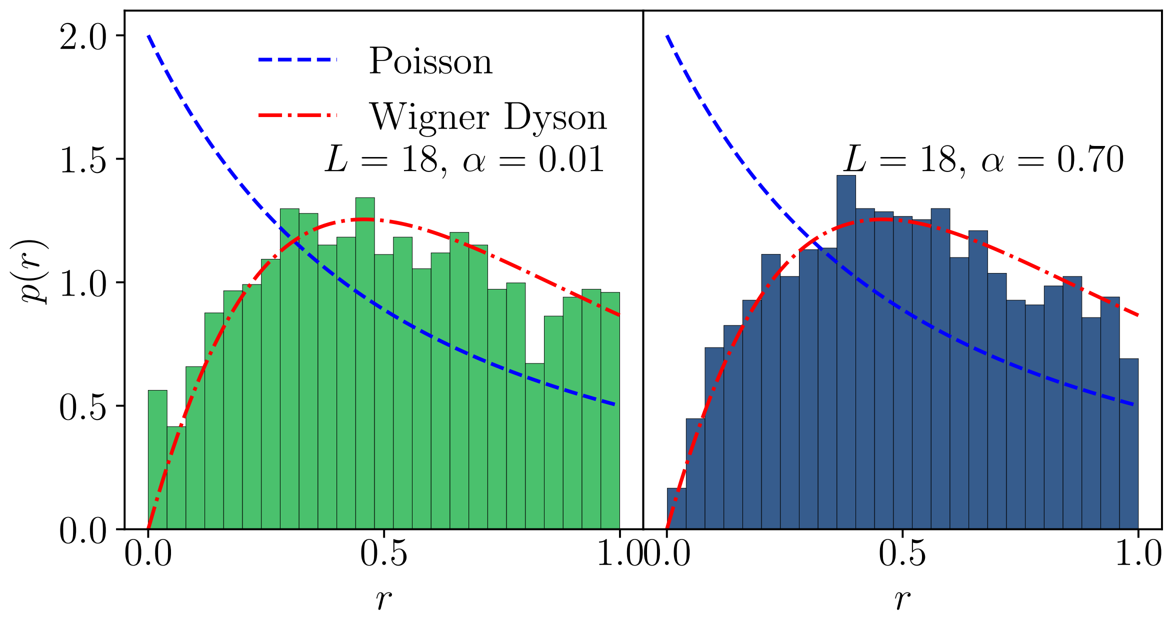
IV. Summary and conclusions

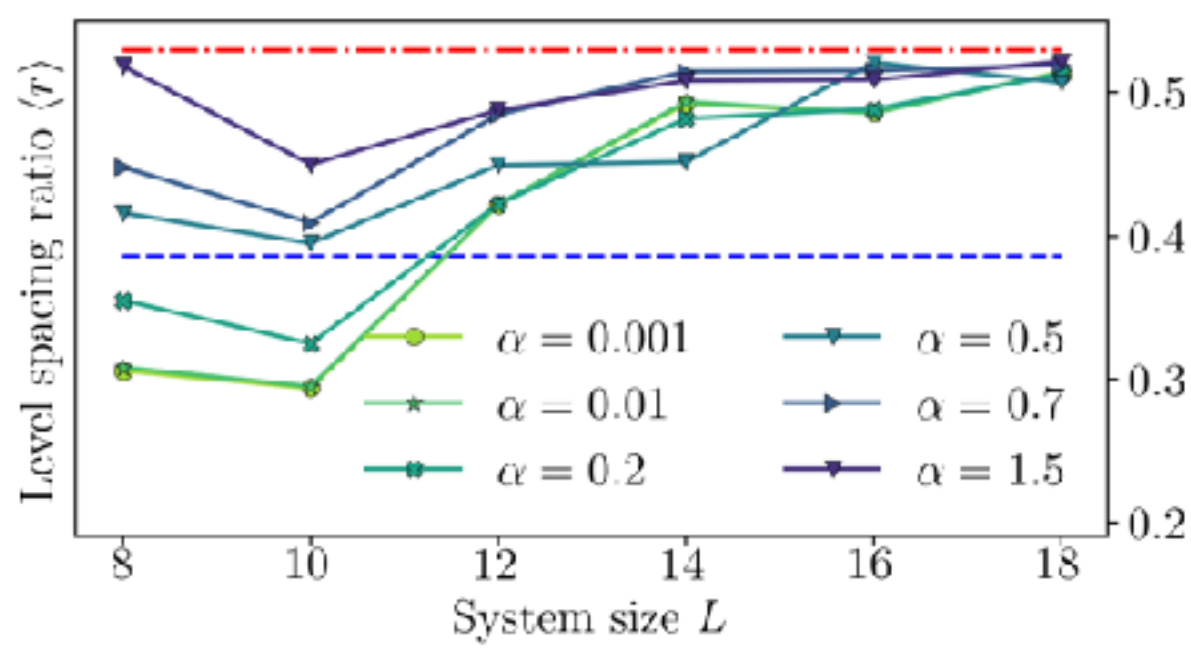
Numerics: Level statistics

Level spacing ratio

$$r_n = \frac{\min(\Delta E_{n+1}, \Delta E_n)}{\max(\Delta E_{n+1}, \Delta E_n)}$$

Std metric of quantum chaos





Level repulsion (quantum chaos) for infinitesimal $\alpha > 0$

cf. Russomanno et al. - PRB 2022

Thermalization metrics

i) Collective spin size depletion

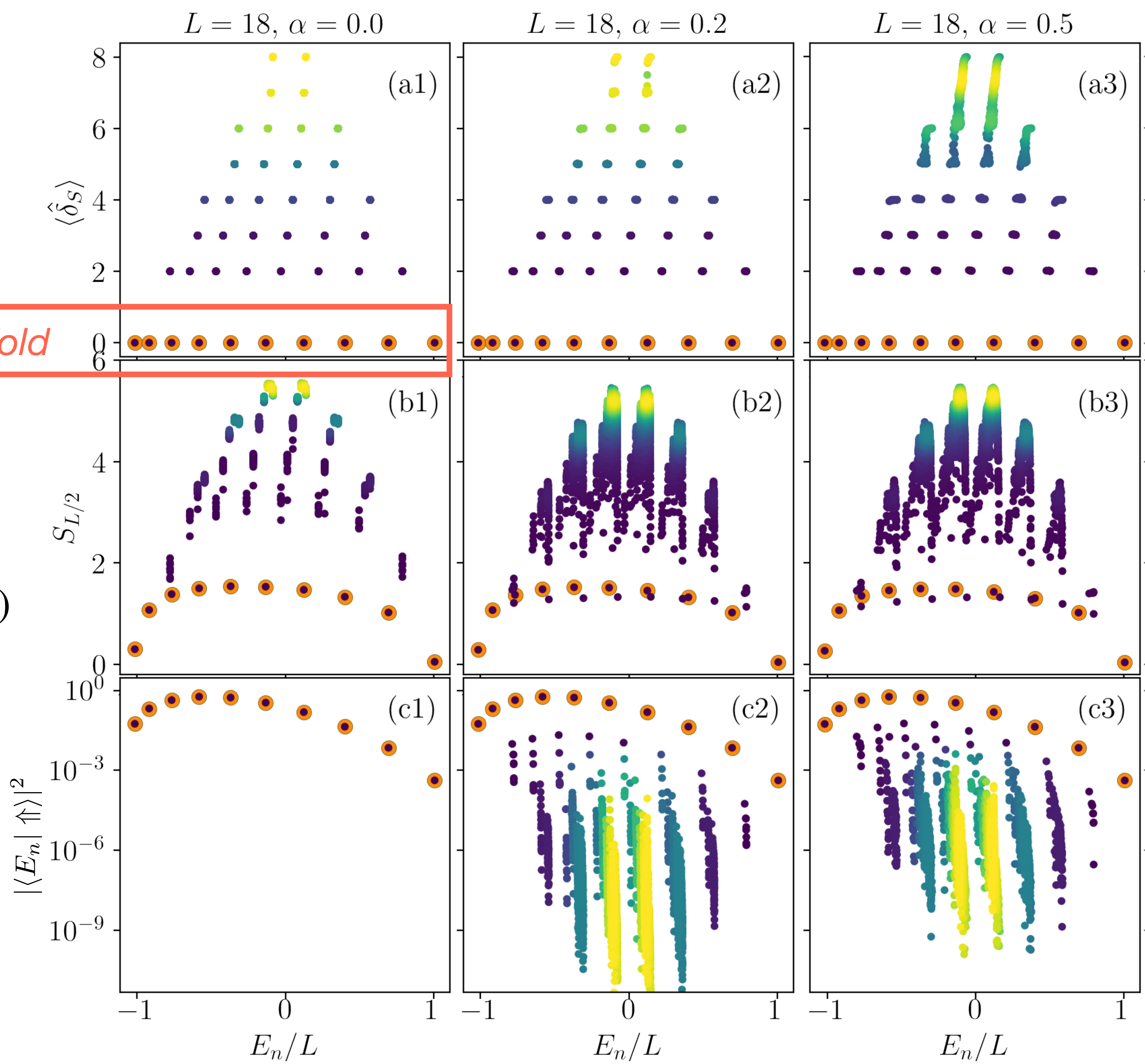
$\delta_S = L/2 - S$

$\delta_S = 0$: *Dicke manifold*

ii) Half-chain entanglement entropy

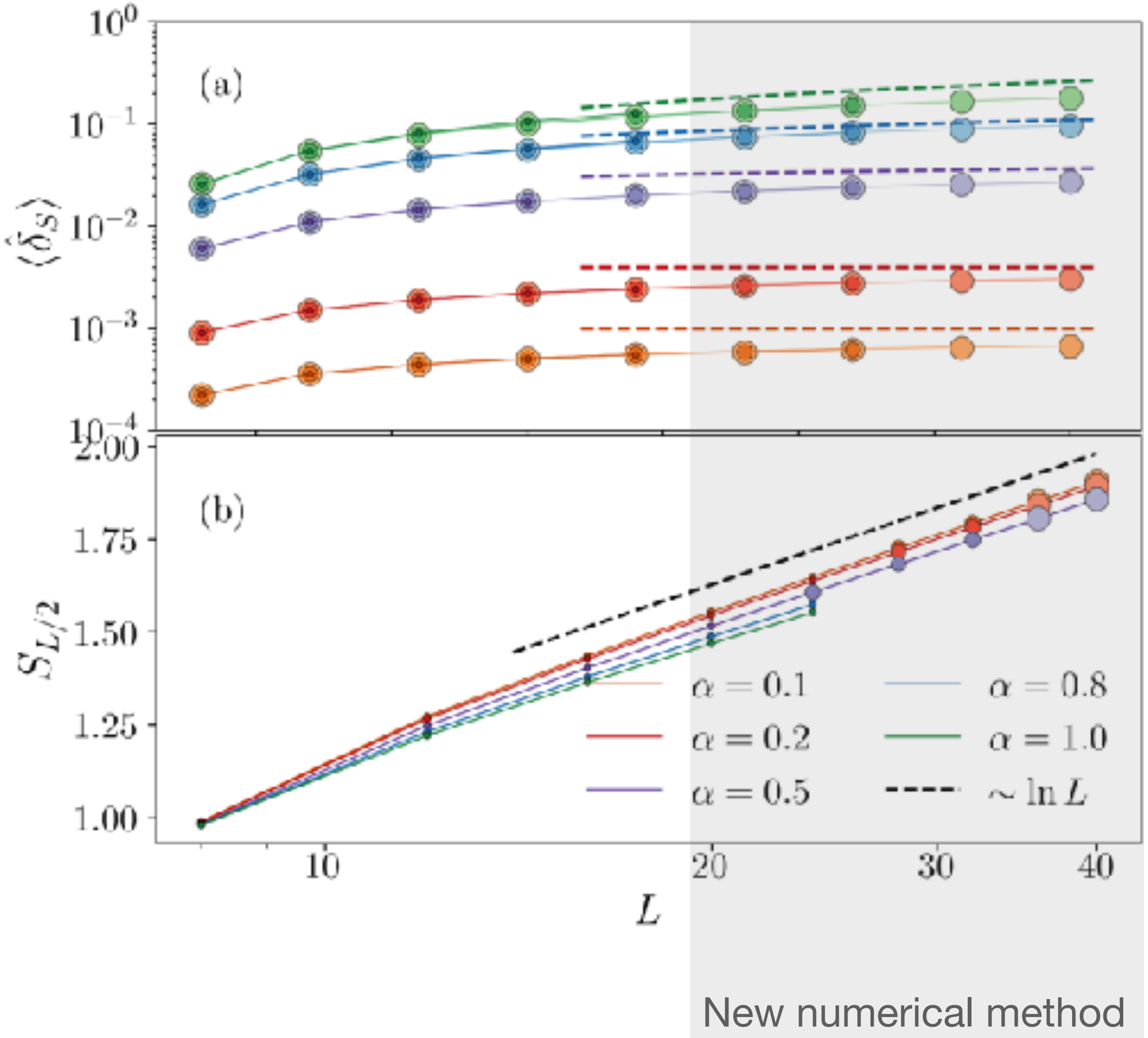
$S_{L/2} = \mathrm{Tr}(\rho_{\mathrm{L/2}} \log \rho_{\mathrm{L/2}})$

iii) Overlap with spin-coherent state

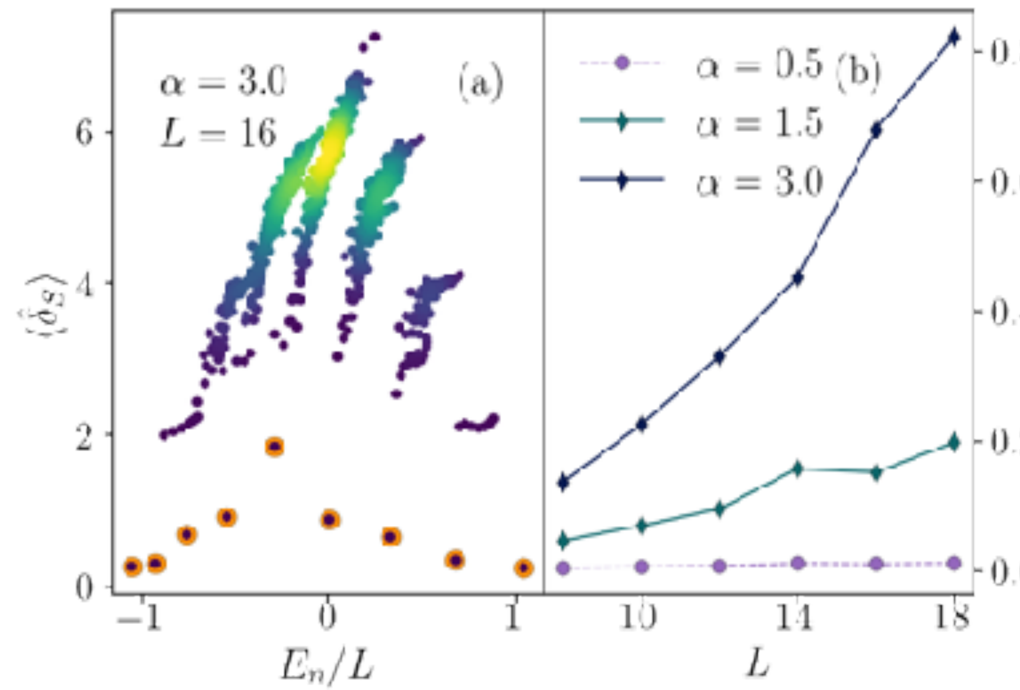


Large-spin scars vs chaotic spectrum

Scaling with system size?



Larger α :



Stability of scars for $0 < \alpha \lesssim 1$?

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Theory: $\alpha = 0$ spectrum

$$H = -J \sum_{i,j} \sigma_i^x \sigma_j^x - g \sum_i \sigma_i^z$$

$\sum_{i,j} \sigma_i^x \sigma_j^x \rightarrow (S^x)^2$

$\sum_i \sigma_i^z \rightarrow S^z$

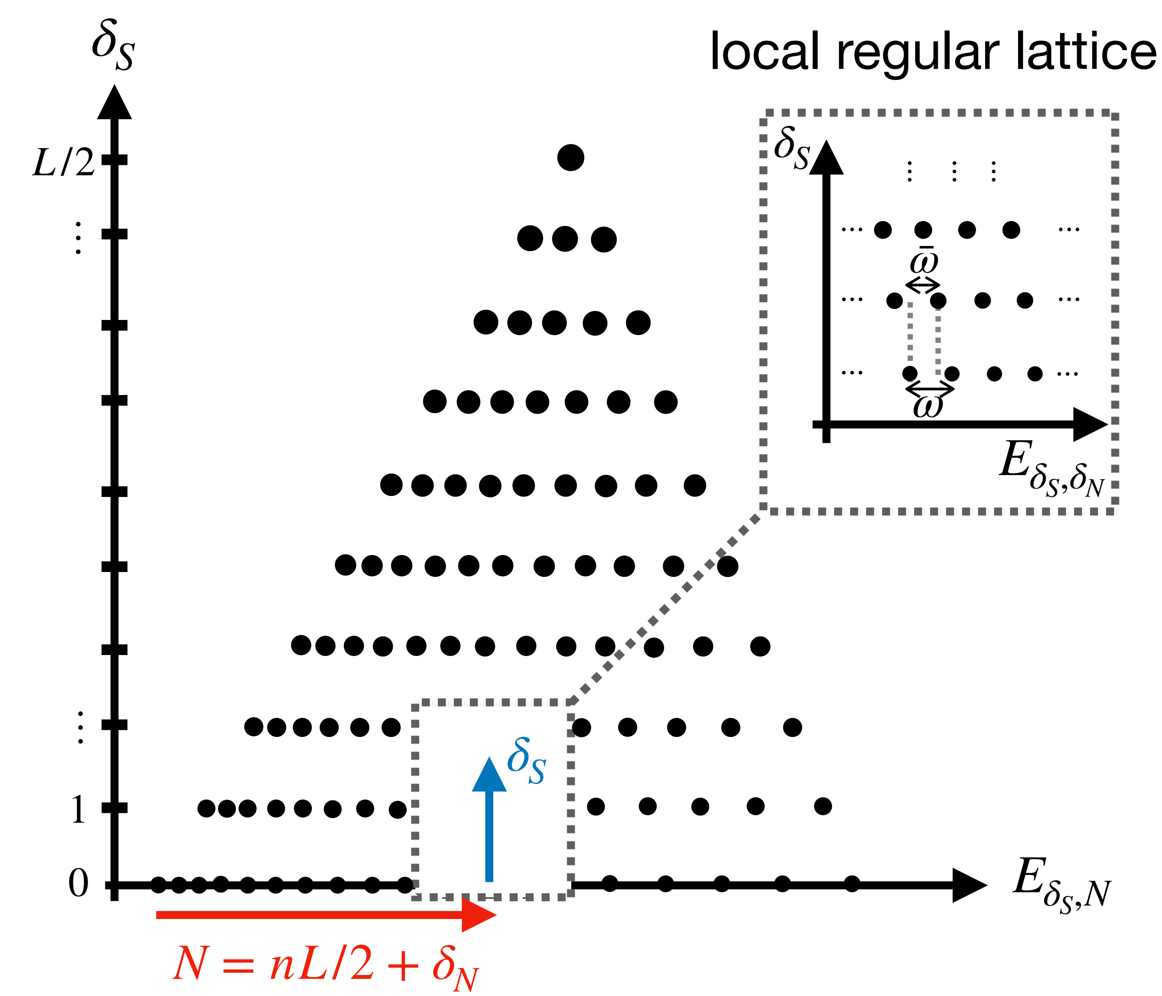
Quantum numbers:

$$\delta_S = L/2 - S$$

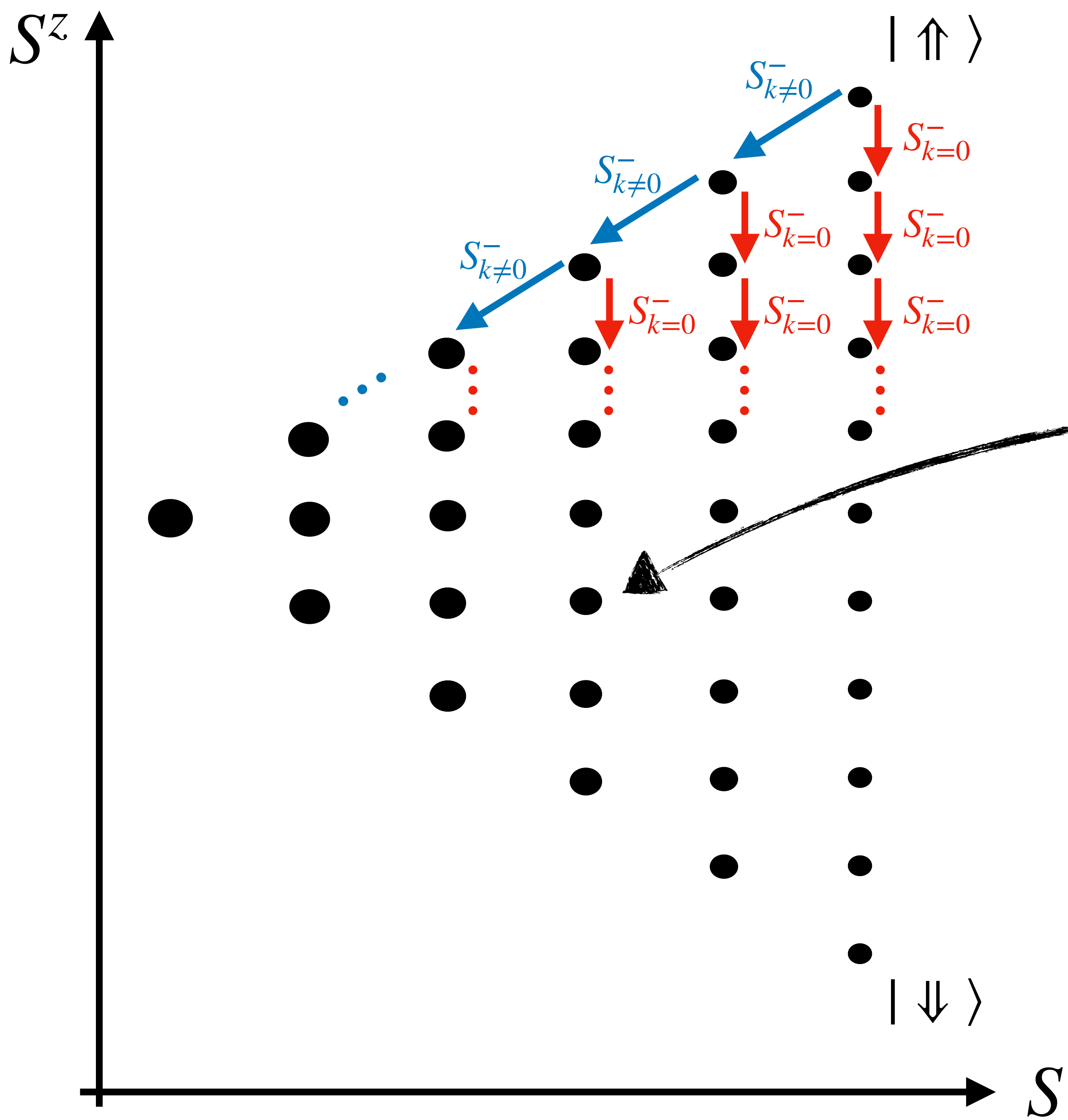
$$N = 0, 1, \ldots, 2S$$

Large-L semiclassical spectrum:

$$E_{\delta_S, N} \sim L\,\mathscr{E}(n) + \omega(n)\,\delta_N + \bar{\omega}(n)\,\delta_S + \mathcal{O}\left(\frac{1}{L}\right)$$



Magnon labelling of unperturbed states



$\implies SU(2)$ towers
labelled by magnon occupation

$|\{k_1, \ldots, k_{\delta_S}\}\ ;\ S^z\rangle$

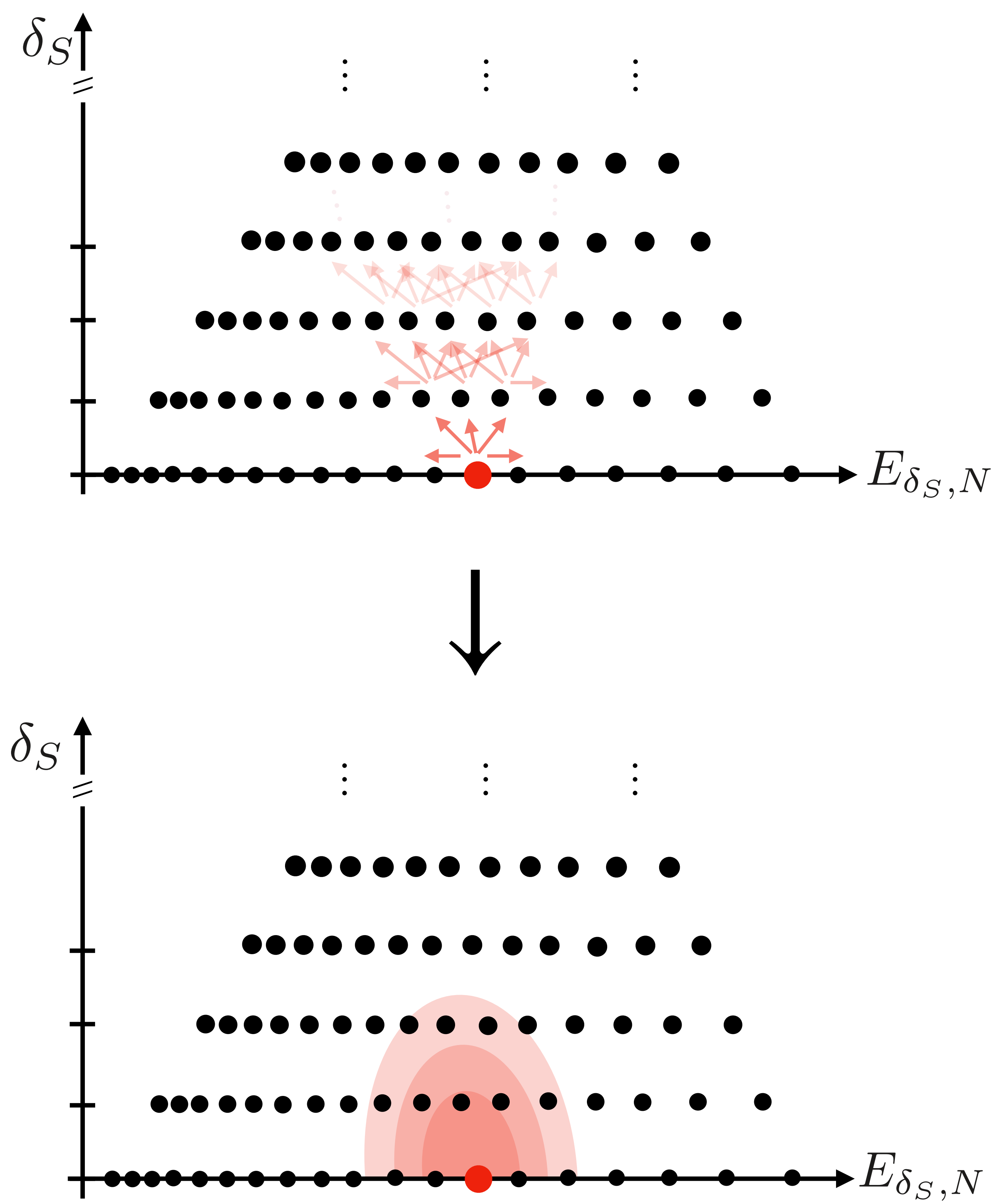
$\alpha = 0$ spectrum:

$S^z \to N$

$|\{k_1, \ldots, k_{\delta_S}\}\ ;\ N\rangle$

$\alpha = 0$ eigenstates labelled by "inert" magnons

Intuition for eigenstate localization



$H_{\alpha\neq0}$:

off-resonant creation/destruction of magnons

small α :

eigenstate localization in subspace

$\delta_S \ll L/2, \quad |\delta_N| \ll L$

Collective spin and spin-waves

$$H_\alpha = -J\sum_{i<j}^{L}\frac{\sigma_i^x\sigma_j^x}{|i-j|^\alpha} - h\sum_i^L \sigma_i^z$$

Fourier space: $H_\alpha = H_{\alpha=0} + V_\alpha$

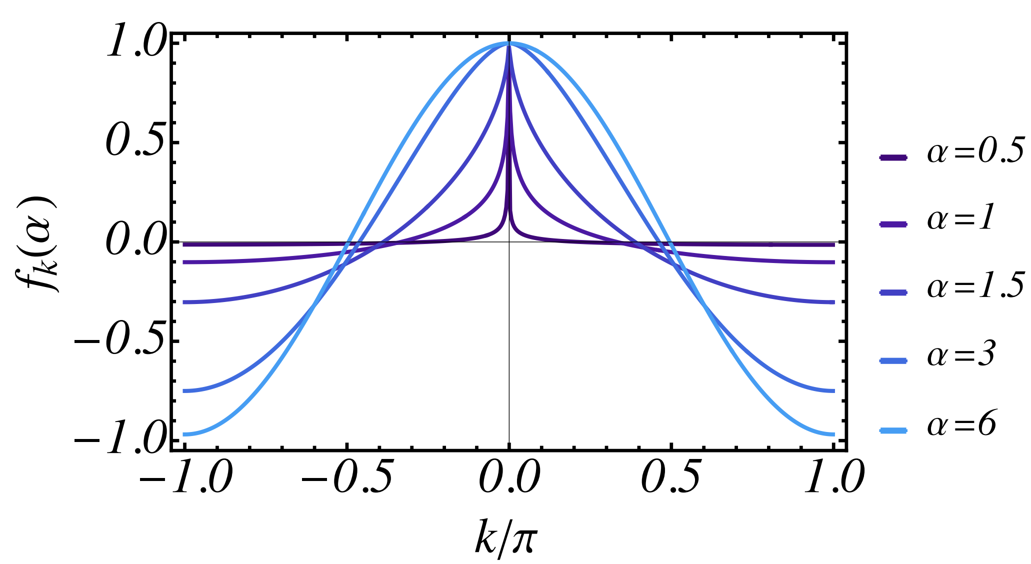
$$H_{\alpha=0} = -J(S^x)^2 - gS^z$$

$k = 0$ part

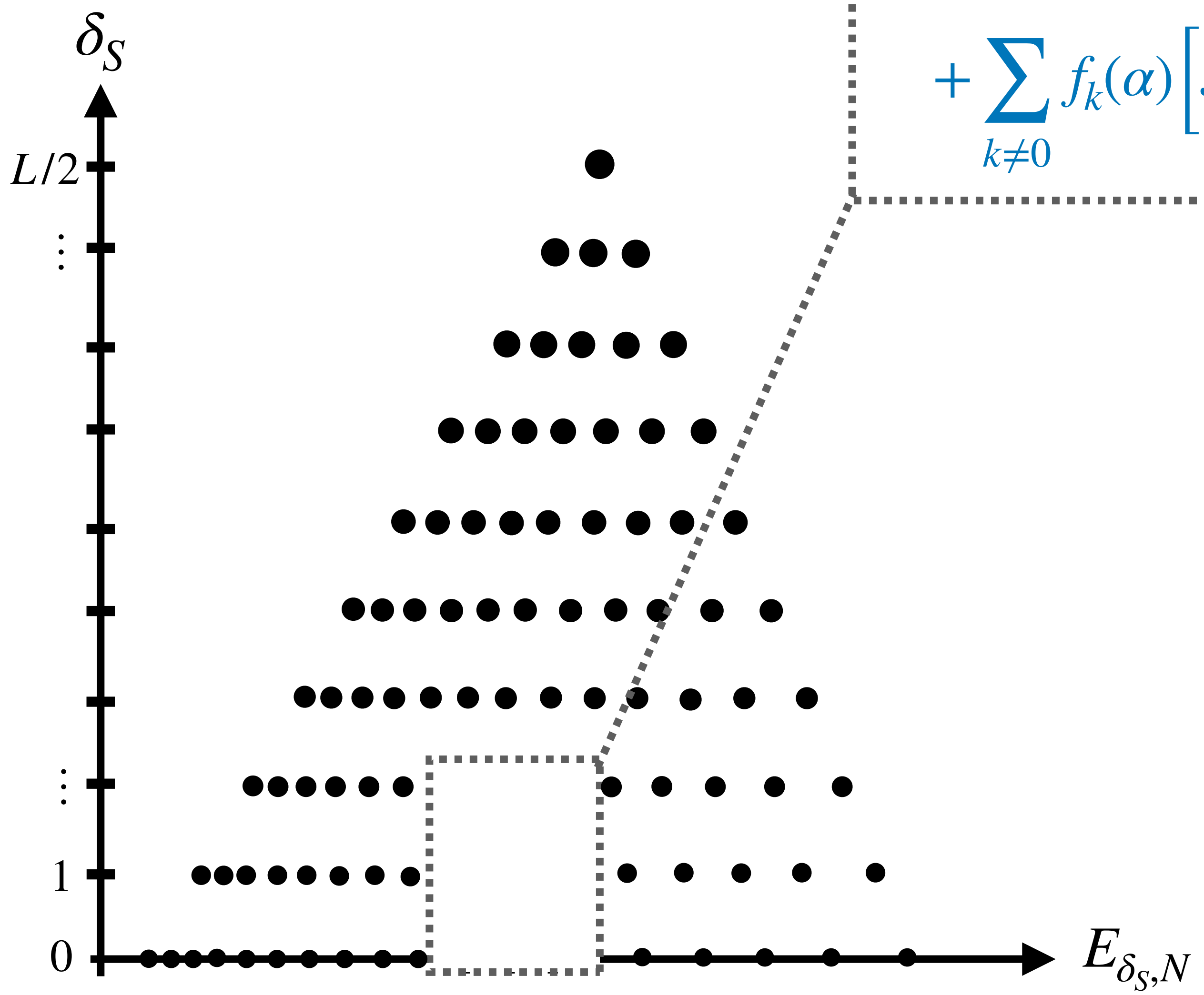
$$V_\alpha = -J\sum_{k\neq 0} f_k(\alpha)\left(\tilde{S}_k^+\tilde{S}_{-k}^- + \tilde{S}_k^-\tilde{S}_{-k}^+ + \tilde{S}_k^+\tilde{S}_{-k}^+ + \tilde{S}_k^-\tilde{S}_{-k}^-\right)$$

$k \neq 0$ part

α-dependent couplings:



Effective rotor-magnon Hamiltonian



$$\hat{H}_{\alpha,\text{eff}} = L\mathscr{E} + \omega\,\hat{\delta}_N + \bar{\omega}\sum_{k\neq 0}\tilde{b}_k^\dagger \tilde{b}_k$$
$$+ \sum_{k\neq 0} f_k(\alpha)\left[J(\hat{\varphi})\left(\tilde{b}_k^\dagger \tilde{b}_k + \tilde{b}_{-k}\tilde{b}_{-k}^\dagger\right) + K(\hat{\varphi})\,\tilde{b}_{-k}\tilde{b}_k + K^*(\hat{\varphi})\,\tilde{b}_k^\dagger \tilde{b}_{-k}^\dagger\right]$$

$\Longrightarrow$ effective **quantum impurity model**

Analytical diagonalization of the rotor-magnon H_{eff}

$\hat{H}_{\alpha,\text{eff}}$ exactly solvable *away from resonances* ($\omega \neq p\bar{\omega}$)

entangling ansatz
$$\hat{S} \;=\; \sum_{k\neq 0}\left[F_k(\hat{\varphi})\left(\tilde{b}_k^\dagger\tilde{b}_k+\tilde{b}_{-k}\tilde{b}_{-k}^\dagger\right)+G_k(\hat{\varphi})\,\tilde{b}_{-k}\tilde{b}_k+G_k^*(\hat{\varphi})\,\tilde{b}_k^\dagger\tilde{b}_{-k}^\dagger\right]$$

$$e^{i\hat{S}}\,\hat{H}_{\alpha,\text{eff}}\,e^{-i\hat{S}} \;=\; L\mathscr{E}(\alpha)+\omega\,\hat{\Delta}_N+\sum_{k\neq 0}\bar{\omega}_k(\alpha)\tilde{\beta}_k^\dagger\tilde{\beta}_k$$

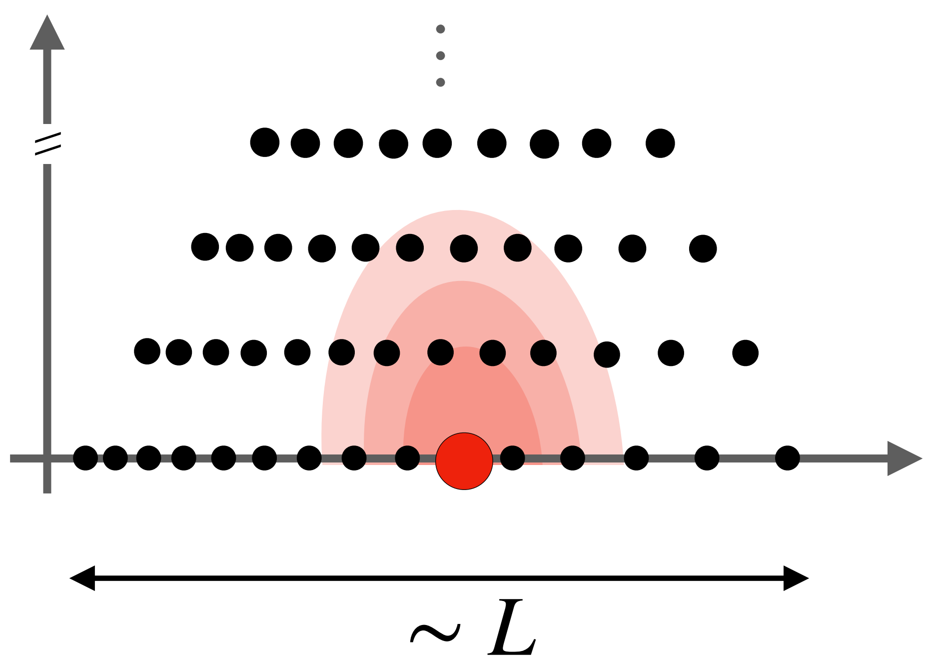
$\Longrightarrow$ Continuity in α !

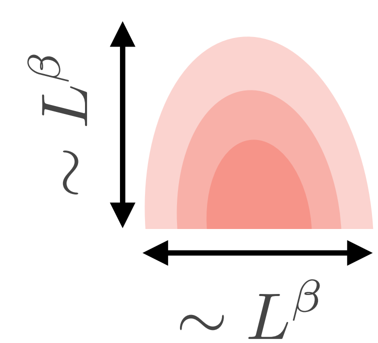
Self-consistency of eigenstate localization

Calculation:

$$\langle \hat{\delta}_S \rangle \sim \sqrt{\langle \hat{\delta}_N^2 \rangle} \sim \sum_{k\neq 0} |f_k(\alpha)|^2 \sim \begin{cases} \text{finite} & \text{for } 0 < \alpha < 1/2\,, \\ \log L & \text{for } \alpha = 1/2\,, \\ L^{2\alpha-1} & \text{for } 1/2 < \alpha < 1\,. \\ c(\alpha)\cdot L & \text{for } \alpha > 1 \end{cases}$$

Eigenstate self-consistency $\delta_S \ll L/2, \quad |\delta_N| \ll L \quad \Longleftrightarrow \quad 0 < \alpha < 1$





Self-consistent quantum many-body scars for $0 < \alpha < d$

I. Numerical analysis of spectrum

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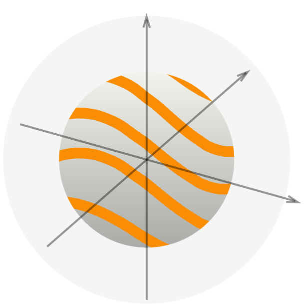
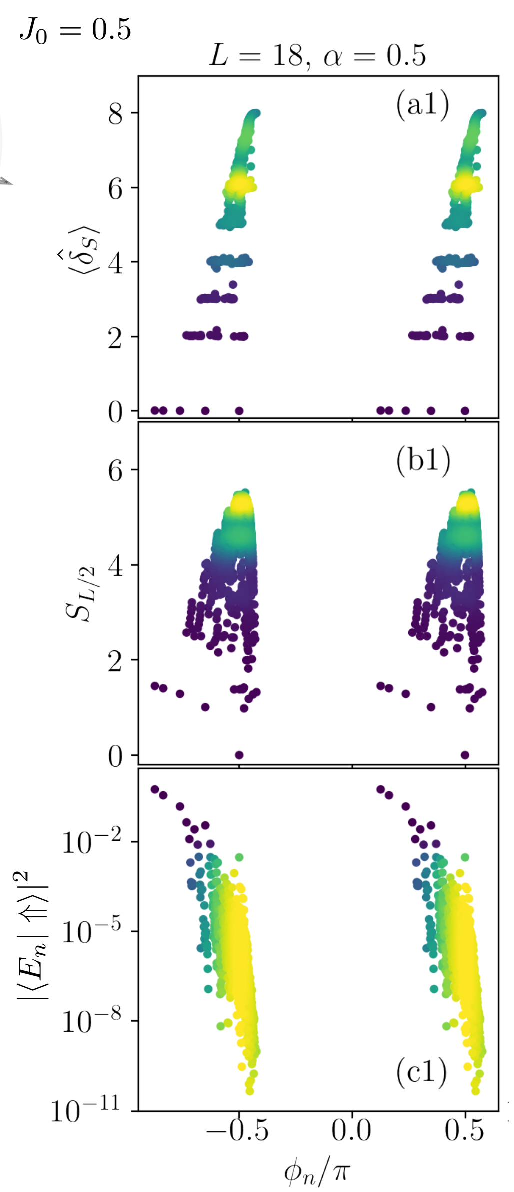
Prediction: Instability of scars from mean-field chaos

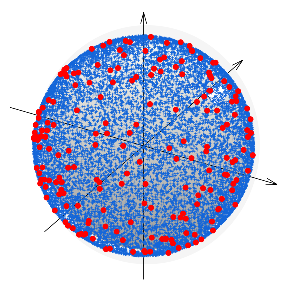
Quantum many-body kicked top:

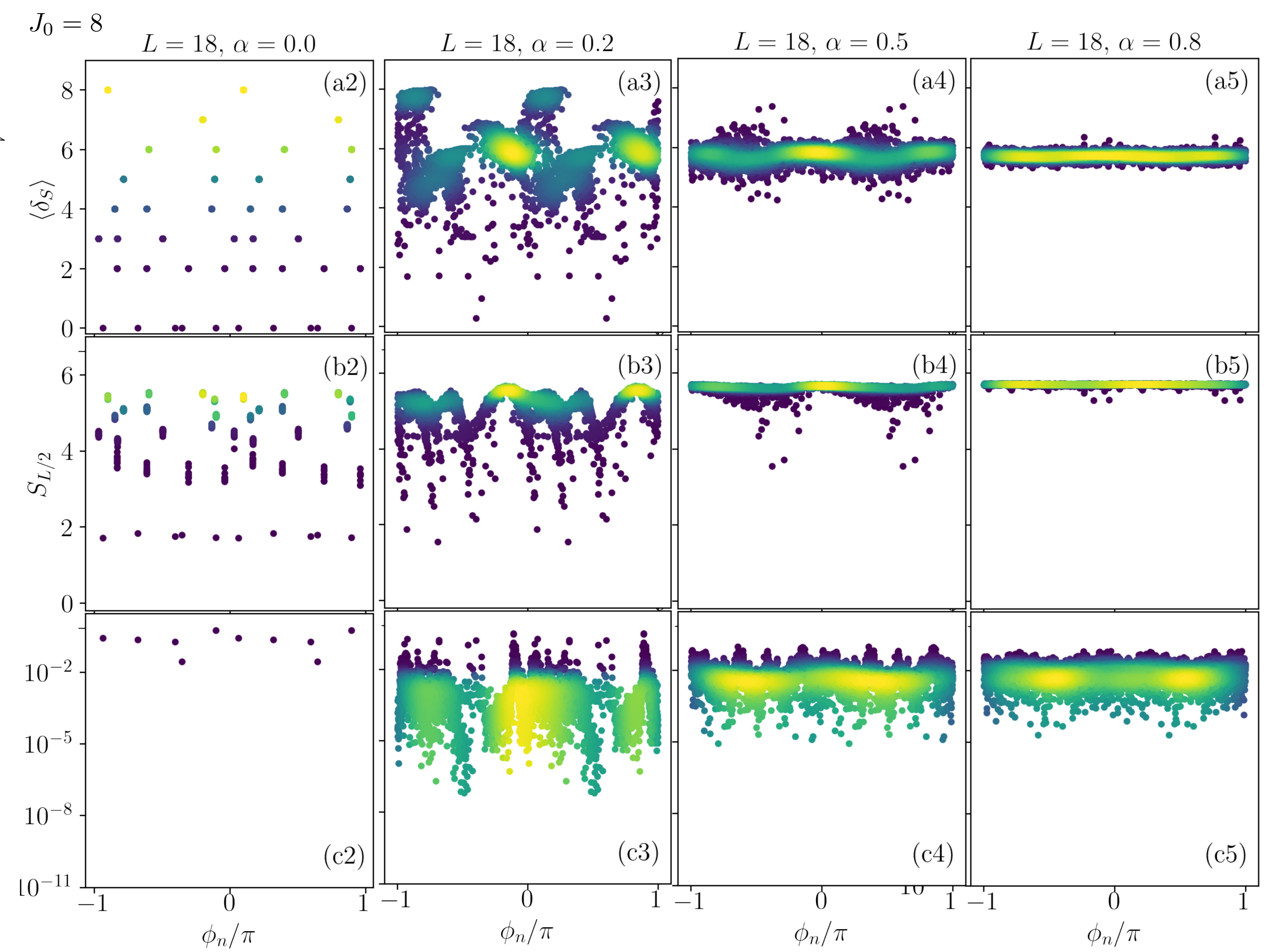
$$\hat{H}_\alpha(t) = \begin{cases} -\dfrac{J_0}{\mathcal{N}_{\alpha,L}} \displaystyle\sum_{j=1}^{L} \sum_{r=1}^{L/2} \dfrac{\hat{\sigma}_j^x \hat{\sigma}_{j+r}^x}{r^\alpha} & t \in \left[-\dfrac{T}{4}, \dfrac{T}{4}\right) \mod T \\ -h \displaystyle\sum_{j=1}^{L} \hat{\sigma}_j^z & t \in \left[\dfrac{T}{4}, \dfrac{3}{4}T\right) \mod T \end{cases}$$

$\alpha = 0$: semiclassical integrability-chaos crossover **Haake, ...**

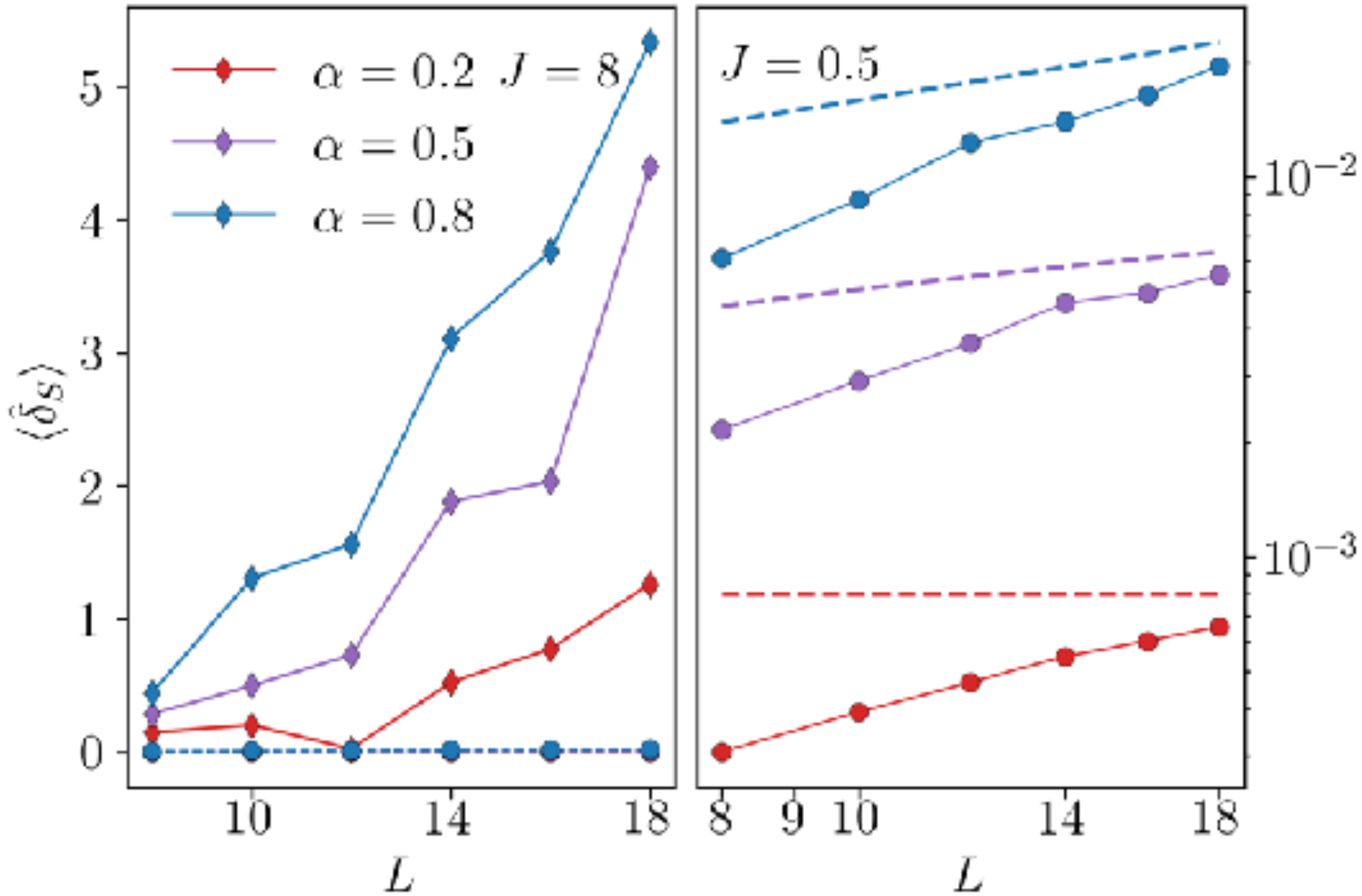








Scaling vs system size



Semiclassical chaos destroys $\alpha \neq 0$ quantum many-body scarring

I. Numerical analysis of spectrum

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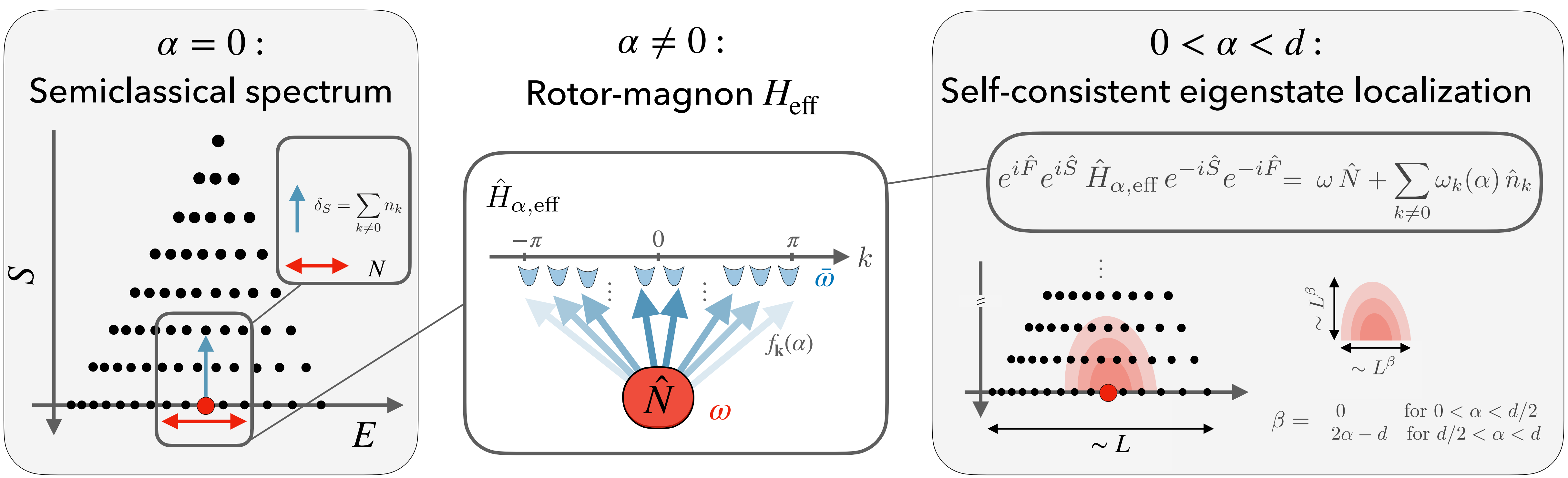
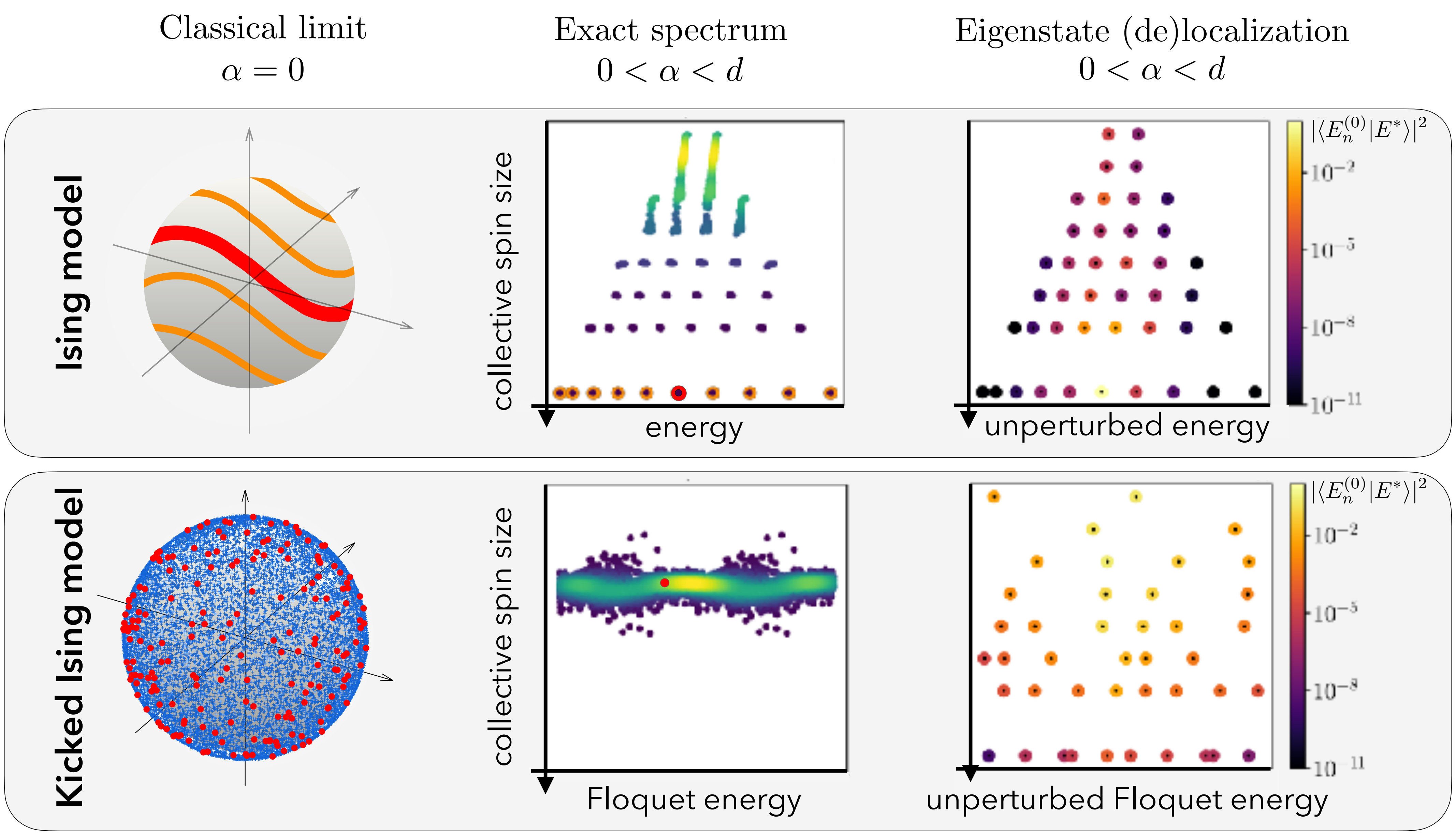
Summary and conclusions (1)

General conditions for robust scars:

- Classical integrability of mean-field Hamiltonian
- Slowly decaying interactions $0 < \alpha < d$



Summary and conclusions (2)



***Robust* Dicke-like states with $\alpha \neq 0$ $\rightarrow$ Useful metrological applications?**