# Verifying the metrological usefulness of Dicke states with collective measurements

## *Precision bound for phase estimation for noisy Dicke states*

**Iagoba Apellaniz**[1]

*Work together with B. Lücke [2], J. Peise [2], C. Klempt [2] & G. Tóth [1,3,4]*

[1] *Department of Theoretical Physics, University of the Basque Country UPV/EHU.*
[2] *Institut für Theoretische Physik, Leibniz Universität Hannover.*
[3] *IKERBASQUE, Basque Foundation for Science.*
[4] *Wigner Research Centre for Physics, Hungarian Academy of Sciences.*

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# Outline

Introduction and motivation

Precision bound with few collective measurements

- The parameter dependence of the precision, $(\Delta\theta)^2(\theta)$
- The optimal parameter value, $\theta_{\mathrm{opt}}$

Application of the result

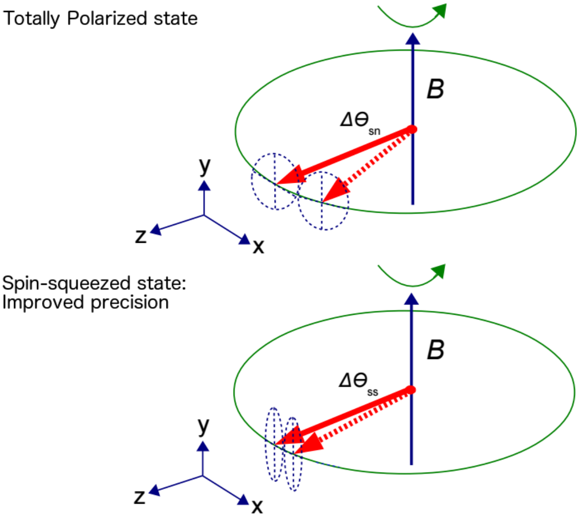
- Testing our approach for various theoretical models
- Using the result for experimental data

Conclusions

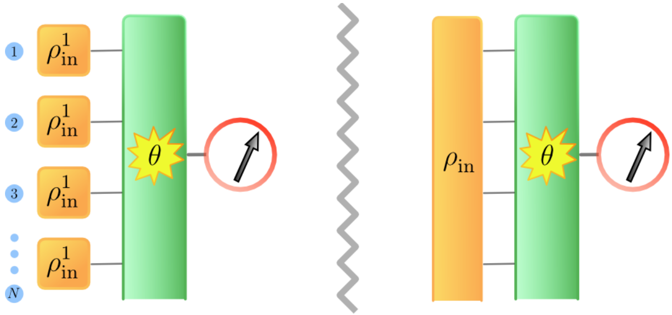
# Introduction

- Quantum Metrology **exploits the quantumness** of a many-body system to improve the precision $(\Delta\theta)$ of the estimation problem.
- **Entanglement** is a resource for such improvement.
- Many experiments create quantum states with **large scale** entanglement.
- It is important to find ways to verify their metrological usefulness with **simple measurements**.

# Metrology with polarized states



# The precision bounds and their scaling




$$\Delta\theta \sim \frac{1}{\sqrt{m}} \quad \rightarrow \quad \Delta\theta \sim \frac{1}{\sqrt{mN}} \quad \rightarrow \quad \Delta\theta \sim \frac{1}{\sqrt{mN^{\beta}}} \quad \rightarrow \quad \Delta\theta \sim \frac{1}{\sqrt{m}N}$$

# Our metrological setup

Basic task in metrology: estimate the homogeneous magnetic field, $B_y$, with N qubits.

- Interaction with the magnetic magnetic field

$$H = \gamma B_y J_y.$$

- Unitary dynamics

$$U = \exp(-i\theta J_y),$$

where $\theta = \gamma B_y t$.

- Collective observables

$$J_l = \sum_{i=1}^{N} \frac{\sigma_l^{(i)}}{2},$$

for $l \in \{x, y, z\}$ and where $\sigma_l^{(i)}$ are Pauli matrices.

# Theoretical precision bounds

- Shot-noise limit

$$\Delta\theta \geq \frac{1}{\sqrt{N}}.$$

- Heisenberg limit

$$\Delta\theta \geq \frac{1}{N}.$$

- Precision bound when local noise affects the system and when $N \gg 1$,

$$\Delta\theta \geq \frac{1}{\alpha\sqrt{N}}.$$

[V. Giovannetti, et al., Science **306** 1330-1336 (2004)]
[B. Escher, et al., Nat. Phys. **7** 406-411 (2011)]
[R. Demkowicz-Dobrzański et al., Nat. Commun. **3**, 1063 (2012)]

# We use unpolarized states

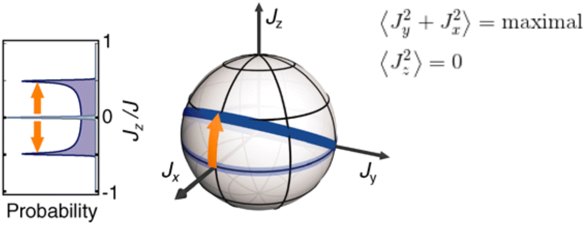
- Non-polarized states are better for metrology

$$\Delta\theta \geq 1 \Bigg/ \sqrt{2N + N^2\left(1 - \frac{\langle J_z\rangle^2}{J_{\max}^2}\right)}\ .$$

[G. Tóth, **IA**, J. Phys. A: Math. Theor. **47**, 424006 (2014)]

- Even if many experiments have been done with polarised ensembles, unpolarised ensembles are becoming trendy

[B. Lücke, et al., Science **334**, 773 (2011)]
[I. Urizar-Lanz, et al., Phys. Rev. A **88**, 013626]




[B. Lücke, et al., Science **334**, 773 (2011)]

- In this work, the measurement of $\langle J_z^2 \rangle$ has been considered to estimate $\theta$,

$$(\Delta\theta)^2 = \frac{(\Delta J_z^2)^2}{|\partial_\theta \langle J_z^2 \rangle|^2}.$$

# Precision Bound, measuring $\langle J_z^2 \rangle$

- Using the error propagation formula

$$(\Delta\theta)^2 = \frac{(\Delta J_z^2)^2}{|\partial_\theta \langle J_z^2 \rangle|^2} = \frac{\langle J_z^4 \rangle - \langle J_z^2 \rangle^2}{|\partial_\theta \langle J_z^2 \rangle|^2}.$$

- We assume that $\langle J_z^2 \rangle$ and $\langle J_z^4 \rangle$ are even functions of $\theta$, which is true in most relevant cases.
- We obtain,

$$\begin{aligned}\langle J_z^2(\theta) \rangle &= \langle J_z^2 \rangle \cos^2(\theta) + \langle J_x^2 \rangle \sin^2(\theta), \\ \langle J_z^4(\theta) \rangle &= \langle J_z^4 \rangle \cos^4(\theta) + \langle J_x^4 \rangle \sin^4(\theta) \\ &\quad + \Big( \langle \{J_z, J_x\}^2 \rangle + \langle \{J_z^2, J_x^2\} \rangle \Big) \cos^2(\theta) \sin^2(\theta).\end{aligned}$$

# Precision dynamics and optimal parameter

- Precision bound written with initial expectation values

$$(\Delta\theta)^2 = \frac{(\Delta J_x^2)^2 f(\theta) + 4\langle J_x^2\rangle - 3\langle J_y^2\rangle - 2\langle J_z^2\rangle(1+\langle J_x^2\rangle) + 6\langle J_z J_x^2 J_z\rangle}{4(\langle J_x^2\rangle - \langle J_z^2\rangle)^2},$$

$$f(\theta) := \frac{(\Delta J_z^2)^2}{(\Delta J_x^2)^2}\frac{1}{\tan^2(\theta)} + \tan^2(\theta).$$

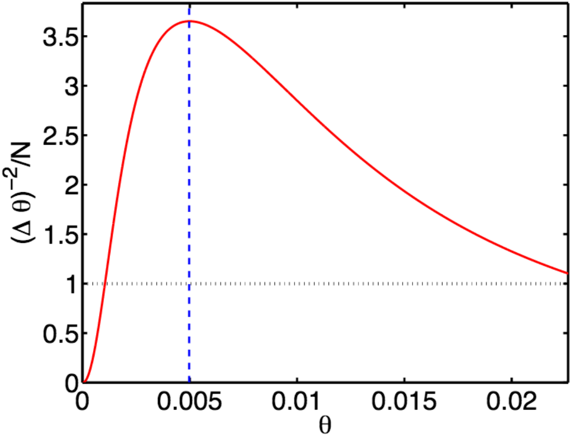
- **Optimal $\theta$** for the precision

$$\tan^2(\theta_{\text{opt}}) = \sqrt{\frac{(\Delta J_z^2)^2}{(\Delta J_x^2)^2}}.$$

- For Dicke states $|\frac{N}{2}, 0\rangle$,

$$(\Delta\theta)^2_{\text{opt}} = \frac{1}{4\langle J_x^2\rangle} = \frac{1}{F_Q[|\frac{N}{2}, 0\rangle, J_y]}.$$

# Simple example for thousands of particles

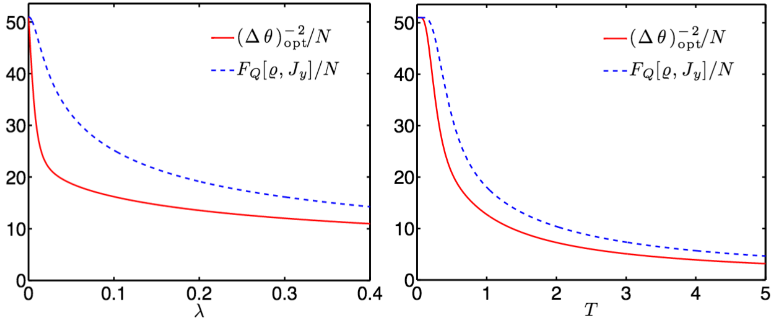



One can see that the optimal value is at $\theta_{\mathrm{opt}} \approx 0.005$.

[**IA**, B. Lücke, J. Peise, C. Klempt and G. Tóth, arxiv:1412.3426 ]

# Testing the bound against CR inequality

$N = 100$ particle system



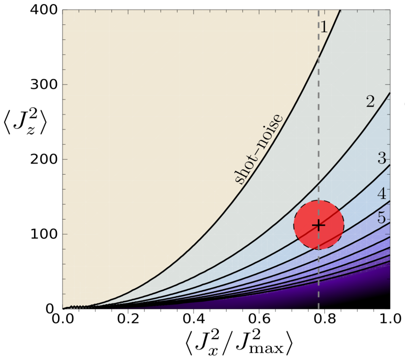

(left) Our bound compared to the Cramér-Rao bound (dashed) for the pure state, ground state of $H = J_z^2 - \lambda J_x$.

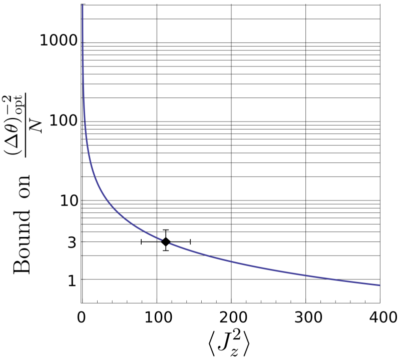
(right) The same for thermal state $\varrho \propto \sum e^{-\frac{m^2}{T}} |\frac{N}{2}, m\rangle\langle\frac{N}{2}, m|$.

# Using the bound with experimental data

We approximate $\langle J_z^4 \rangle \approx 3\langle J_z^2 \rangle^2$ and we bound the 4$^{\text{th}}$ moments with 2$^{\text{nd}}$ order ones.

$$N = 7900, \quad \langle J_x^2 \rangle = 6.1 \times 10^6 \pm 0.6 \times 10^6, \quad \langle J_z^2 \rangle = 112 \pm 31$$



# Conclusions

- We have developed a method to estimate the metrological precision for Dicke states.
- Our method needs the the second and forth moments of collective angular momentum components.
- We can also get a somewhat worse lower bound with second order moment only.
- We tested our method for an experiment with 8000 particles, creating a Dicke states in cold gases.

# Acknowledgements

**IA**, Bernd Lücke, Jan Peise, Carsten Klempt & Géza Tóth
*Verifying the metrological usefulness of Dicke states with collective measurements*
arxiv.org:1412.3426.

**THANK YOU FOR YOU ATTENTION !**

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