Multistability in quantum systems and related cavity QED experiments

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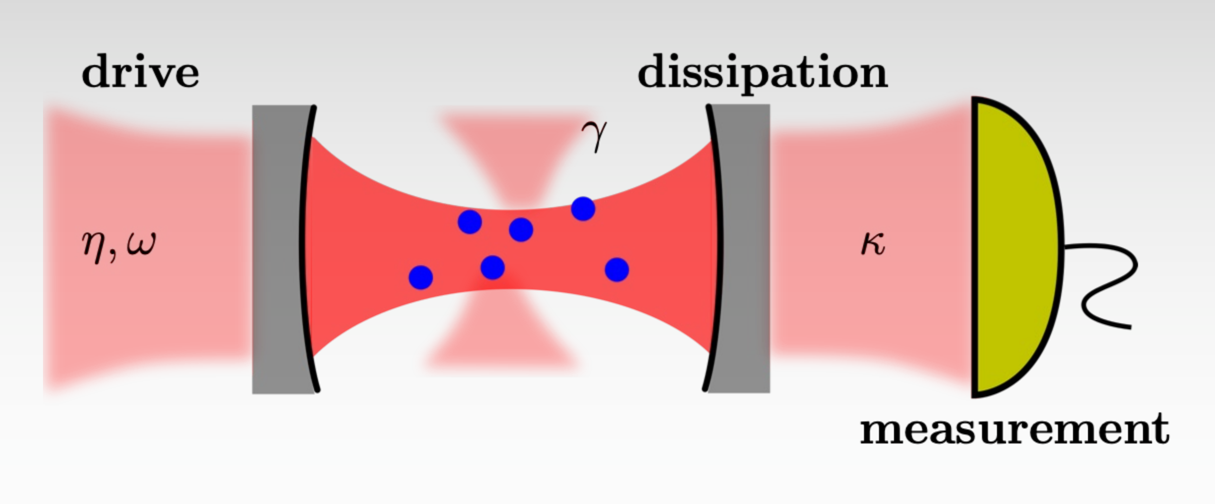
National Research, Development and Innovation Office Hungary
PROJECT FINANCED FROM THE NRDI FUND
MOMENTUM OF INNOVATION

Cavity QED: a driven-dissipative open quantum system



"many-body"

few atoms but large cooperativity

$$\mathcal{C} = \frac{\mathcal{F}}{\pi} \times N \times \frac{\sigma_A}{\mathcal{A}}$$

$$\mathcal{C} \gg 1$$

Phases

- **stationary states: dynamical equilibrium of driving and dissipation**
- **continuous measurement due to dissipation**
- **order parameter: macroscopic observable**

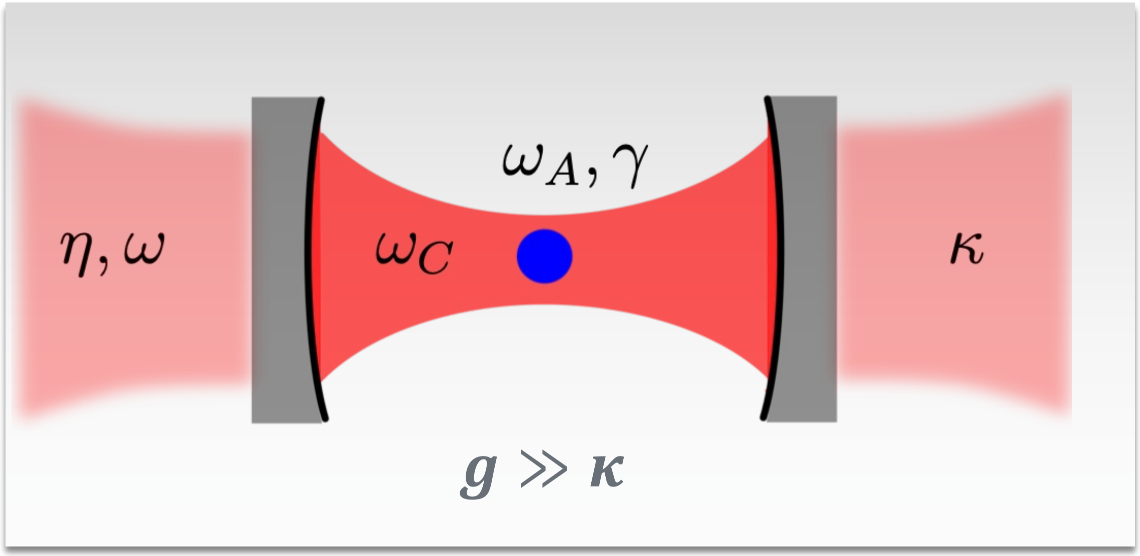
Phase transition

- **non-analytic change of the order parameter as a**
- **control parameter is continuously tuned through**
- **a critical point**

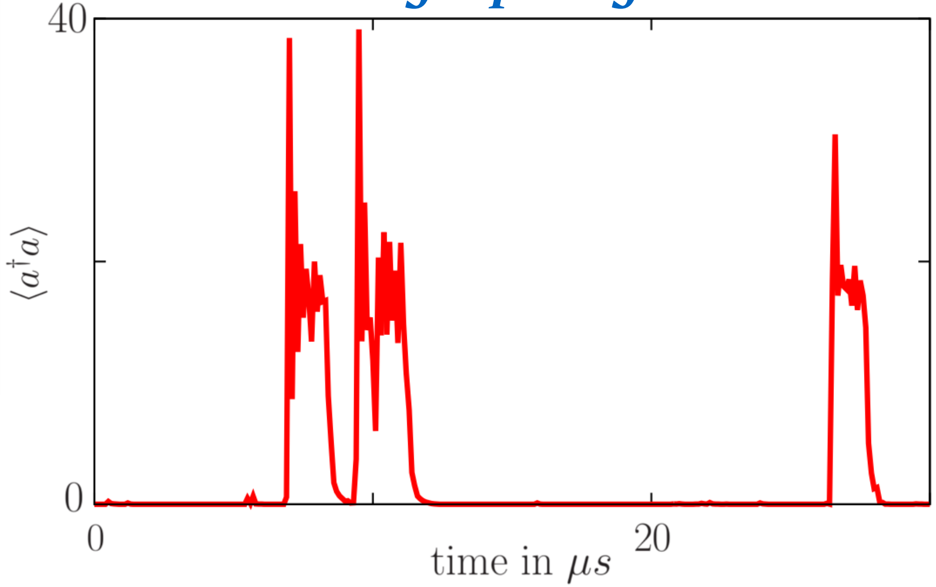
A. Vukics, A. Dombi, J. Fink, P. Domokos, Quantum (2019)

Quantum bistability

Continuously driven cavity with a strongly-coupled single atom



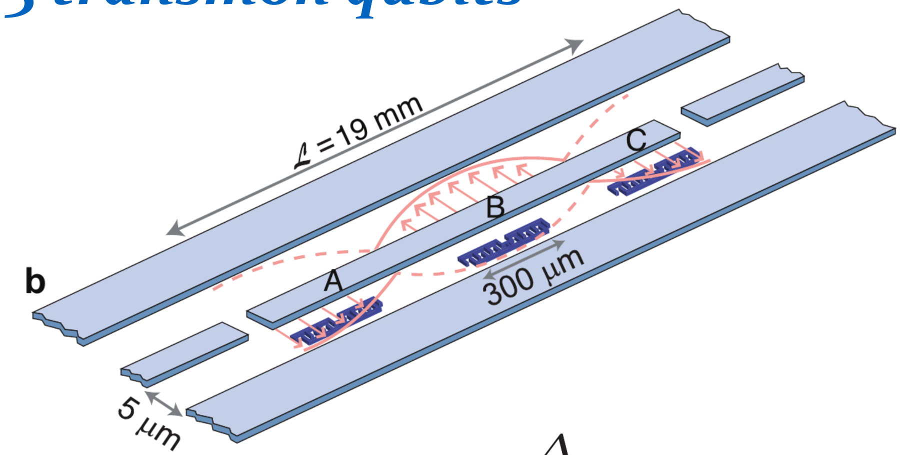
Transmission output is a random telegraph signal



H. J. Carmichael, Phys. Rev. X 5, 031028 (2025)

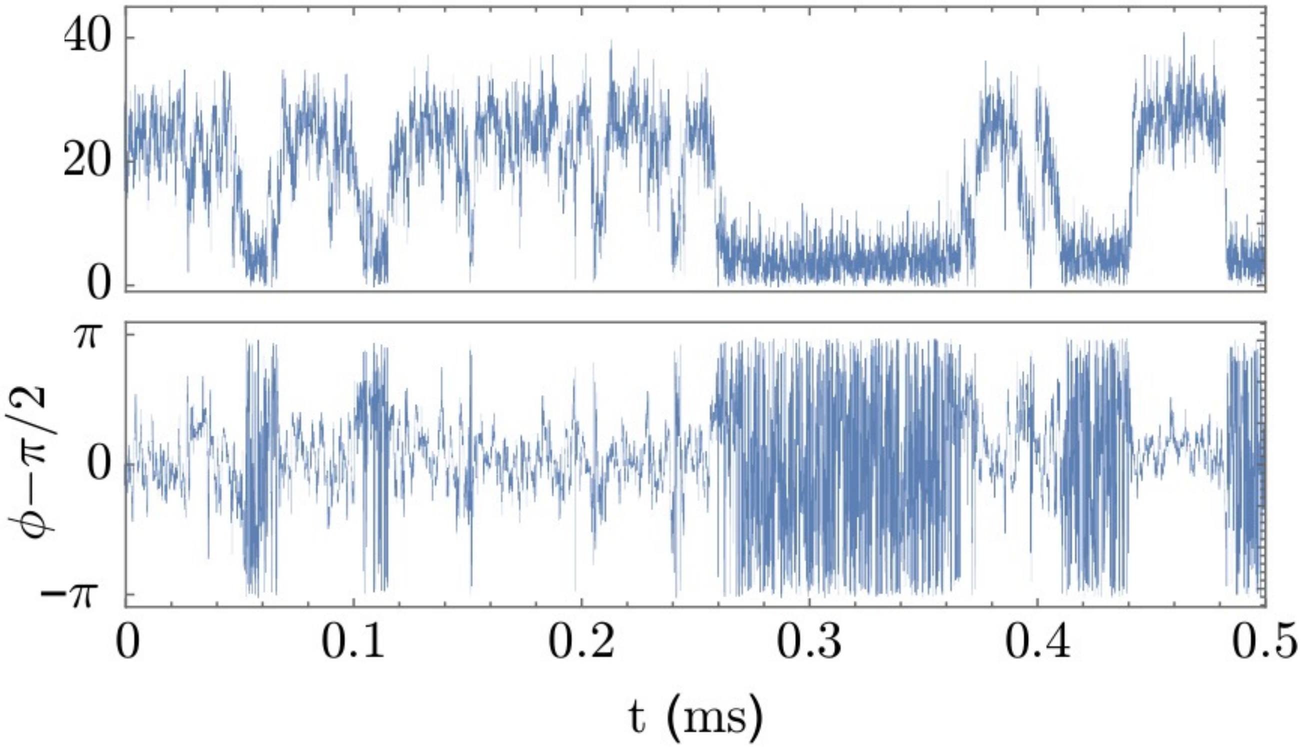
Quantum bistability observed in circuit QED

3 transmon qubits



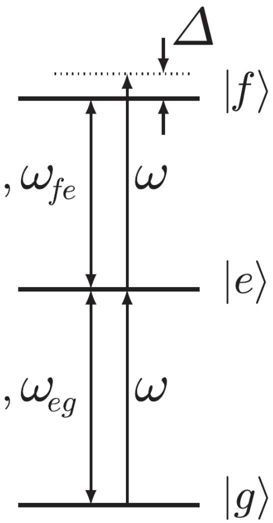
Johannes Fink

Experimental observation



$\Delta = 2\text{GHz}$

$g_2 = \sqrt{2}\, g_1$



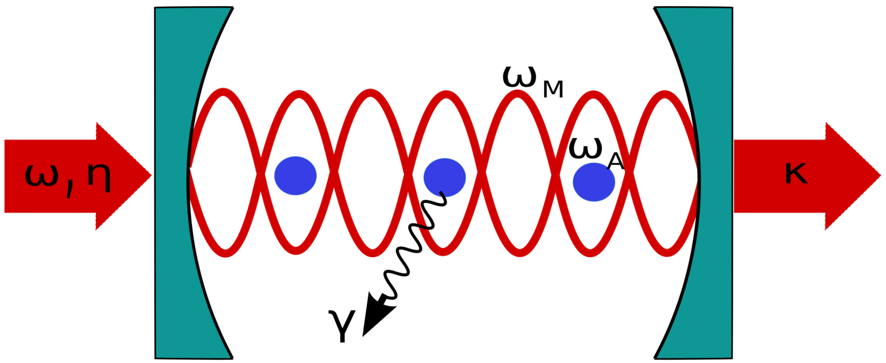
Driven Jaynes-Cummings model with damping

$$H = \omega_M\, a^\dagger a \,+\, \omega_A\, \sigma^\dagger \sigma + \, ig\left(a^\dagger \sigma - \sigma^\dagger a + a^\dagger \sigma^\dagger - \sigma a\right)$$

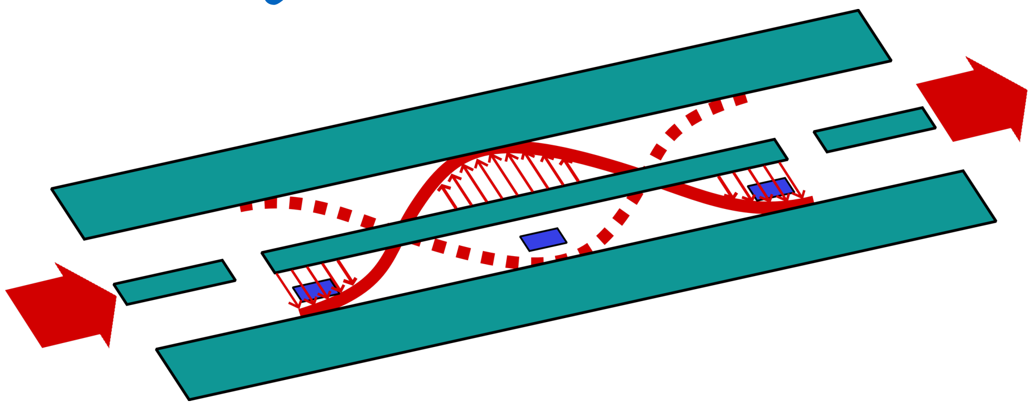
$$+\, i\eta\left(a^\dagger\, e^{-i\omega t} - a\, e^{i\omega t}\right)$$

$$\mathcal{L}\left[\hat{\rho}\right] = \kappa\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right)$$ dissipation

Cavity QED



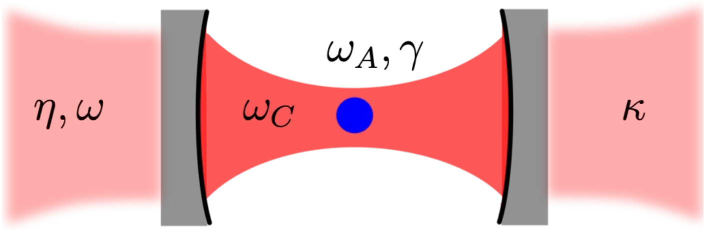
Circuit QED



Transmission spectrum of single-atom CQED systems

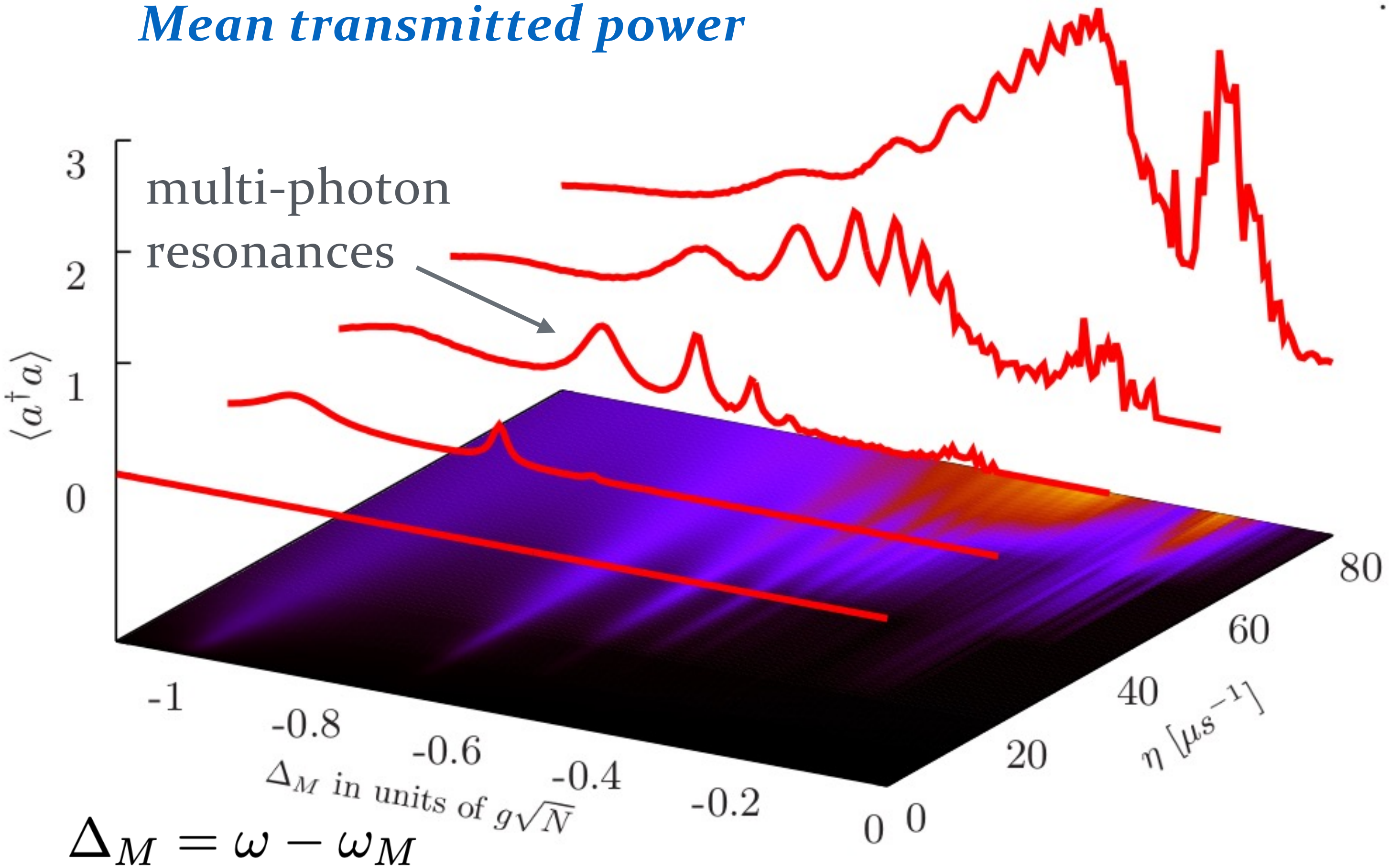
Strong coupling

$$g = 100\kappa$$



$$\omega_A = \omega_M$$

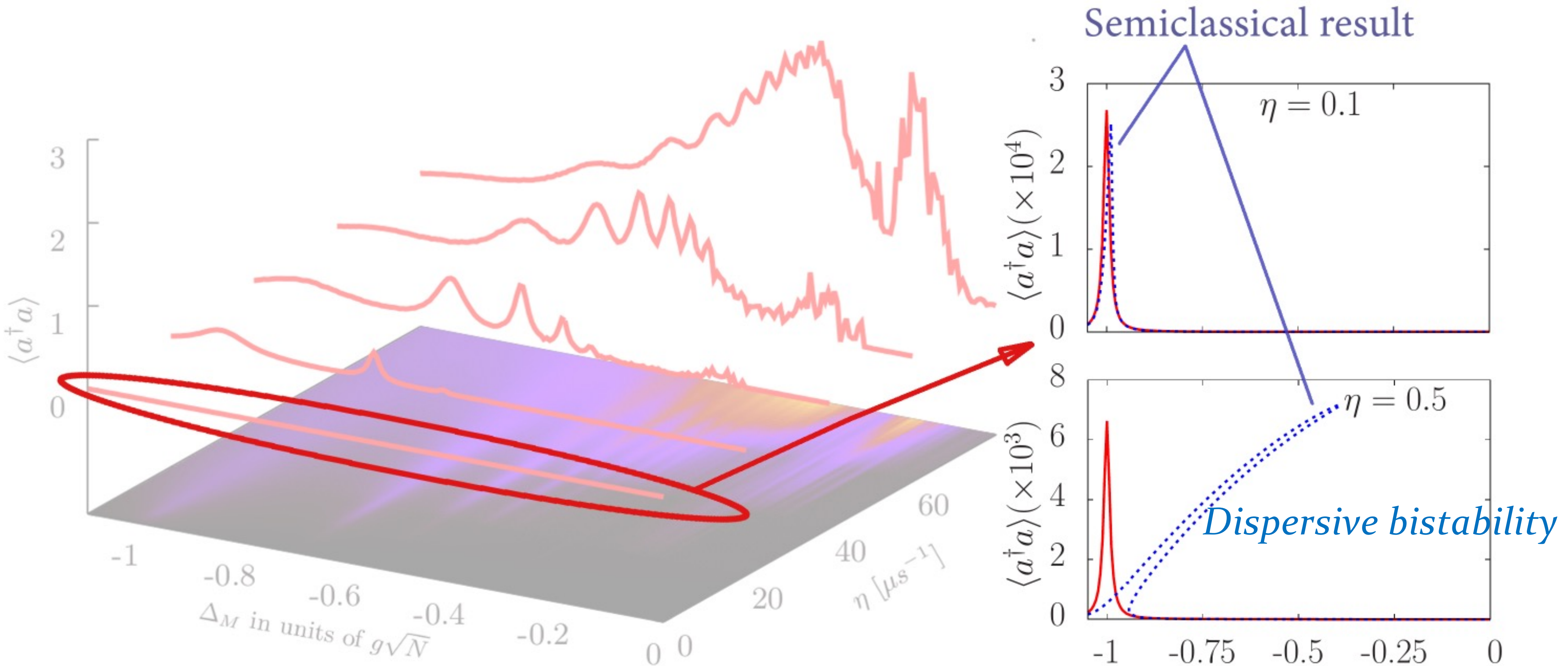
Mean transmitted power



$$\Delta_M = \omega - \omega_M$$

Dombi, Vukics, Domokos, Eur. Phys. J. D (2015)

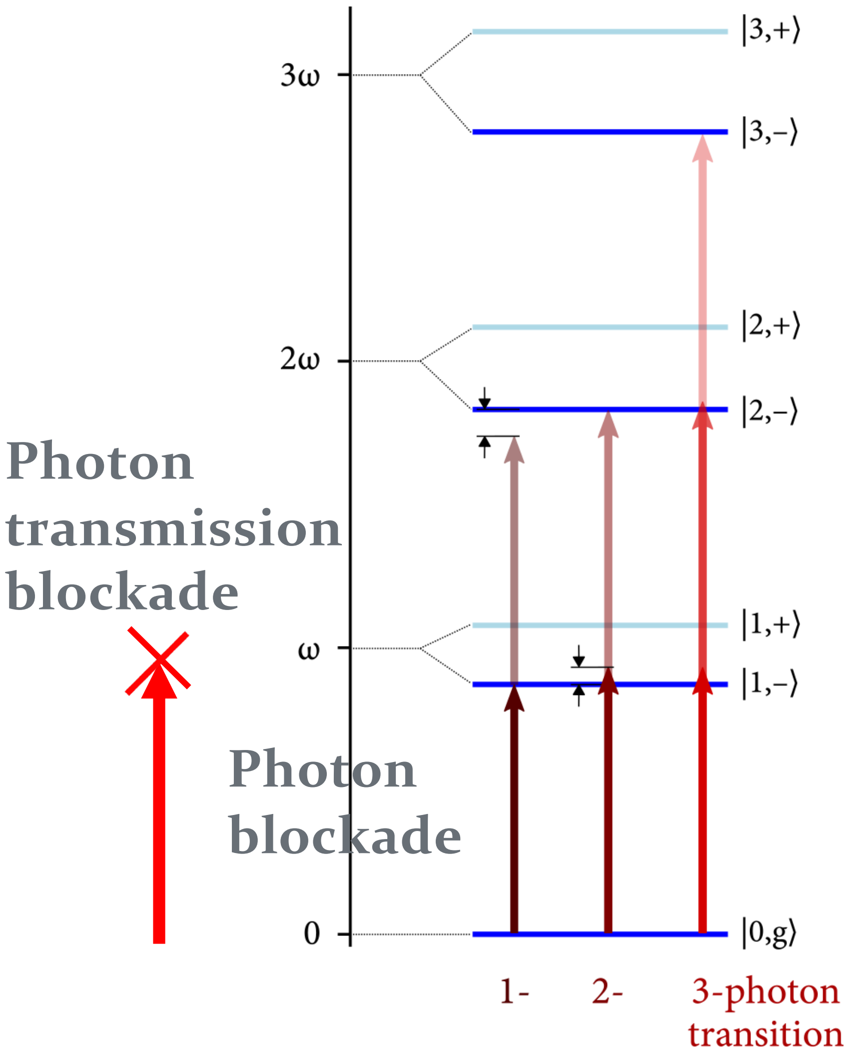
Photon-blockade

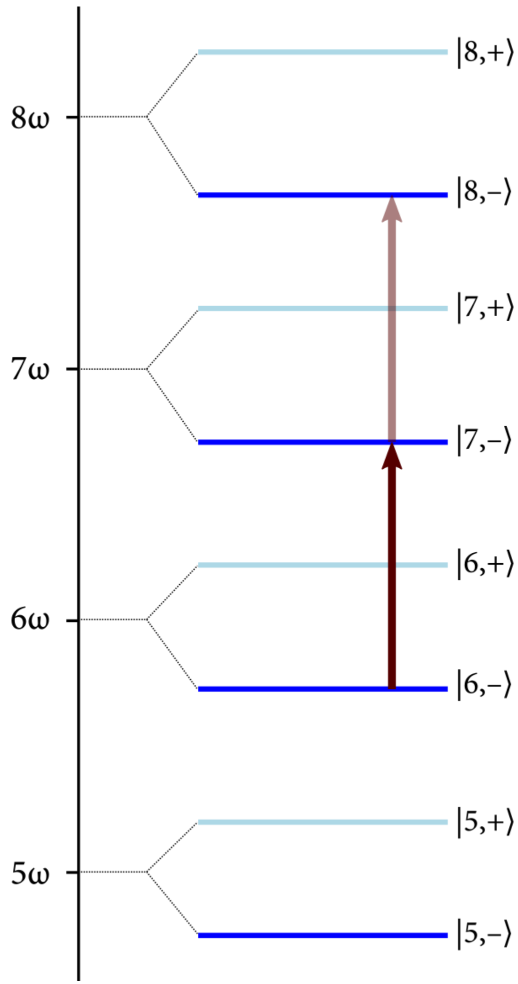


$\Delta_M = \omega - \omega_M \qquad \omega_A = \omega_M$

Dombi, Vukics, Domokos, Eur. Phys. J. D (2015)

Photon-blockade and its breakdown mechanism



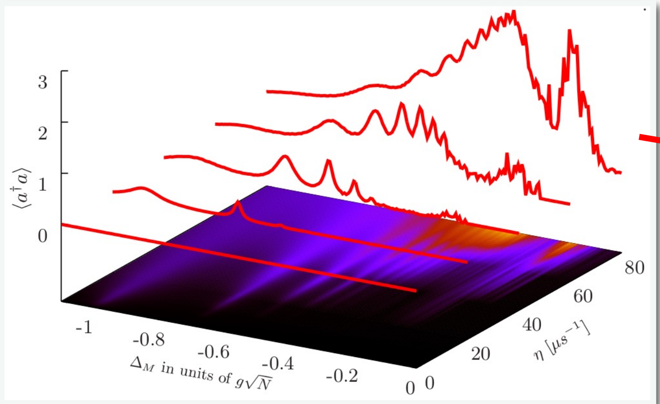


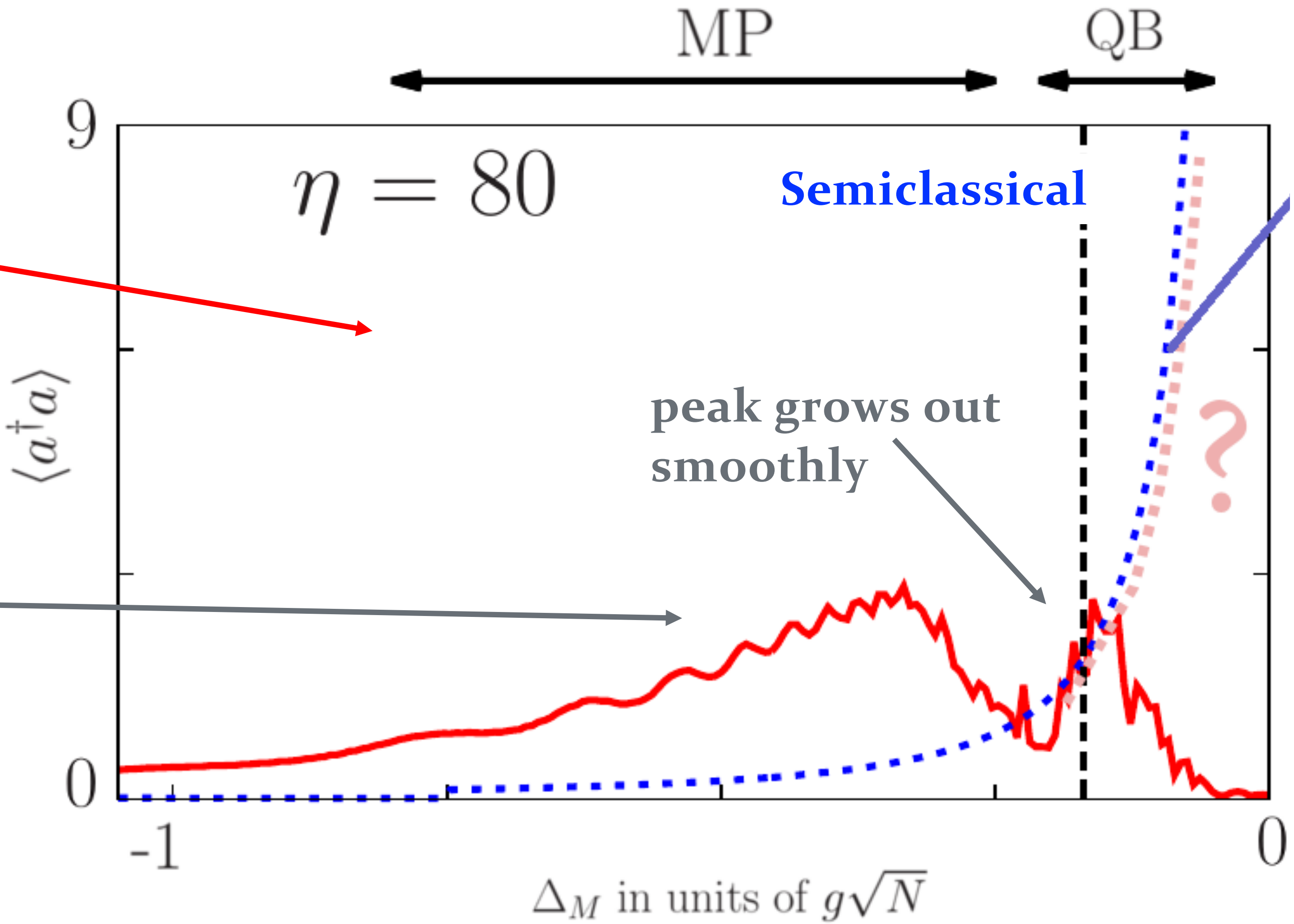
$$E_{n,\pm} = n\,\hbar\omega \pm \sqrt{n}\,g$$

$$\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \propto n^{-3/2} \to 0$$

equidistant ladder
→ hosts quasi-coherent states
→ attractor of a driven lossy oscillator

Photon-blockade-breakdown at strong drive



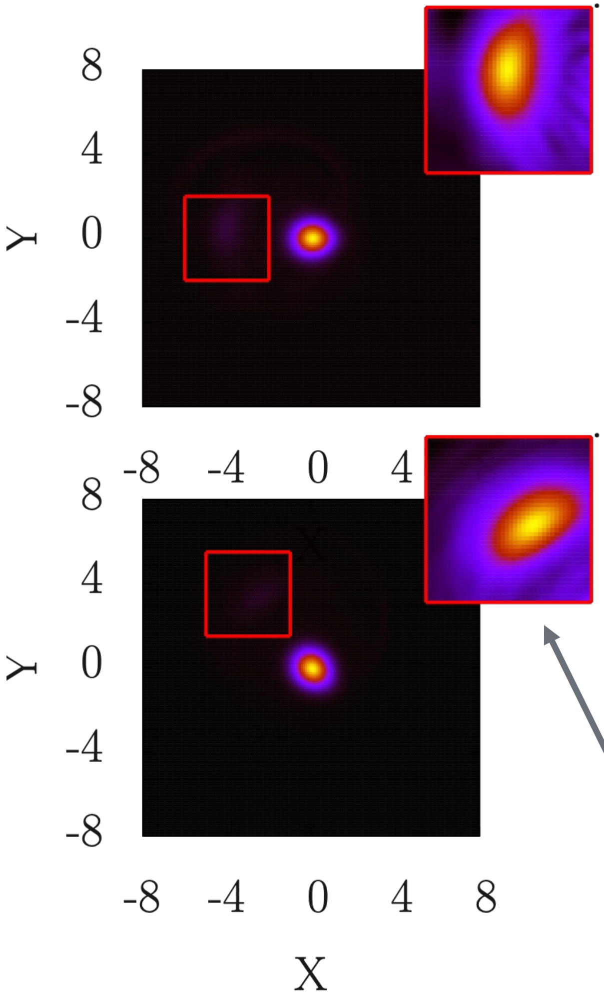


merging power broadened multi-photon resonances

Dombi, Vukics, Domokos, Eur. Phys. J. D (2015)

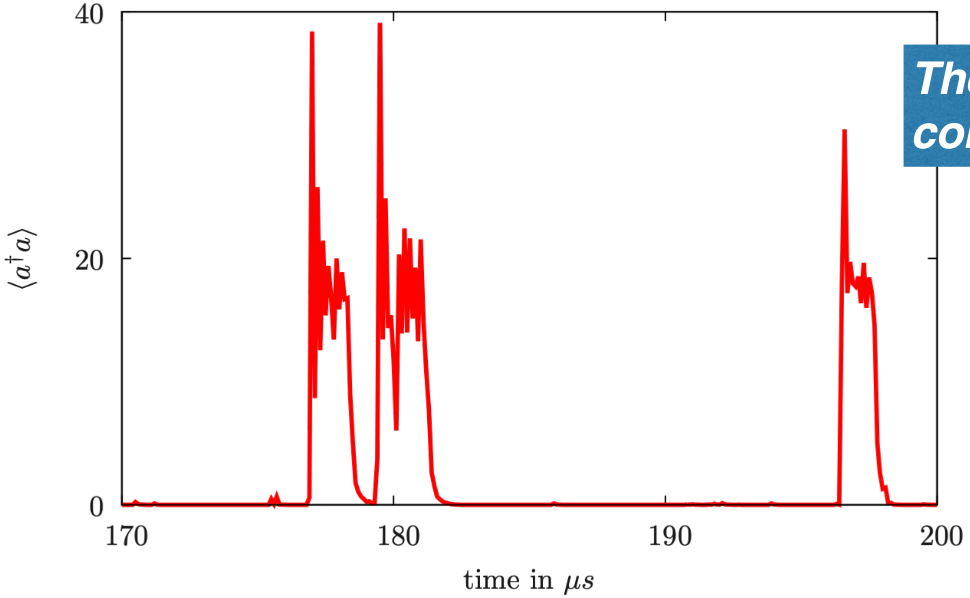
Bimodal density matrix

Phase space distribution: mixture of two semiclassical attractors



drive phase is reflected

Random telegraph signal



These states are continuously measured

Dombi, Vukics, Domokos, Eur. Phys. J. D (2015)

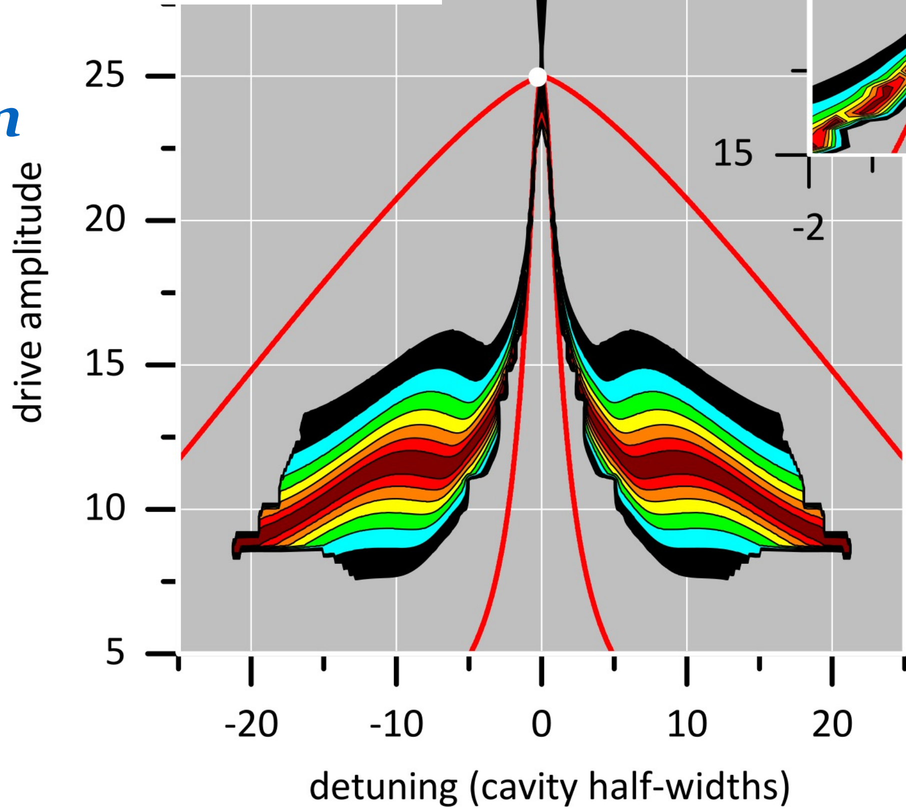
PHYSICAL REVIEW X **5,** 031028 (2015)

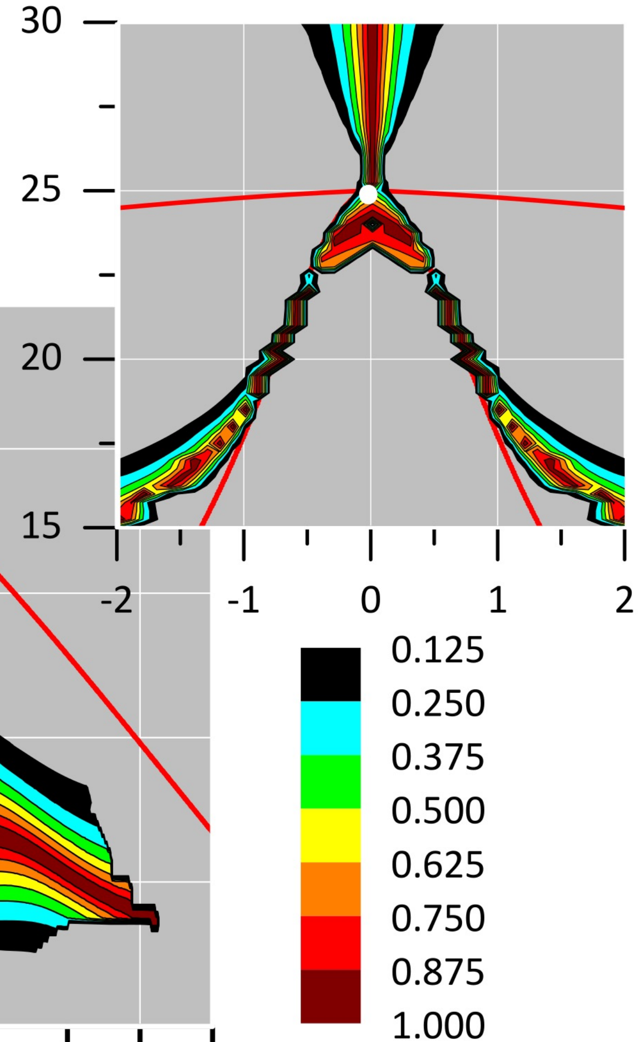
Breakdown of Photon Blockade: A Dissipative Quantum Phase Transition in Zero Dimensions

H. J. Carmichael

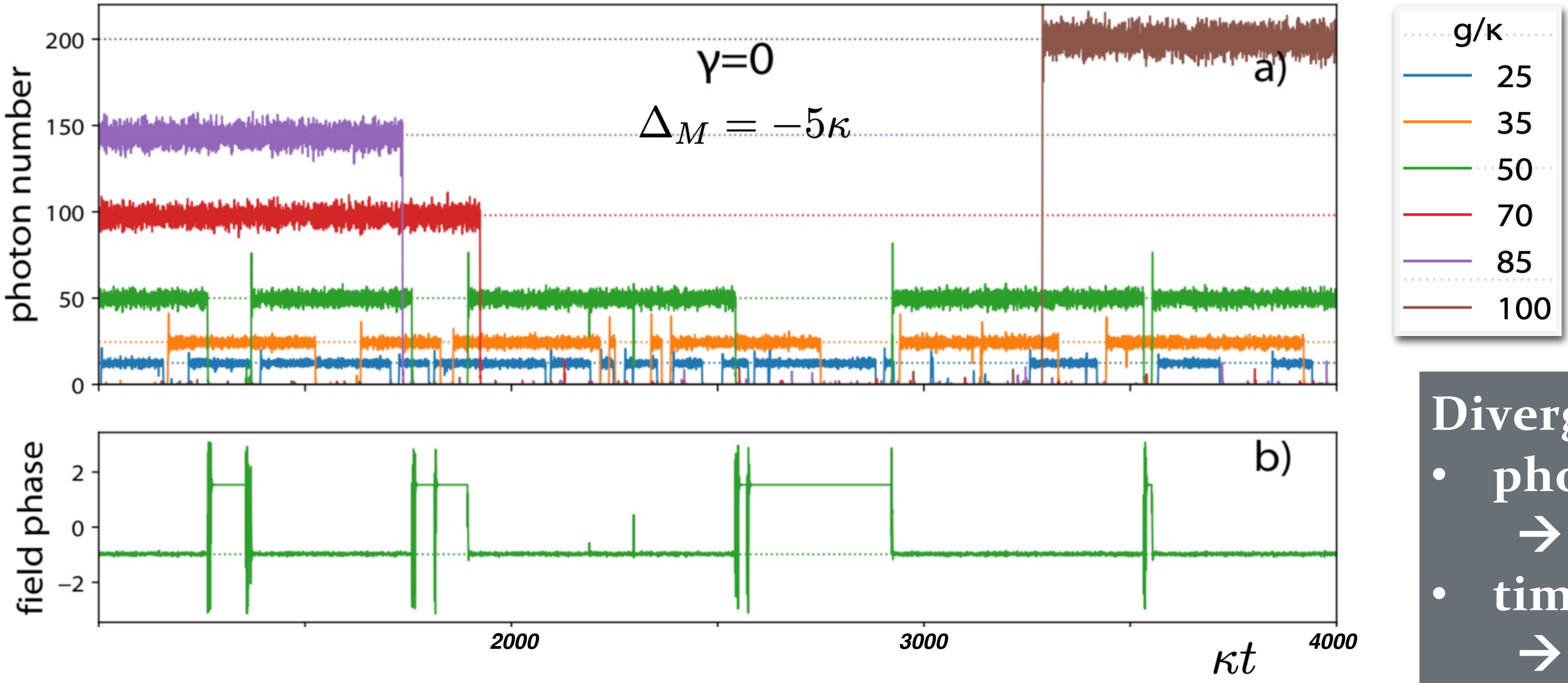
The Dodd-Walls Centre for Photonic and Quantum Technologies, Department of Physics, University of Auckland, Private Bag 92019 Auckland, New Zealand

Phase diagram





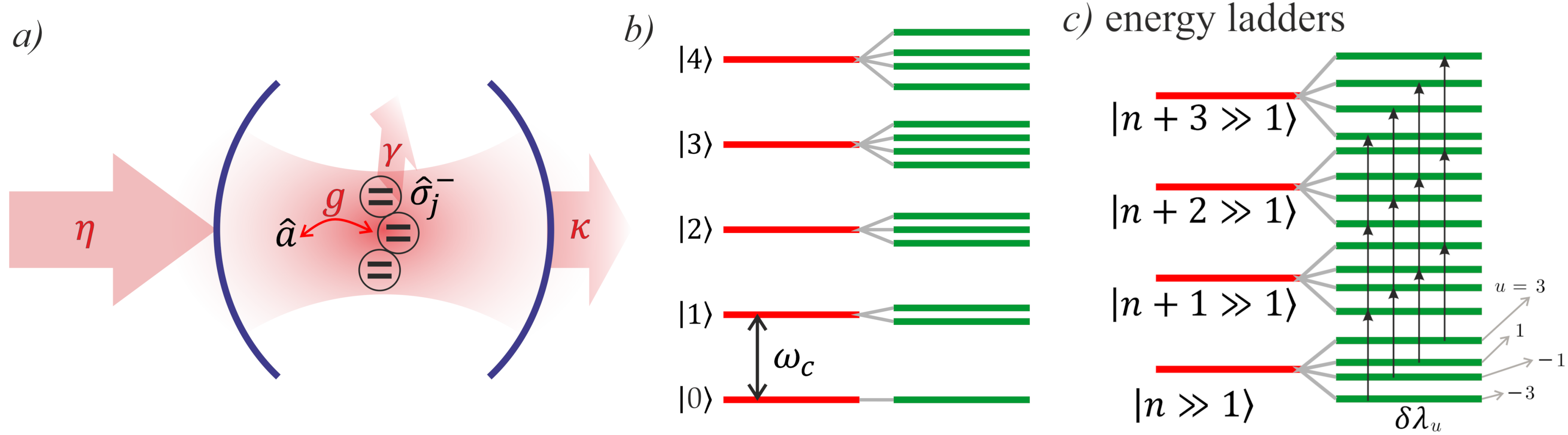
Thermodynamic limit: $g \to \infty, \quad \eta / g = const.$



Diverging
- **photon number** → **phases**
- **time scale** → **hysteresis**

‚Zero dimension' : no increase in system size

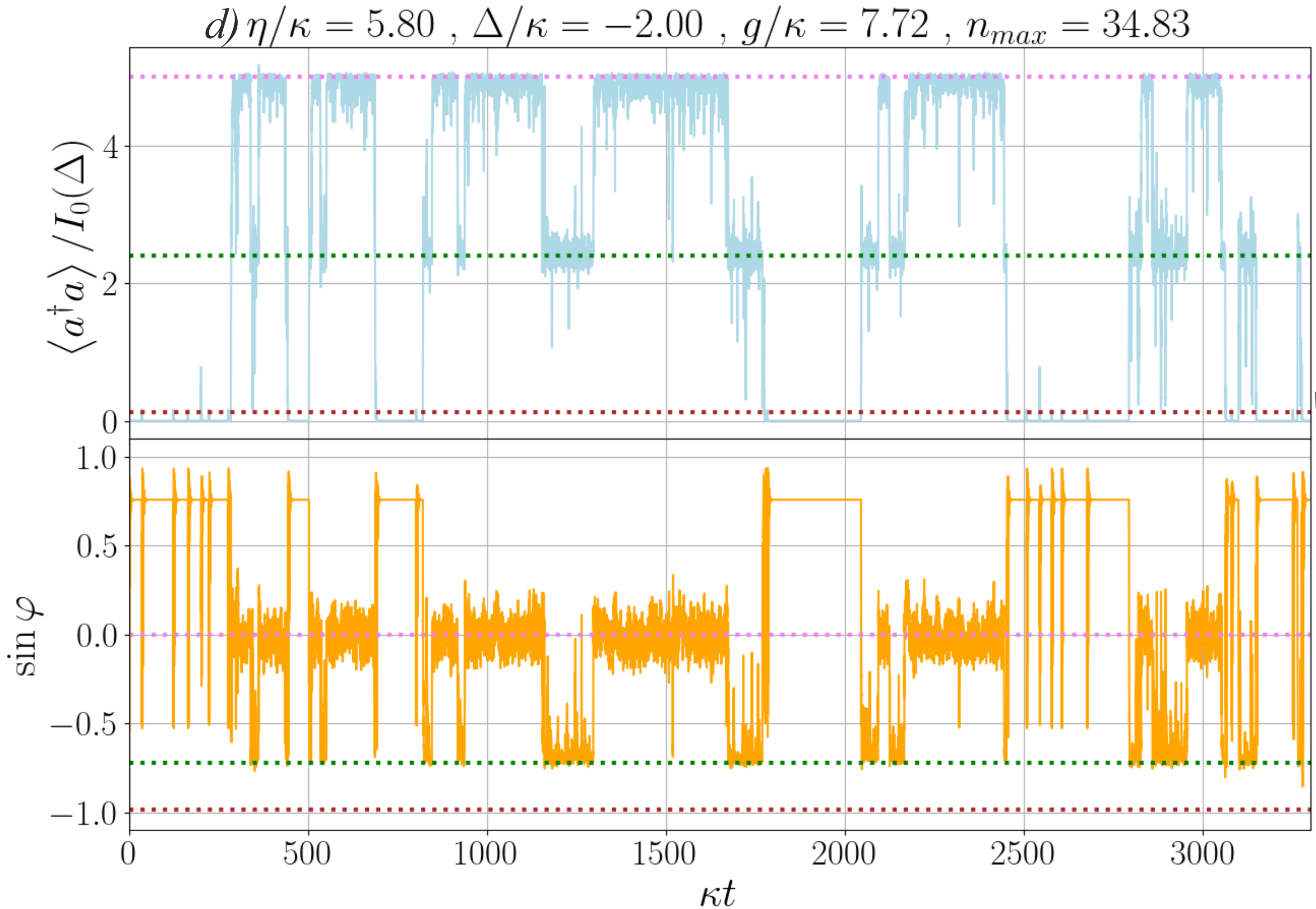
Super-quantisation rule for multistability



Self-consistent equation for the quasi-coherent state amplitude

$$\alpha_u = \frac{\eta/\kappa}{1 - i\left(\delta - \frac{ug}{2\kappa|\alpha_u|}\right)}$$

Quantum trajectories in the multistability domain



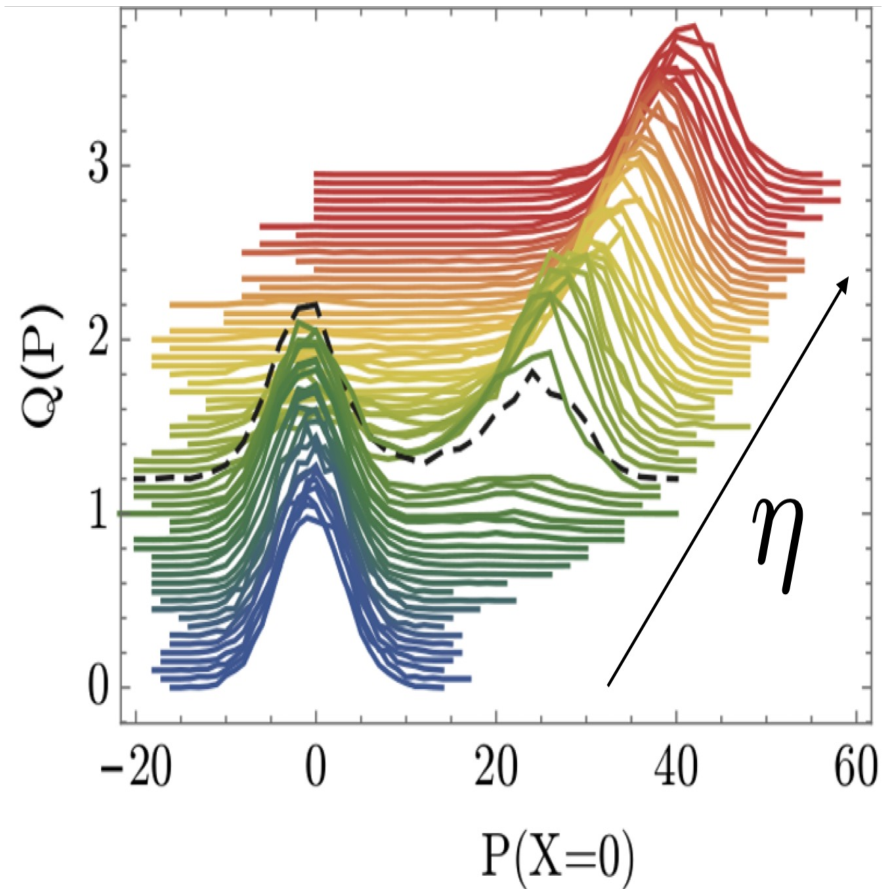
Super-quantisation rule predictions

Német, Kurkó, Vukics, Domokos, New Journal of Physics 2024

Quantum bistability without singularity

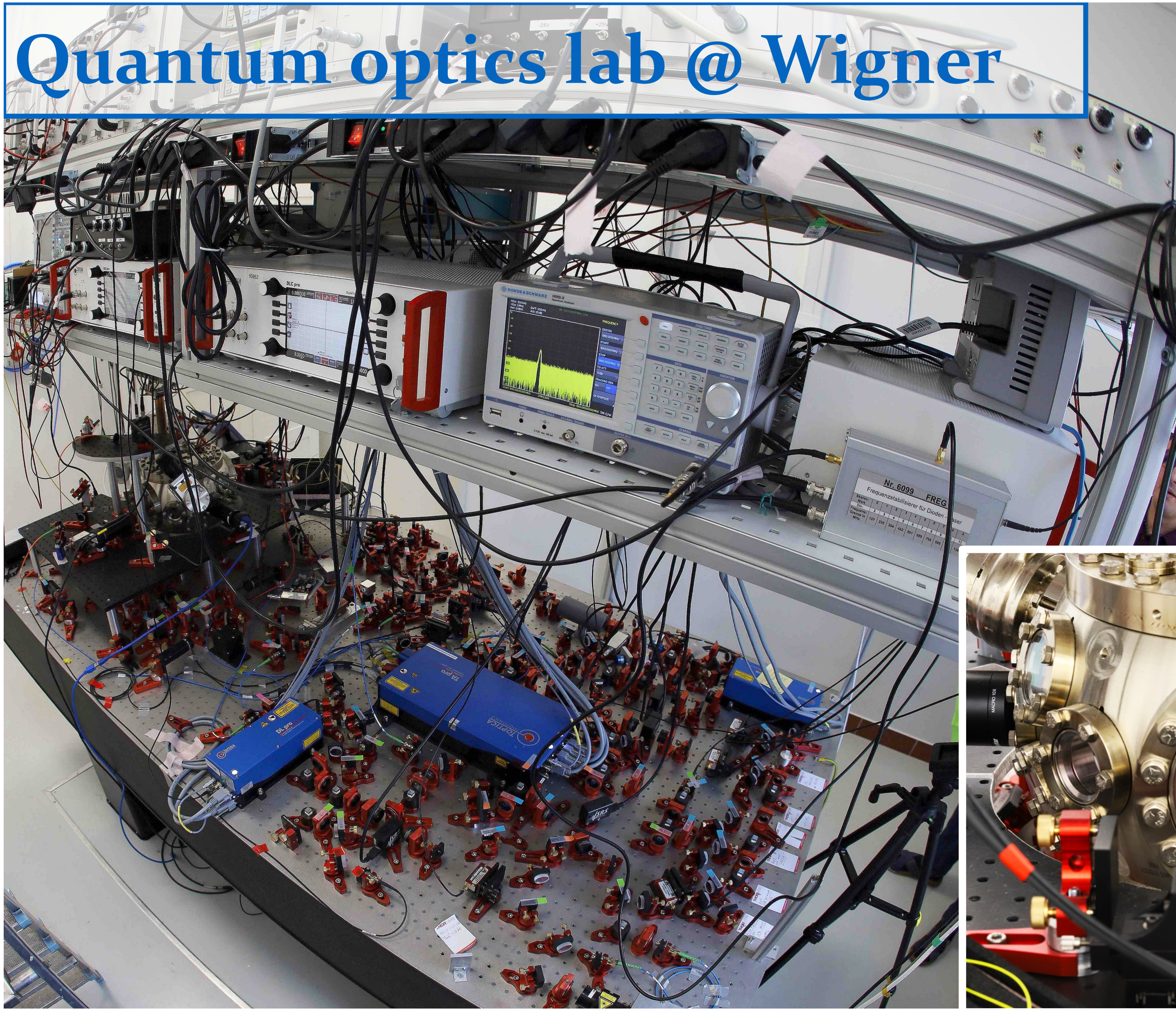
- *1st order ~ discontinuity*
- $\frac{d}{dt}\rho = L\,\rho = 0$
- *density matrix is a continuous function of all system parameters*
- *Co-existence of phases*
- $\rho_{ss} = (1 - F) \cdot \rho_{dim} + F \cdot \rho_{bright}$

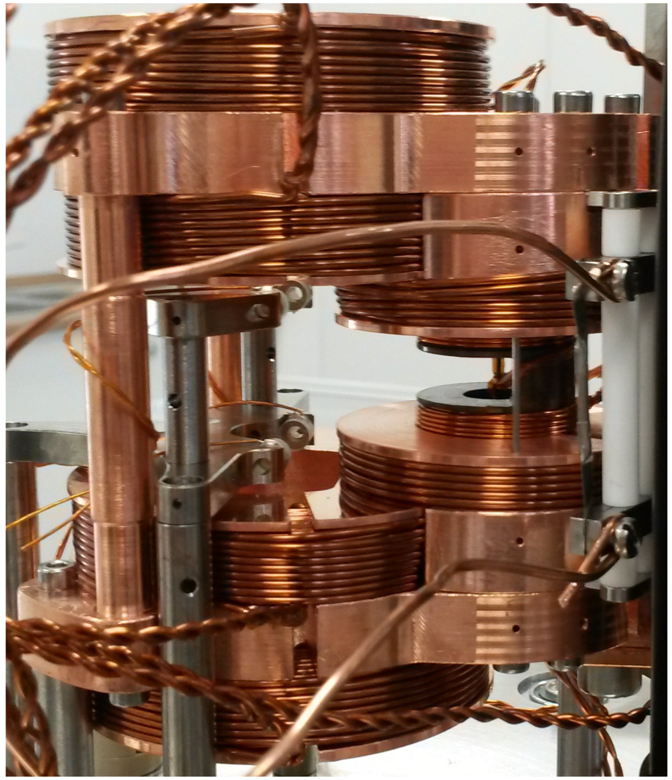
Bimodal density matrix

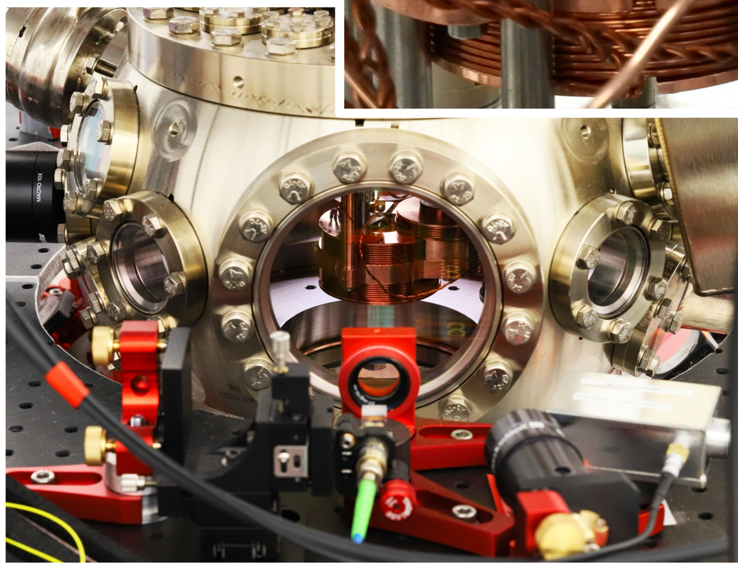


Fink, Dombi, Vukics, Walraff, Domokos, Phys Rev X 2017

Quantum optics lab @ Wigner



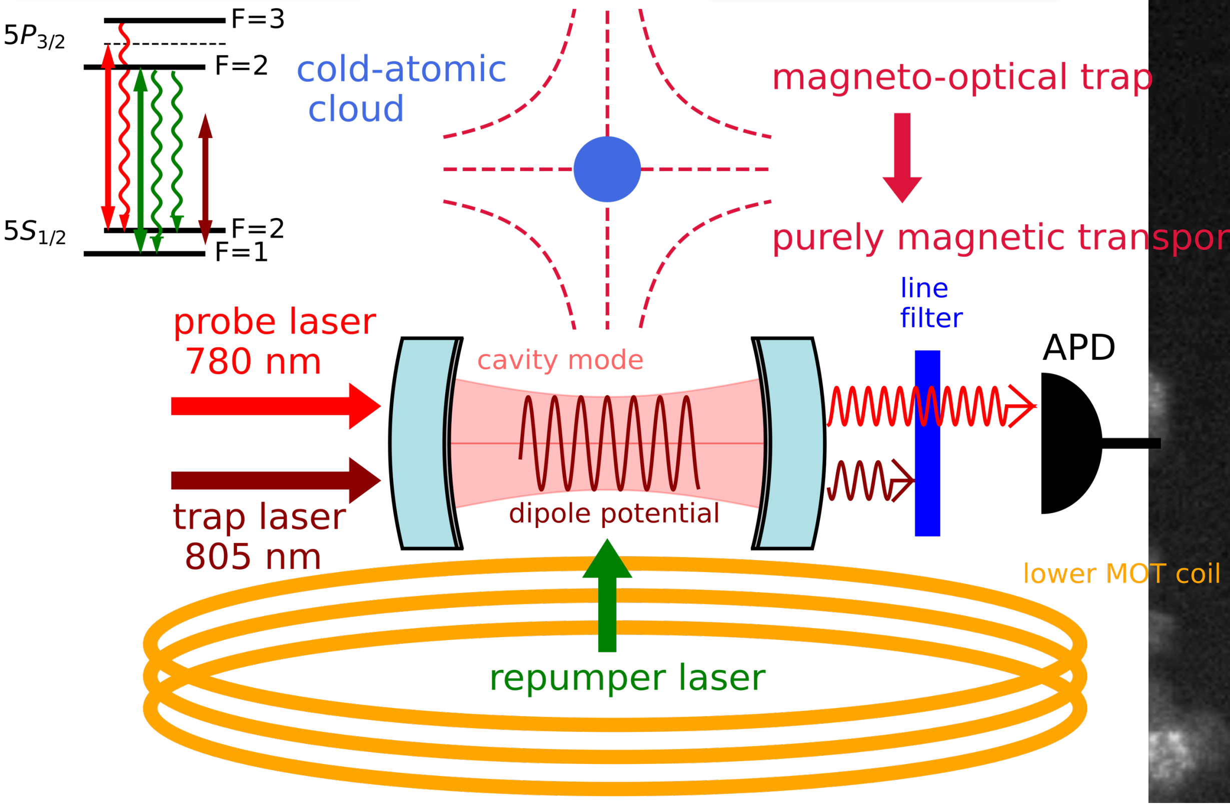


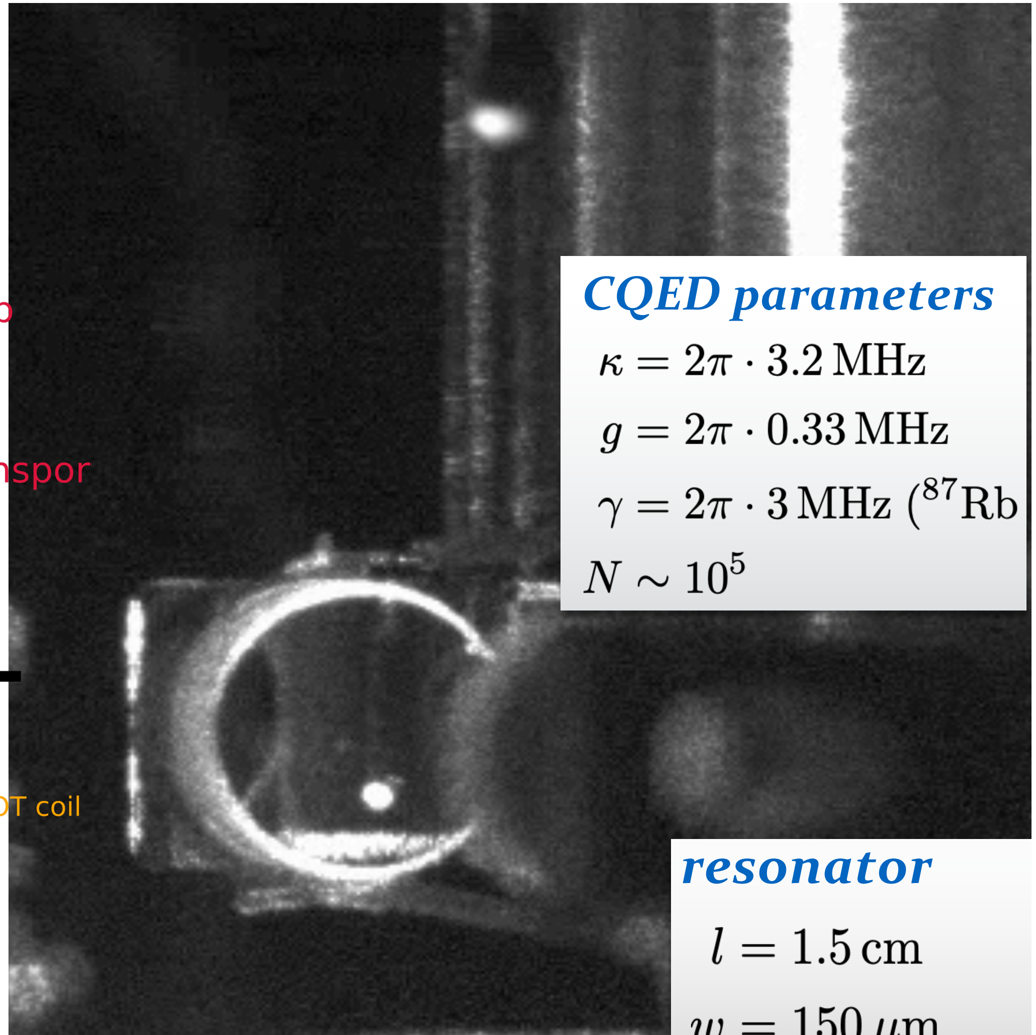


Experimental setup

^{87}Rb

$T \geq 10\ \mu K$





CQED parameters

$\kappa = 2\pi \cdot 3.2\,\mathrm{MHz}$

$g = 2\pi \cdot 0.33\,\mathrm{MHz}$

$\gamma = 2\pi \cdot 3\,\mathrm{MHz}$ (^{87}Rb

$N \sim 10^5$

resonator

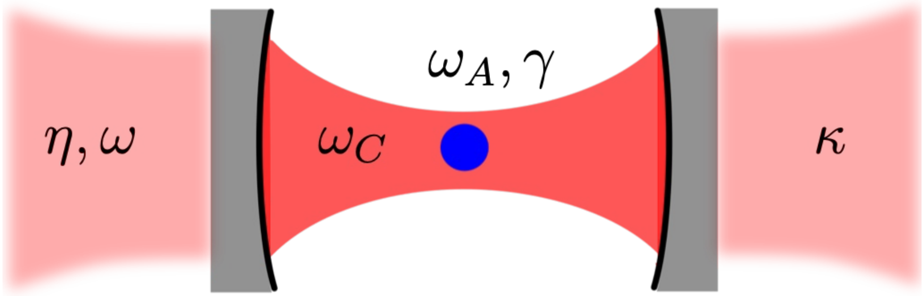
$l = 1.5\,\mathrm{cm}$

$w = 150\,\mu\mathrm{m}$

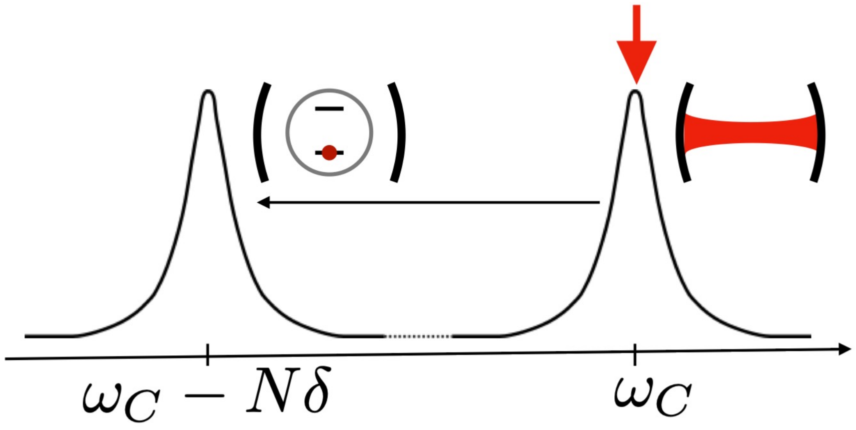
plano-concave

D. Varga et al, Physics Letters A, 2024

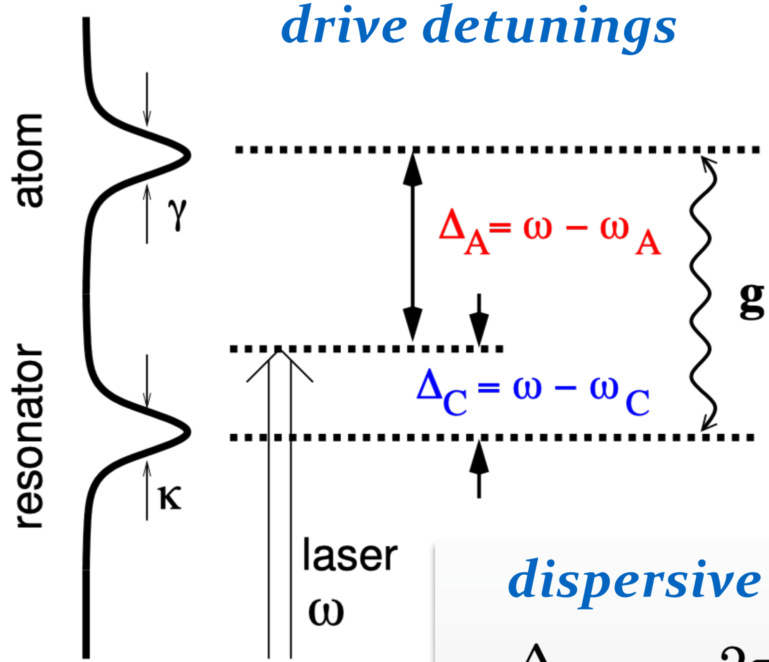
Simple transmission blockading mechanism



resonance shift



drive detunings



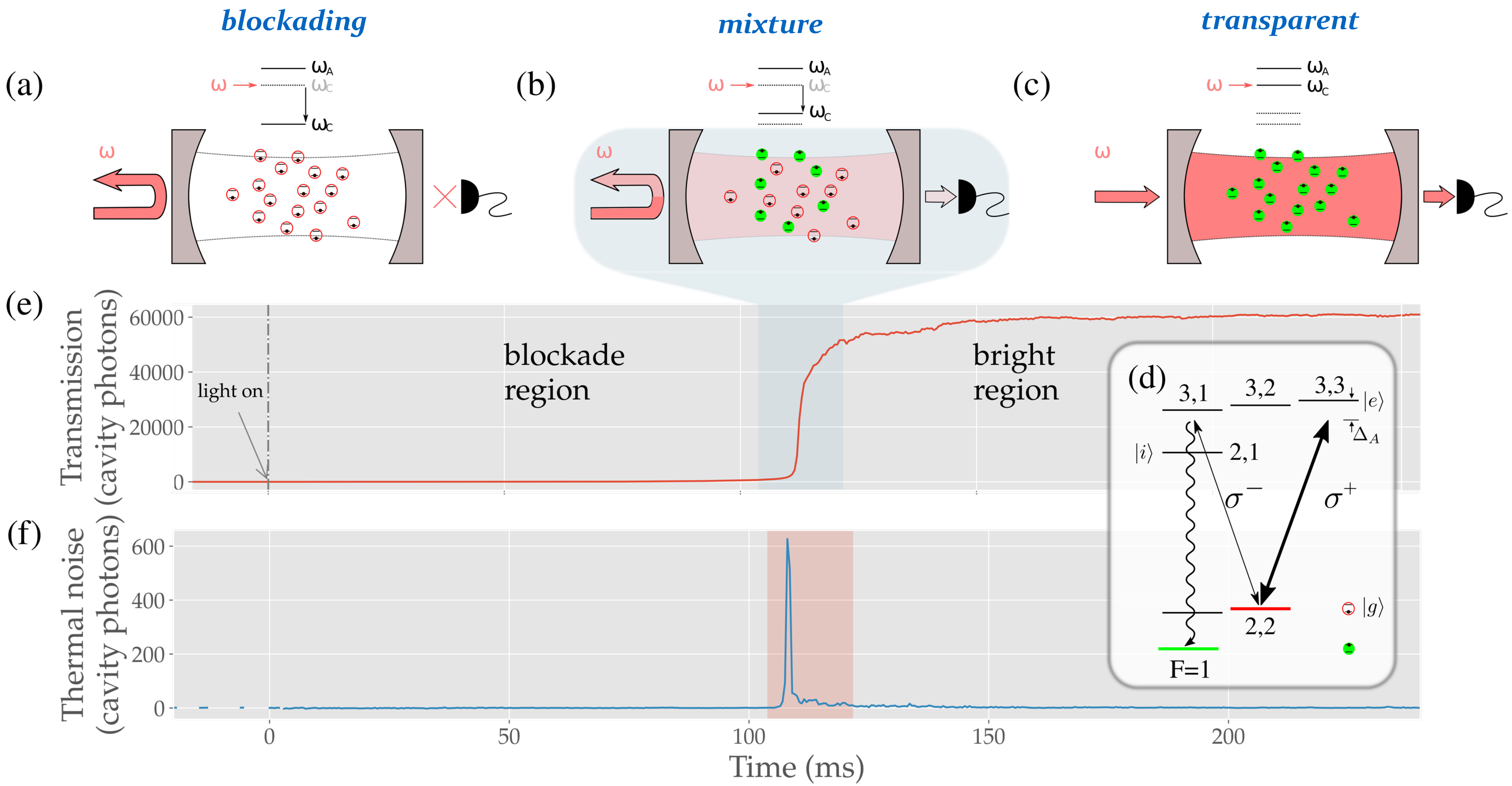
dispersive limit

$$\Delta_A = -2\pi \cdot 35\,\mathrm{MHz}$$

$$\delta \approx \frac{g^2}{\Delta_A} = -2\pi \cdot 3\,\mathrm{kHz}$$

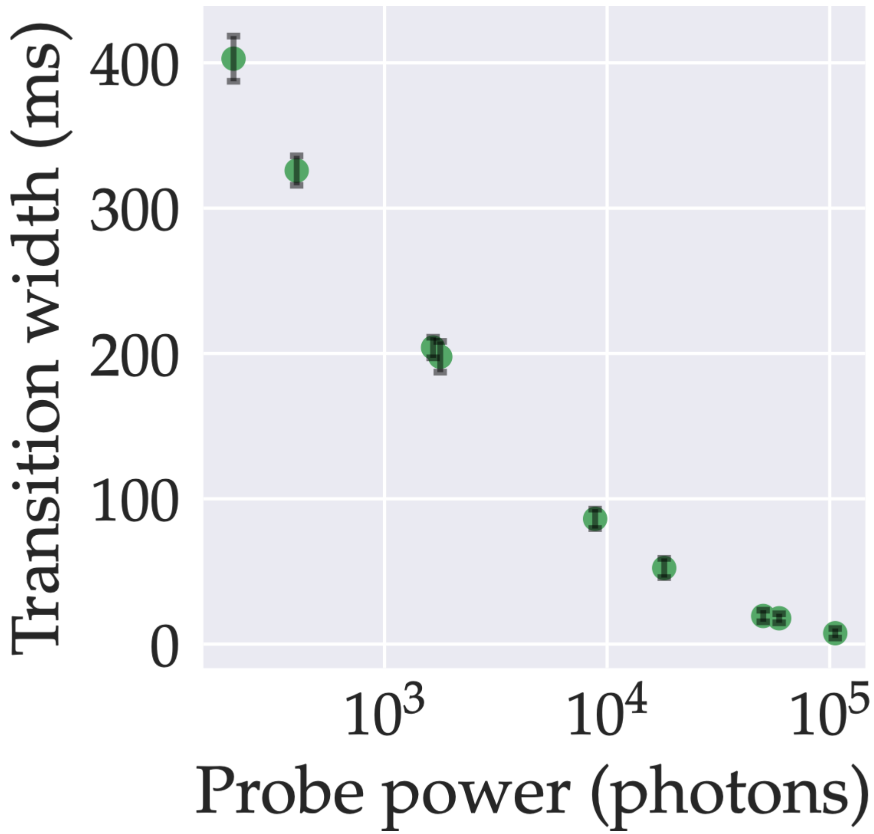
$$N \approx 10^4$$

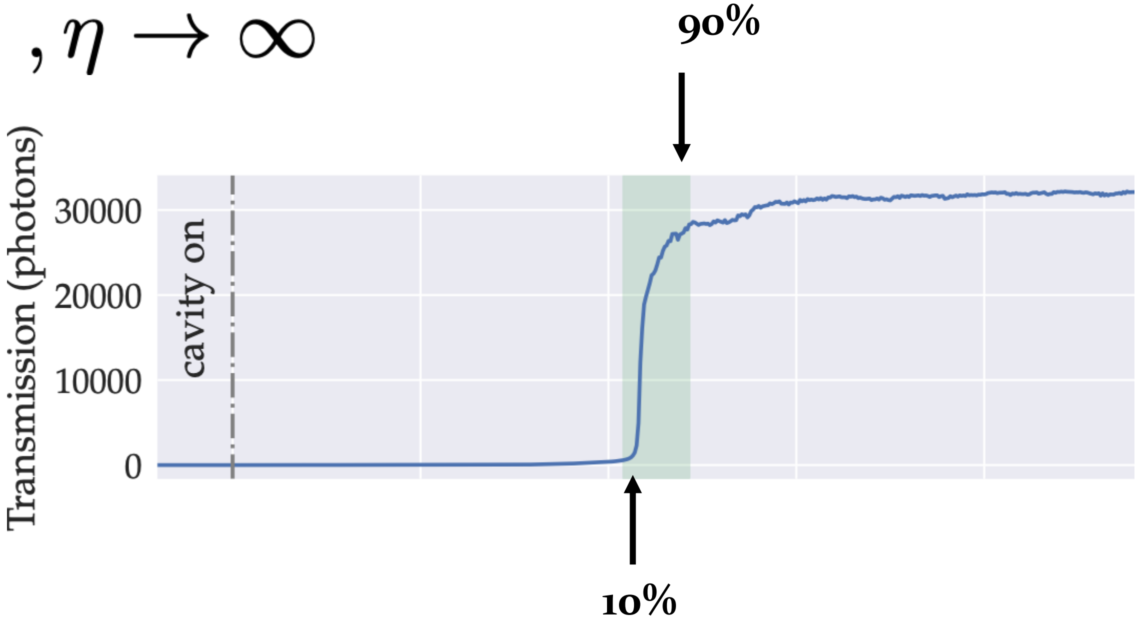
Time-resolved observation of the transmission blockade breakdown



Defining and calibrating a finite-size measure

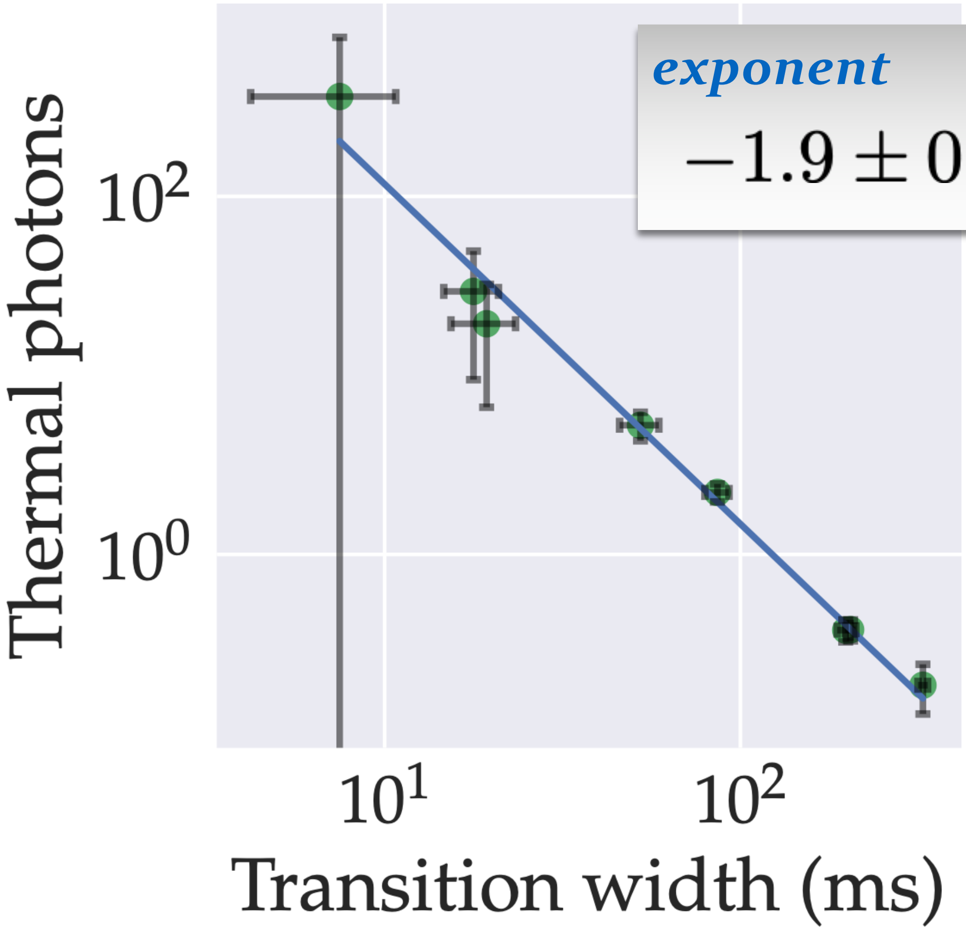
Thermodynamic limit: $\mathcal{C} \to \infty \,, \eta \to \infty$





Two orders of magnitude covered!

Finite-size scaling of fluctuations



exponent

$$-1.9 \pm 0.1$$

measured
photo-current noise

displaced thermal state

$$P_{\mathrm{th,disp}}(\alpha) = \frac{1}{\pi n_{\mathrm{th}}} \exp\left(-|\alpha - \beta|^2 / n_{\mathrm{th}}\right)$$

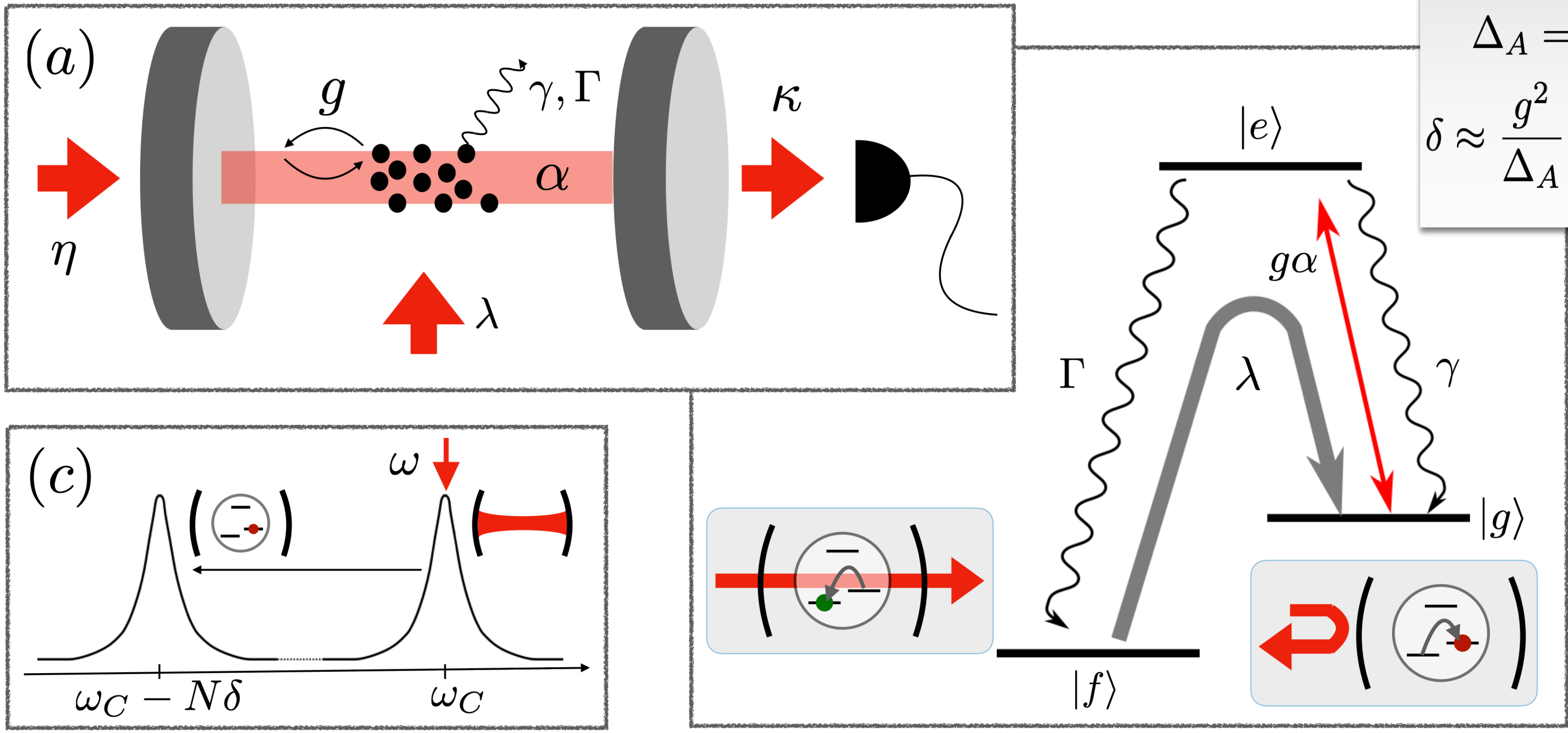
phase transition $\beta : 0 \to \eta/\kappa$

$$g^{(2)}(0) = 2 - \frac{|\beta|^4}{(n_{\mathrm{th}} + |\beta|^2)^2}$$

derived from measurement

Competing optical pumping processes

Three-level scheme with repumper



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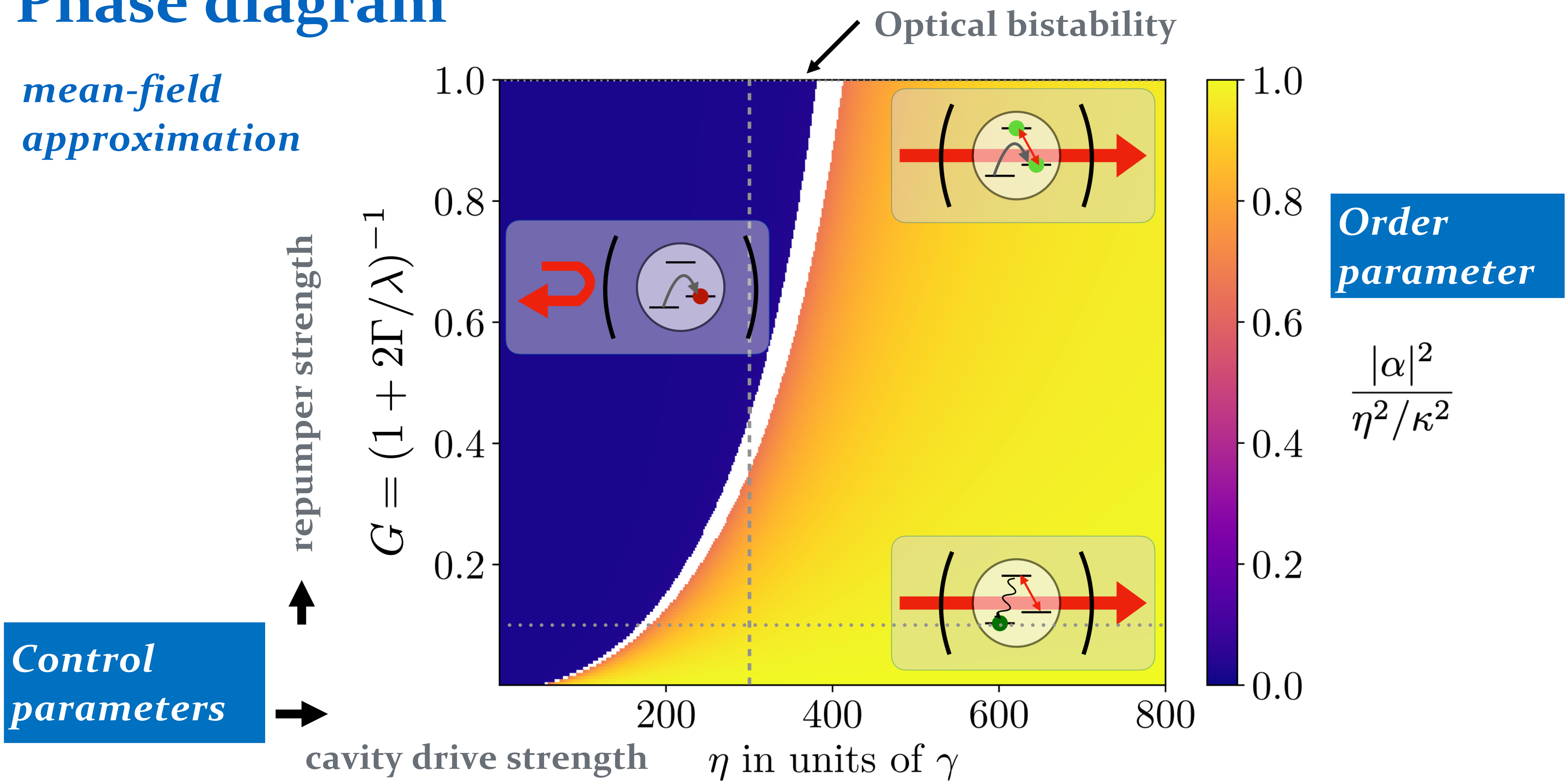
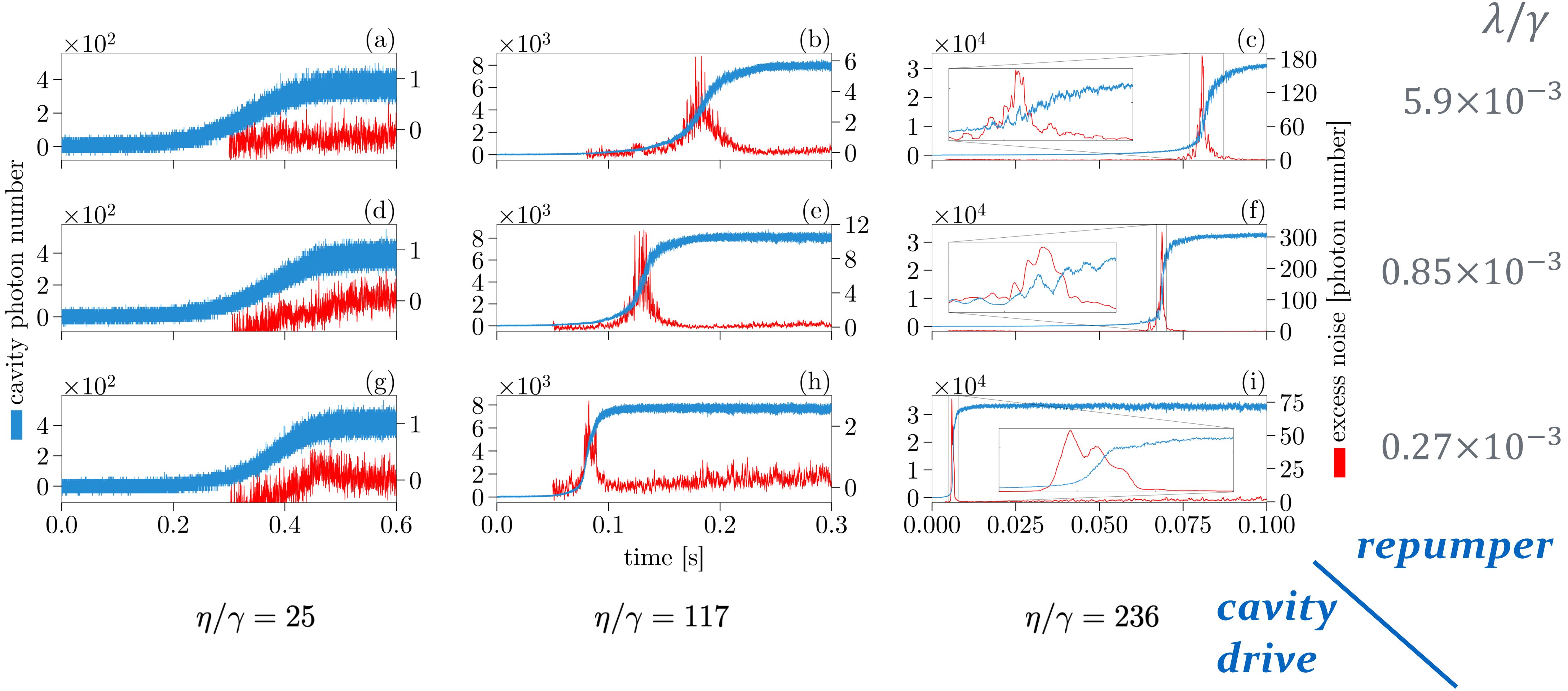
B. Gábor et al, Phys. Rev. A 107, 023713 (2023)

Phase diagram

mean-field approximation



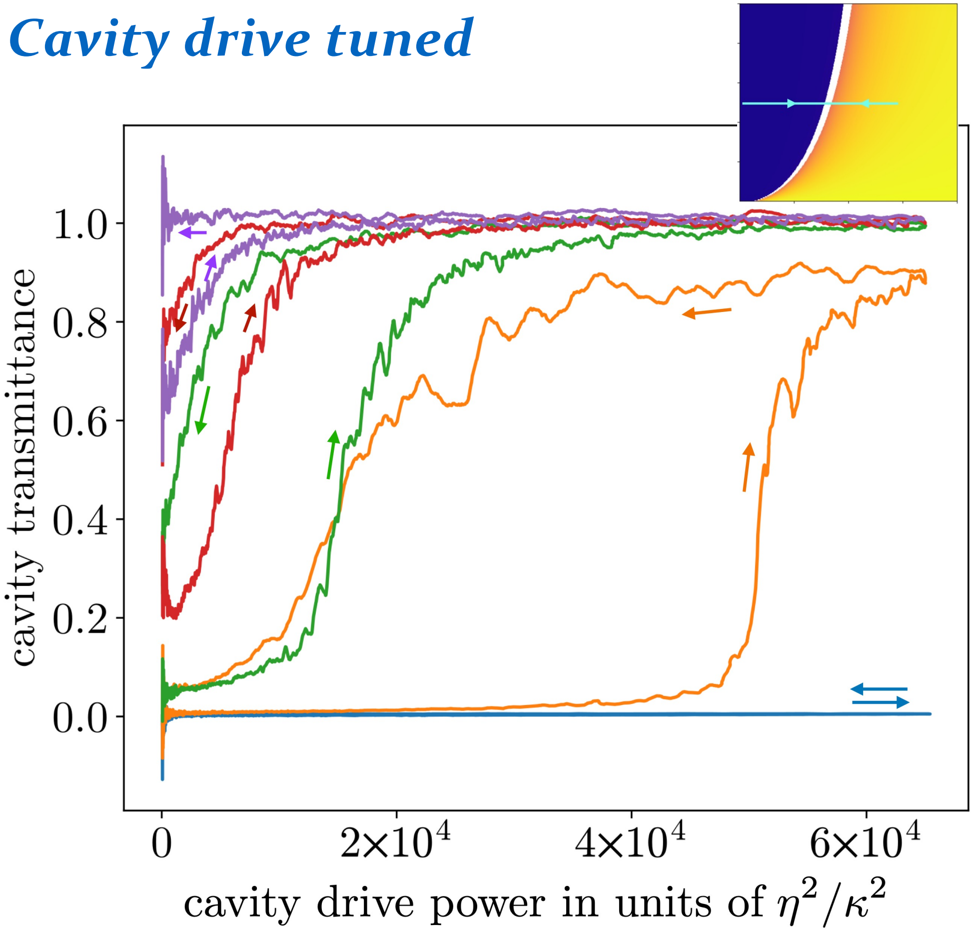
Time evolution from different points in phase space



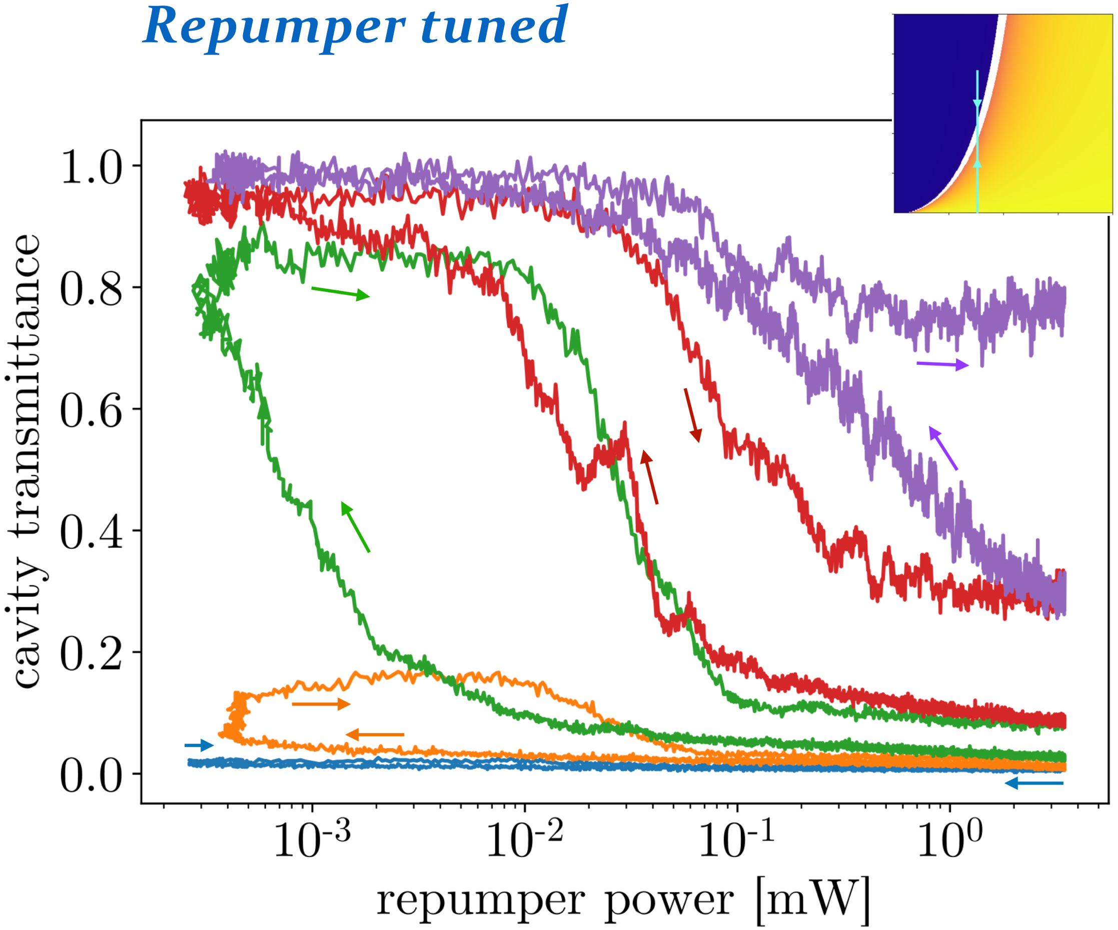
B. Gábor et al, Phys. Rev. A 107, 023713 (2023)

Demonstration of the hysteresis

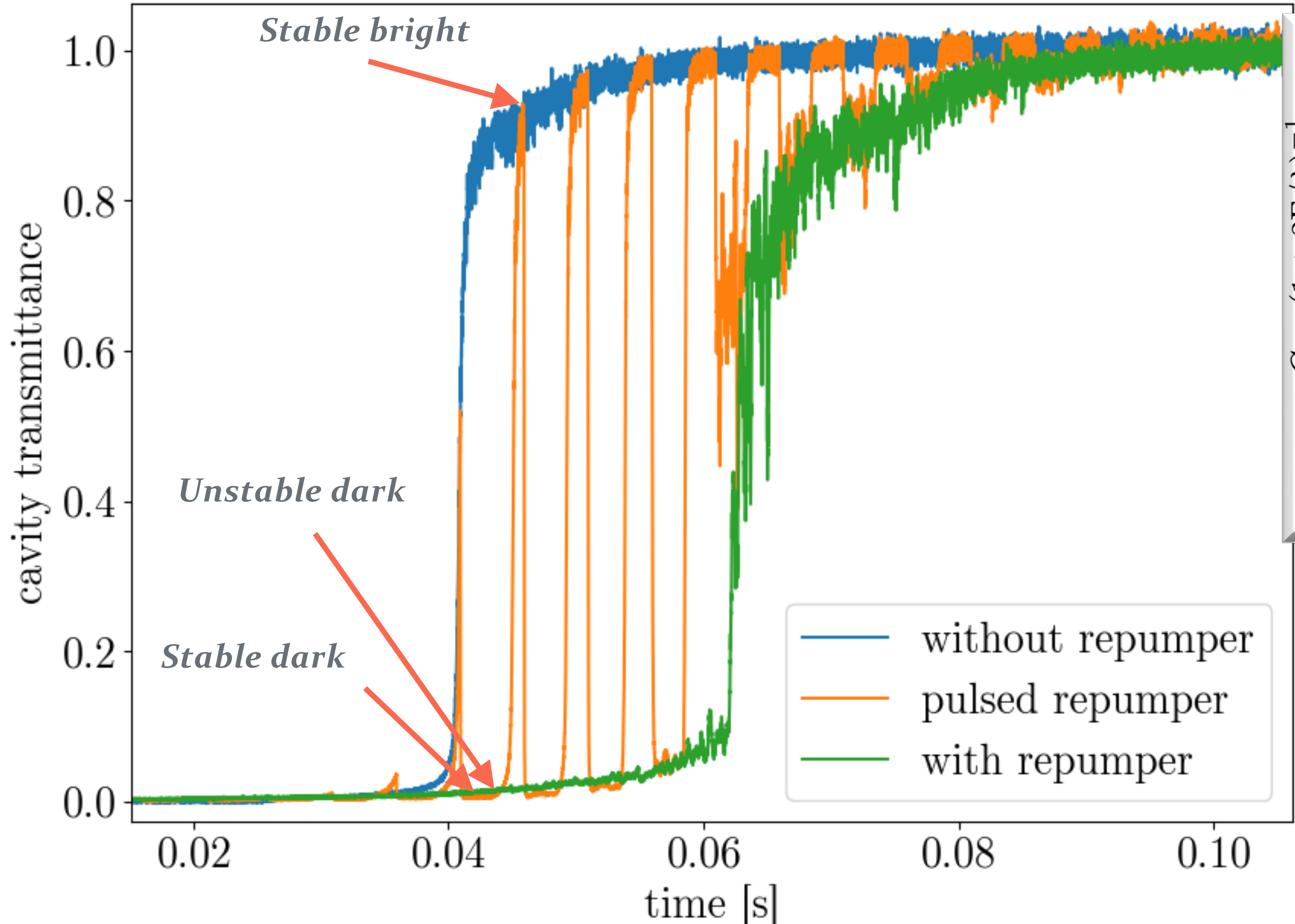
Cavity drive tuned

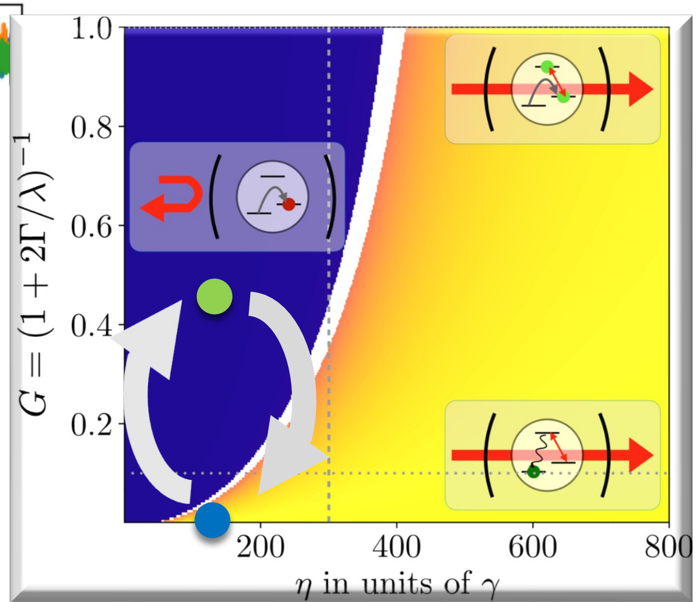


Repumper tuned

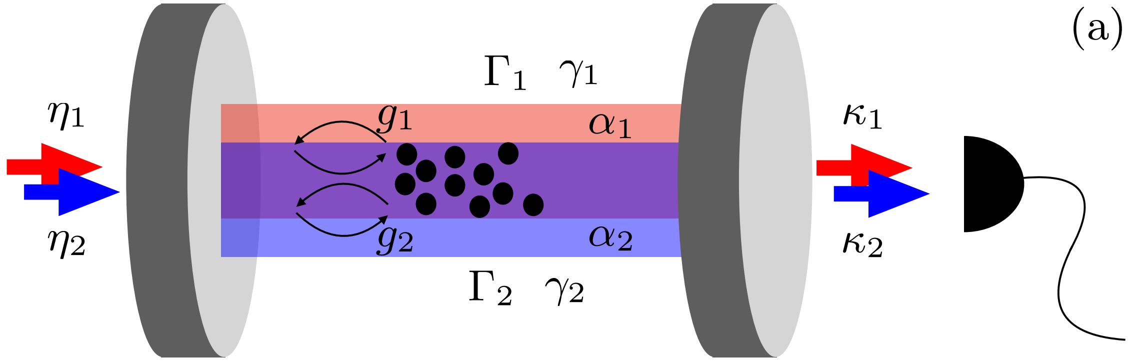


Switching between stable and unstable phases

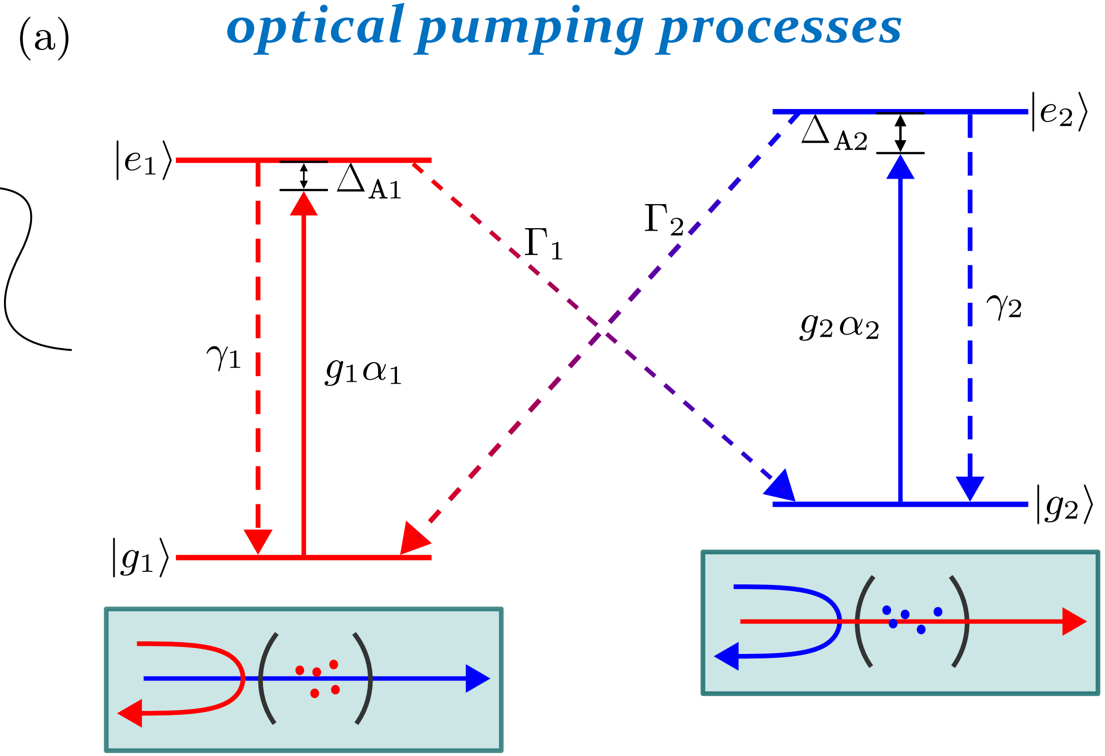




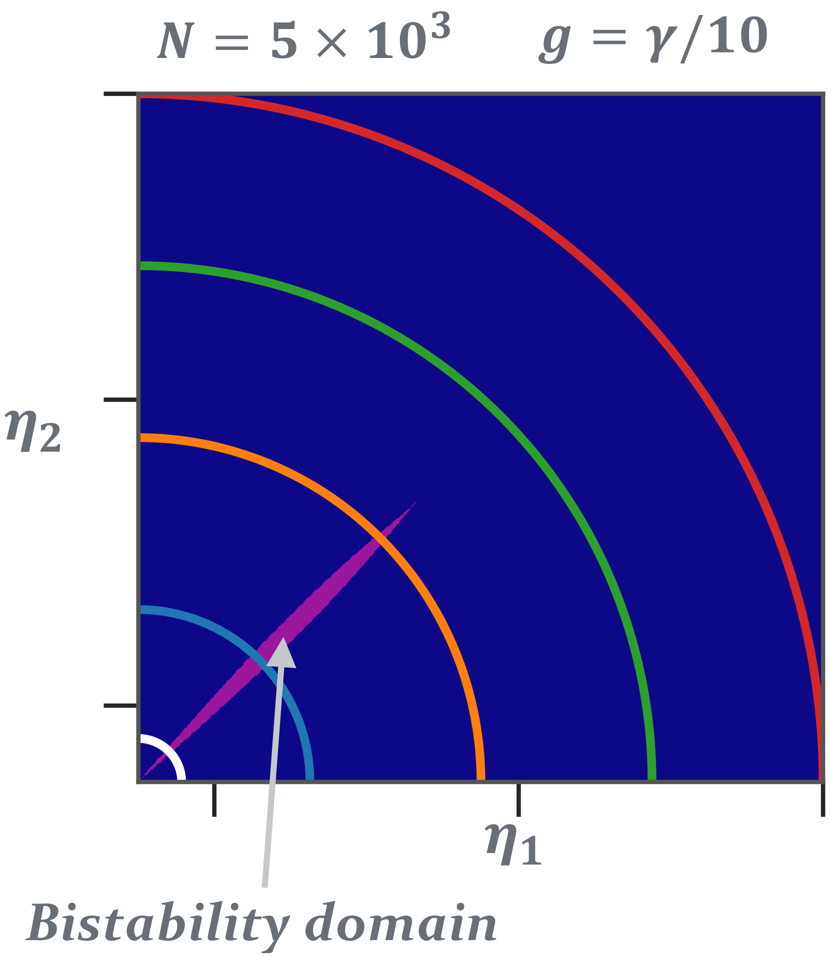
Ground state bistability with two cavity modes



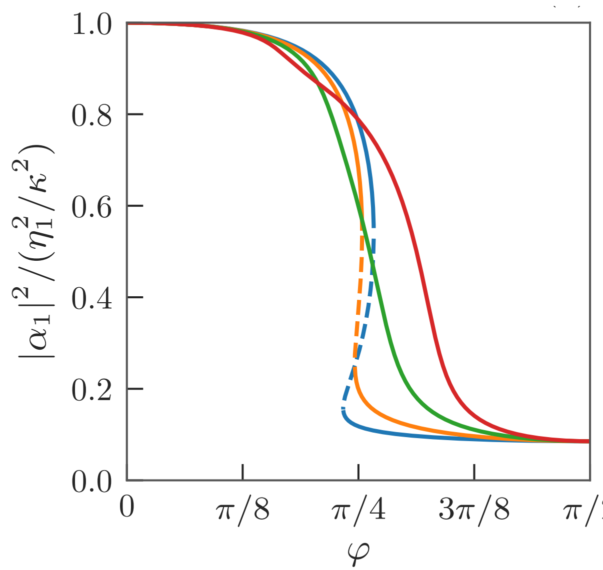
Competing non-linear optical pumping processes

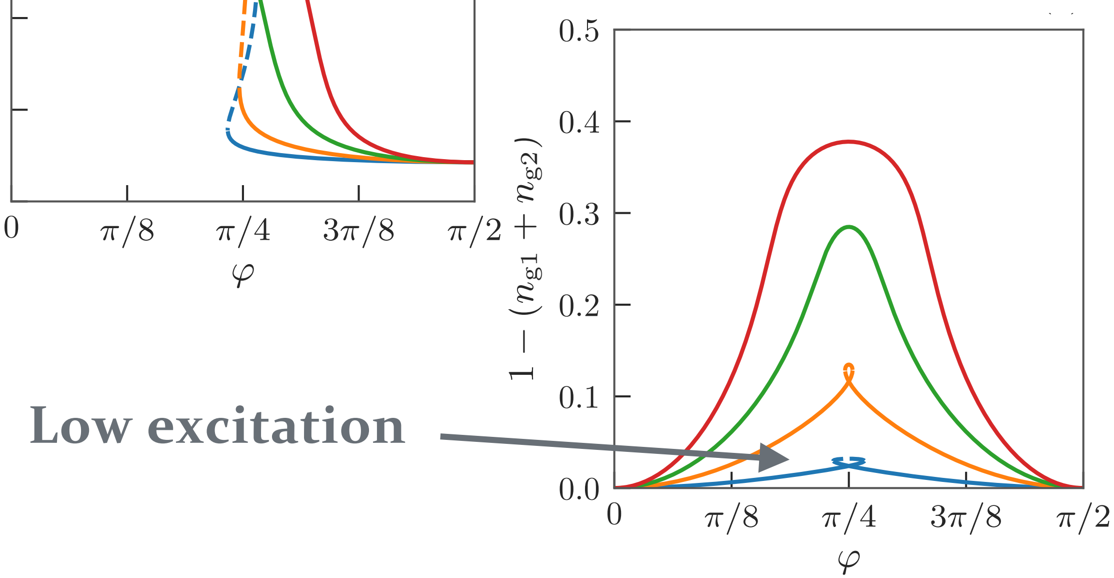


Phase diagram of the ground state bistability



Varying the relative drive stregths at fixed total power

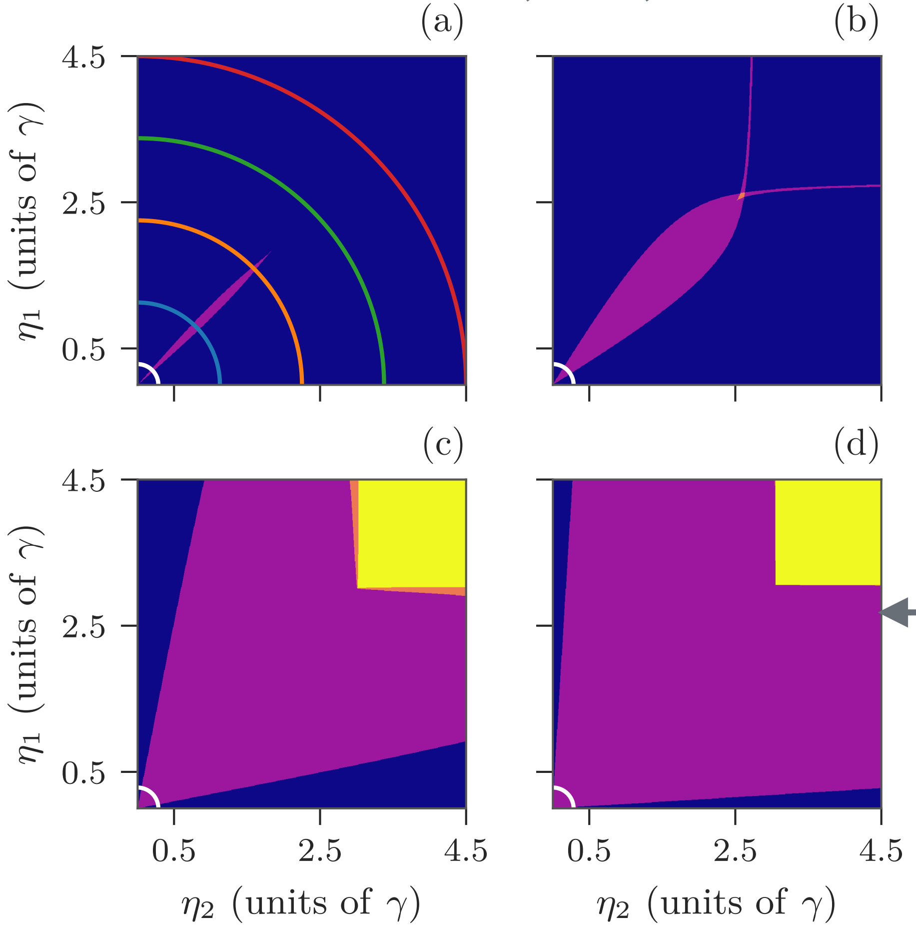


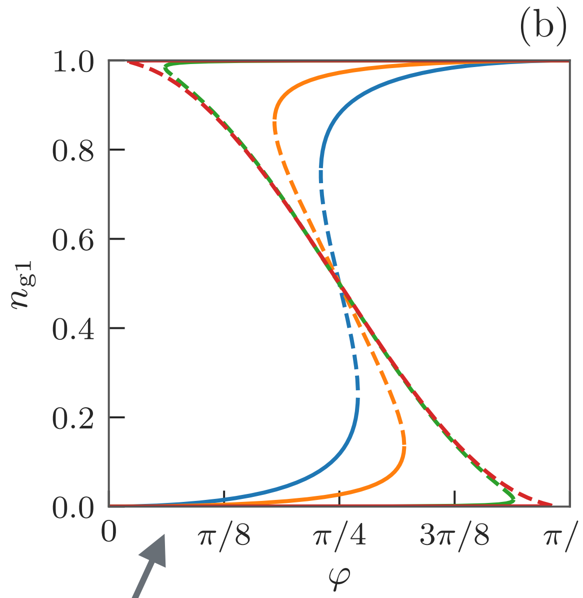


B. Gábor, D. Nagy, A. Vukics, P. Domokos, Phys. Rev. Research 5, L042038 (2023)

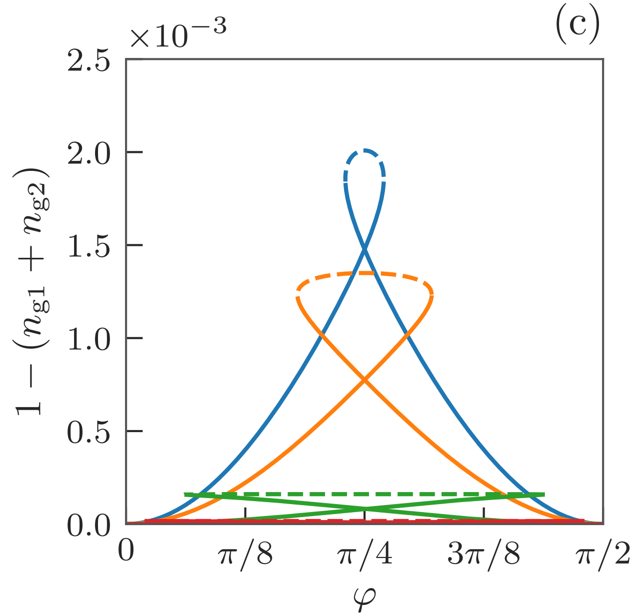
Thermodynamic limit: cooperativity C $\to \infty$

$N = 5 \times 10^3, 10^4, 10^5, 10^6$





Vanishing excitation



Both ground states are stable

B. Gábor, D. Nagy, A. Vukics, P. Domokos, Phys. Rev. Research 5, L042038 (2023)

Conclusions

- Zero dimensional quantum systems under continuous measurement can host `macroscopic’ phases and can undergo phase transitions
- Cavity QED systems are paradigmatic driven-dissipative open quantum systems where a single or a few atoms in strongly coupled to a cavity mode can produce bistability
- The breakdown of the transmission blockade has been observed with time resolution and finite-size scaling of the fluctuations has been performed
- We demonstrated experimentally hysteresis in a first-order phase transition
- There is a limit of cavity-induced bistability in whihc the phases correspond to pure ground states

Acknowledgements

QNL Quantum Information National Laboratory HUNGARY

National Research, Development and Innovation Office Hungary

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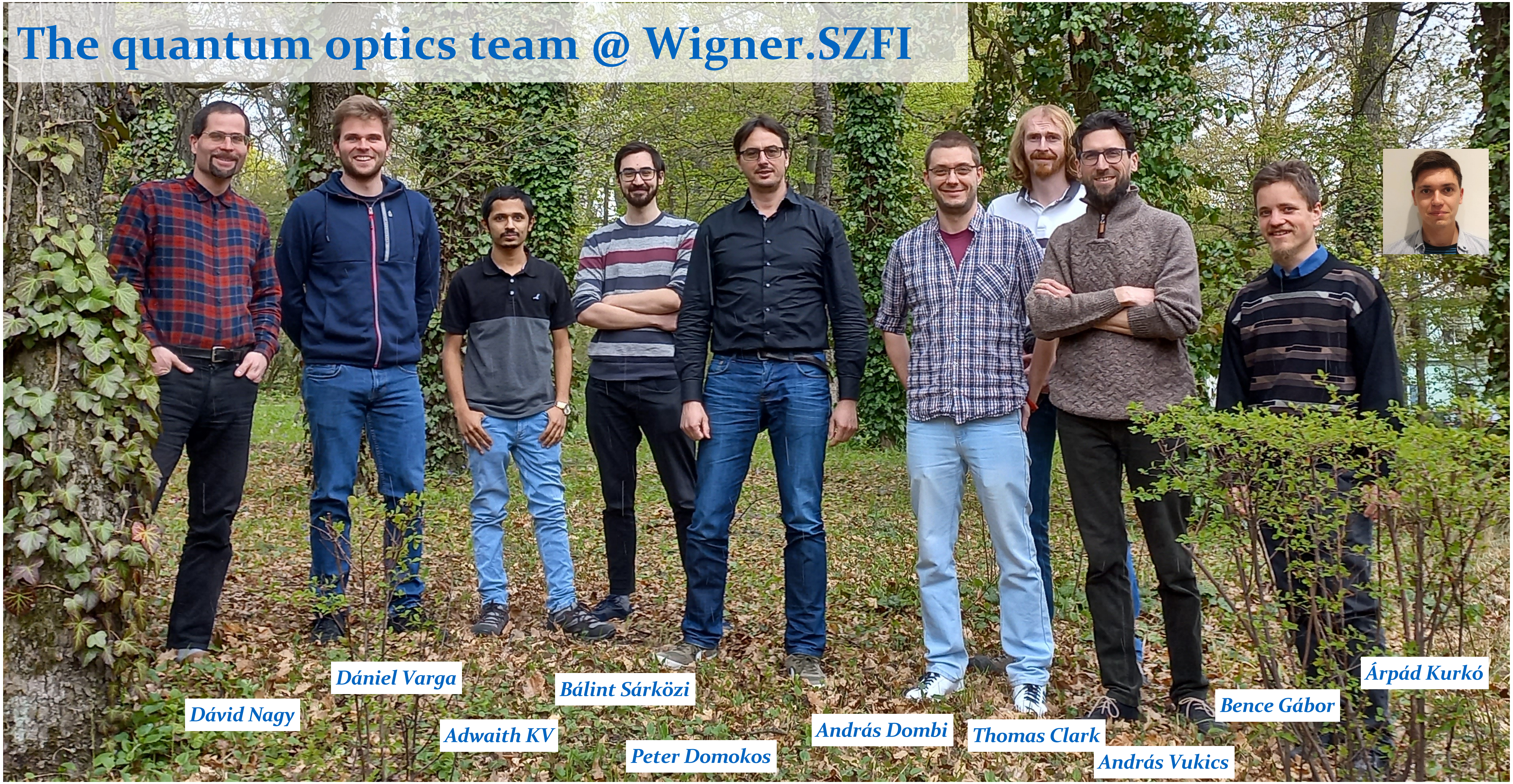
The quantum optics team @ Wigner.SZFI
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