Investigation of high-precision phase estimation with trapped ions in the presence of noise

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Karte
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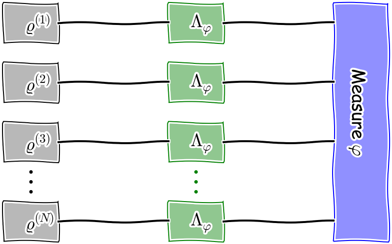
Motivation: Why Quantum Metrology?

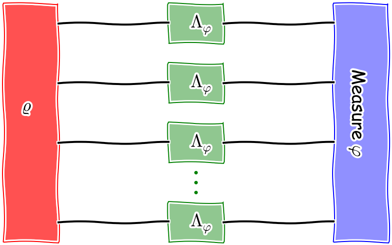
- **Classical setup**:
  - N particles in a classical state,
  - The Standard Quantum Limit (SQL):

$$(\Delta\varphi)^2 \propto 1/N$$



- **Quantum setup**:
  - N particles in a quantum state.
  - The Heisenberg Limit (HL):

$$(\Delta\varphi)^2 \propto 1/N^2$$



Motivation: Why Quantum Metrology?

The Cramer-Rao bound

$$(\Delta\varphi)^2 \geq 1/F_Q[\varrho, \Lambda_\varphi]$$

With the tool;Quantum Fisher Information (QFI) $F_Q[\varrho, \Lambda_\varphi]$

- **But noise destroys the advantage of using quantum states.**

The Question:

For a given set-up, with a given noise model;
Which quantum state should be used?

1. Motivation
2. The Quantum Fisher-Information
3. Quantum Metrology with Ions
4. Metrology with different states
5. Conclusions and next steps

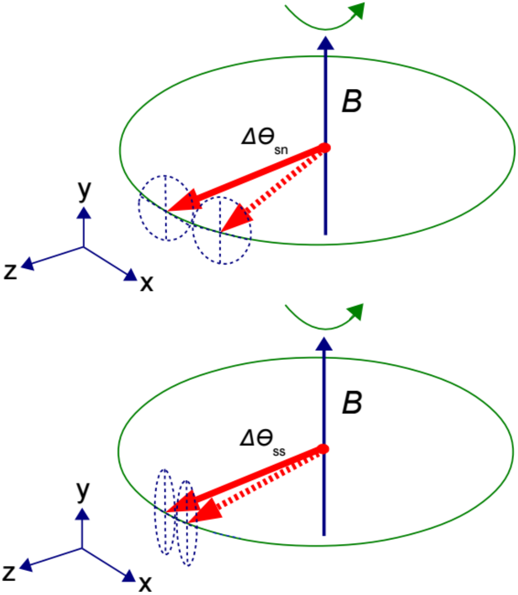
The Quantum Fisher-Information

The QFI is proportional to the statistical velocity of the evolution of a given state

$$(\partial_t D_H)^2 \propto F$$

with the Hellinger Distance D_H (distance in probability space).

Picture from G. Tóth and I. Apellaniz, J. Phys. A **42**, 424006 (2004)



The Quantum Fisher-Information

- Initial state in its eigendecomposition $\varrho = \sum_i \lambda_i |i\rangle\langle i|$
- Unitary time evolution $U = \exp\left[-i\theta M\right]$

Definition of the Quantum Fisher-Information

$$F(\varrho, M) = 2\sum_{\alpha,\beta} \frac{(\lambda_\alpha - \lambda_\beta)^2}{\lambda_\alpha + \lambda_\beta} \left|\langle\alpha| M |\beta\rangle\right|^2$$

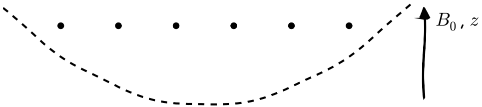
Example:

- The fully mixed state $\varrho = \mathbb{1}/d$ is insensitive to any kind of unitary time evolution, so that $F(\varrho, M) = 0$.

Quantum Metrology with Ions

Setup:

- N trapped ions in a chain.
- Magnetic field $\vec{B_0} = B_0 \vec{e}_z$



Dynamics:

- Ideal dynamics

$$H = \underbrace{\gamma B_0 t}_{\varphi} S_z$$

- Noise: Magnetic field fluctuations $B = B_0 + \Delta B(t)$

$$\eta = \varphi + \underbrace{\gamma \int_0^t \mathrm{d}\tau \Delta B(\tau)}_{\delta\varphi(t)}$$

- Estimate φ for a given time $t = T$

The noise model[1]

- Dynamics

$$U = \underbrace{\exp[-i\varphi S_z]}_{U_\varphi} \underbrace{\exp[-i\gamma \int_0^T \mathrm{d}\tau \Delta B(\tau) S_z]}_{U_{noise}}$$

- The state ϱ_0 evolves in a noisy state $\varrho_T = U_{noise}\varrho_0 U_{noise}^\dagger$
- Time correlation function for the magnetic field fluctuations $\Delta B(\tau)$

$$\langle \Delta B(\tau)\Delta B(0)\rangle = \exp[-\frac{\tau}{\tau_c}] \langle \Delta B^2\rangle$$

- Assume Gaussian phase fluctuations with $\langle \delta\varphi(T)\rangle = 0$
- Average $\langle \varrho_T\rangle$ and calculate the QFI $F_Q[\langle \varrho_T\rangle, S_z]$
- For frequency estimation with $\varphi = \omega T$:

$$F_Q^\omega = T^2 F_Q^\varphi$$

[1] T. Monz et al.: *14-Qubit Entanglement: Creation and Coherence*, PRL **106** (2011)

The product-state

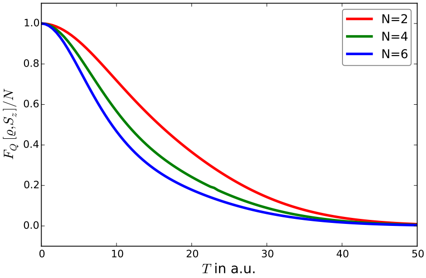
The classical state....

...is a Product-state:

$$|P_N\rangle = \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)^{\otimes N}$$

Observations:

- For $T = 0$ we find $F_Q = N$, this is the SQL.
- For $T > 0$ the QFI decreases.
- The more ions the faster the QFI decreases.



The GHZ-state

The GHZ-state...

...is defined as

$$|\Psi\rangle_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}}(|000...0\rangle + |111...1\rangle).$$

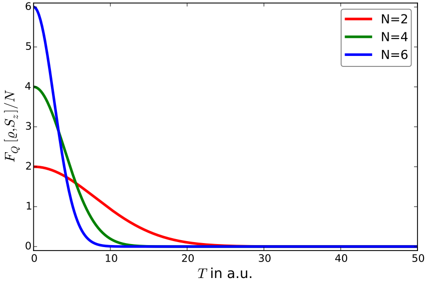
- The QFI is

$$F_Q = N^2 \mathrm{e}^{-N^2 C(T)}.$$

- Solving the master equation leads to

$$F_Q = N^2 \mathrm{e}^{-N^2 \tilde{C}(T)}.$$

[F.Fröwis et al., New Journal of Phys. **8** (2014)]



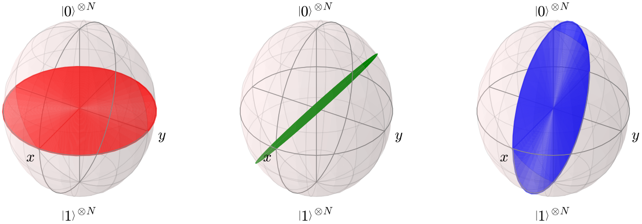
The sym. Dicke-state

Definition: Dicke-state

$$|D^N_{sym.}\rangle \propto \sum_j P_j\{|0\rangle^{\otimes N/2} \otimes |1\rangle^{\otimes N/2}\}$$

- Rotation around the X-axis: $U^N_x(\alpha)\,|D^{sym.}_N\rangle$

$\alpha = 0$ $\quad$ $\alpha = \pi/4$ $\quad$ $\alpha = \pi/2$

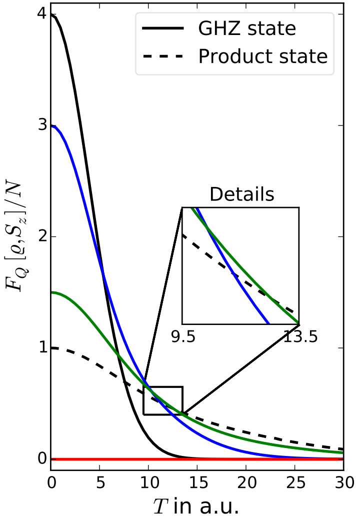


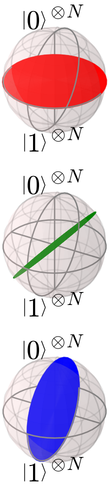
The sym. Dicke-state

For $N = 4$:

- For $\color{red}{\alpha = 0}$, $F_Q = 0$
- For $\color{blue}{\alpha = \pi/2}$ and $T = 0$, $F_Q = N(N+2)/2$
- For $\color{green}{\alpha = \pi/4}$, somewhere in between

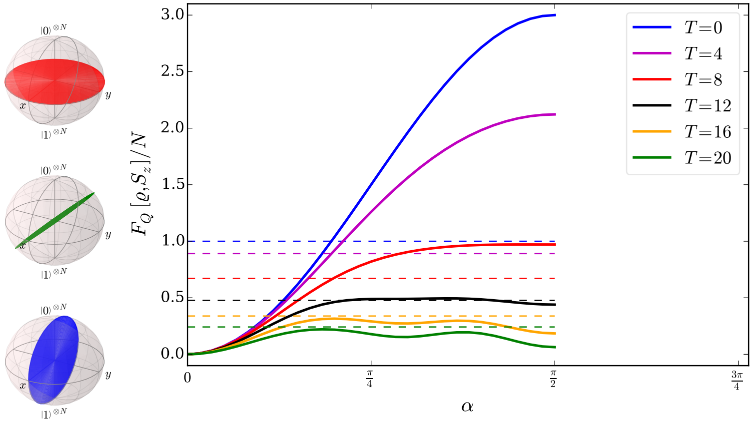
For a given T and N, we are able to tell which state gives the best precision for phase and frequency estimation.





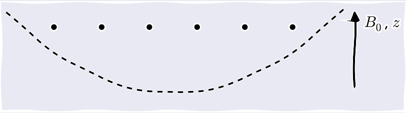
Dicke state under any rotation angle α

- $N = 4$
- Rotation around the X-axis: $U_x^N(\alpha)\,|D_N^{sym.}\rangle$

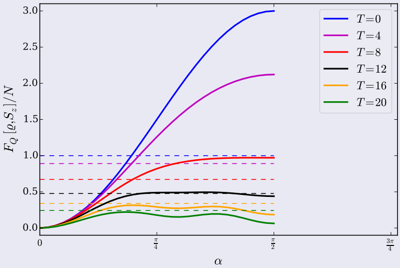


Conclusions

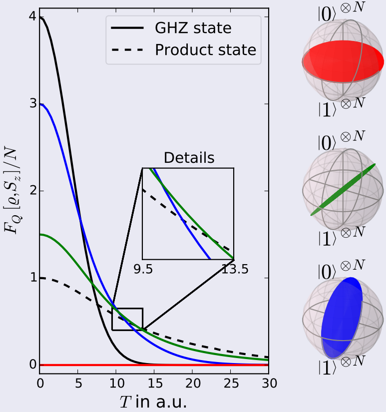
Trapped ions in the presence of noise



Sym. Dicke state under noise

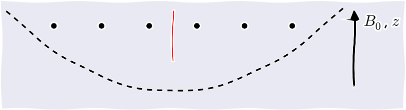


Optimization over states



Next steps

Differential interferometry with trapped Ions



- Trapped ions with Hamiltonian

$$H = \gamma \int_0^t \mathrm{d}\tau \Delta B(\tau)(S_z \otimes S_z) + \gamma\phi(\mathbb{1} \otimes S_z).$$

- For photons and atoms see M. Landini et al., New J. of Phys. **11**, 113074 (2014)

Other states

Investigation of the usefulness of other Dicke-states, e.g. the W-state.

Collaboration with Ch. Wunderlich (Siegen)

I. Baumgart et al., arXiv:1411.7893 (2014)

- Trapped ions with Hamiltonian

$$H = \gamma \int_0^t \mathrm{d}\tau \Delta B(\tau) S_z + \gamma(\Omega + \epsilon) t S_x.$$

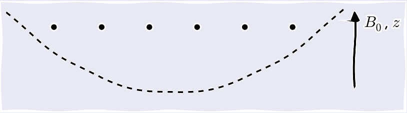
- Optimization for the estimation of ϵ.

Acknowledgements

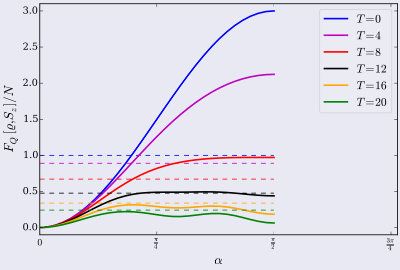
This work was founded by the Friedrich-Ebert-Foundation.

Thank you for your Attention! Questions?

Trapped ions in the presence of noise



Sym. Dicke state under noise



Optimization over states