Quantifying clumsiness in a Leggett-Garg test

G. Vitagliano[1]

[1] Theoretical Physics, University of the Basque Country UPV/EHU, E-48080 Bilbao, Spain

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Invasivity of measurements in phase space

- Assuming an initial definite phase-space point $\mathbf{r}_{\rm in} = (x, p)$
- We formalize the effect of measurement as

$$\mathbf{r}_{\rm in} = (x, p) \mapsto \mathcal{M}(x, p) = (x', p') := \mathbf{r}_{\rm out} \ ,$$

- For initial *uncertainty* we have a PDF $\rho(x, p)$ and

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$$Q = \operatorname{sgn}(x) \qquad\qquad P = \operatorname{sgn}(p)$$

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Macrorealism conditions

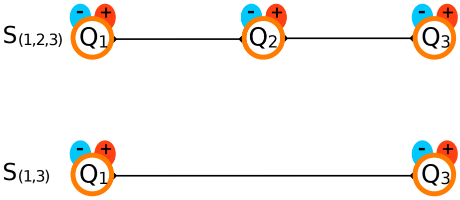
- *Macrorealism* (MR) means that $\mathcal{M}(\mathbf{r}) = \mathbf{r}$ and in particular

$$\mathrm{sgn}(\mathcal{M}(\mathbf{r})) = \mathrm{sgn}(\mathbf{r}) \tag{1}$$

- Necessary conditions follow

$$W := \sum_{Q_1, Q_3 = \pm 1} \left| \Pr(Q_1, Q_3)_{\mathcal{S}_{1,2,3}} - \Pr(Q_1, Q_3)_{\mathcal{S}_{1,3}} \right| = 0 \tag{2}$$

- On joint probabilities $\Pr(Q_i, Q_j)_{\mathcal{S}_{(i,j)}}$ of $\mathcal{S}_{i,j} = \mathcal{M}_j \circ \mathcal{M}_i$



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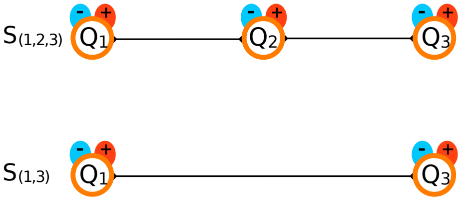

Clumsiness loophole of Macrorealism tests

- A test can witness failure of MR due to

1. Non-existence of a definite state $\mathbf{r}(t)$ at t_1, t_2, t_3
2. Invasive effect of measurement $\mathcal{M}_i(\mathbf{r}) \neq \mathbf{r}$

- QM assumes both (1) and (2)

- However, also *clumsy* measurements are invasive!

- One must distinguish clumsiness from **invasivity** and quantify the former (Remember:macroscopic observables should be considered)

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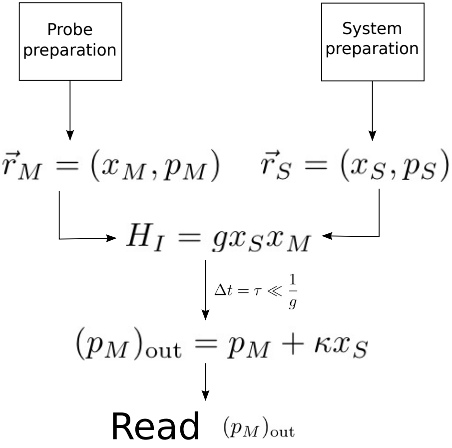
Our proposal: perform QND measurements...



$$[H_I, x_S] = 0$$

$\Longrightarrow$ Quantum Non-Demolition[1] measurement of x_S

- It is suitable for macroscopic variables

[[1] P. Grangier, J. A. Levenson, and J.-P. Poizat, Nature 396, 537 (1998); V. B. Braginsky, Y. I. Vorontsov, and K. S. Thorne, Science 209, 547 (1980)]

Where does MR fail?

$$p_S^{(\mathrm{out})} = p_S^{(\mathrm{in})} + \kappa x_M^{(\mathrm{in})}$$

there is a **back action** on p_S

Where does MR fail?

$$(\Delta x_S)^2_{(\mathrm{out})} = \chi(\Delta x_S)^2_{(\mathrm{in})} + (1-\chi)(\Delta x_S)^2_{(\mathrm{noise})}$$

and also **noise** directly in x_S

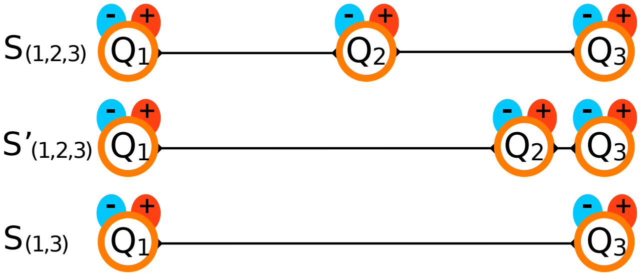
Where does MR fail?

- We can make an explicit distinction

$$\mathcal{M}(\mathbf{r}) = \mathcal{M}_X \circ \mathcal{M}_P \ ,$$

where $\mathcal{M}_X(x,p) = (x',p)$ and $\mathcal{M}_P(x,p) = (x,p')$

... then quantify the direct disturbance on x_S

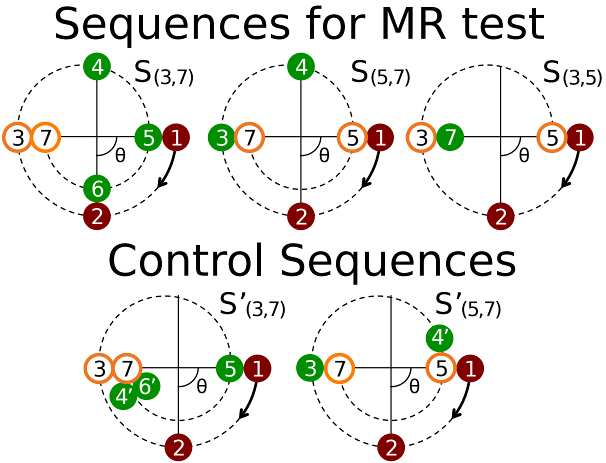


We define a quantifier of $\mathcal{M}_X(x,p)$ for the second measurement

Clumsiness parameter

$$\mathcal{I}(\mathcal{M}_2) = \sum_{Q_1, Q_3 = \pm 1} \left| \Pr(Q_1, Q_3)_{\mathcal{S}_{(1,2',3)}} - \Pr(Q_1, Q_3)_{\mathcal{S}_{(1,3)}} \right|$$

A proposed test in atomic ensembles



Modified LG inequality

$$\langle Q_7 Q_3 \rangle + \langle Q_5 Q_3 \rangle + \langle Q_7 Q_5 \rangle + 1 + \mathcal{I}_{37} + \mathcal{I}_{57} \geq 0$$

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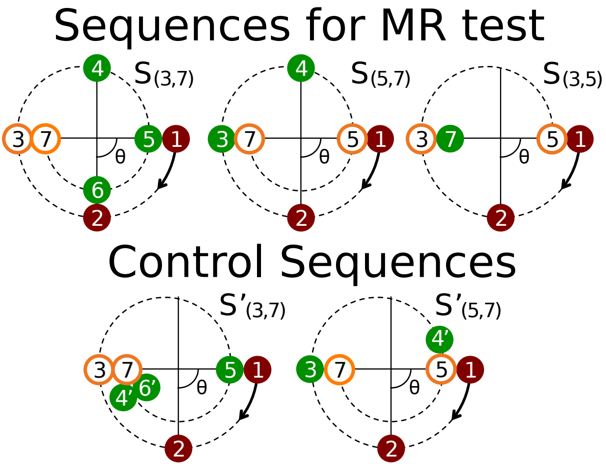

$$\mathcal{J}_{57} = \sum_{Q_5, Q_7 = \pm} \left| \Pr(Q_5, Q_7)_{\mathcal{S}'_{(5,7)}} - \Pr(Q_5, Q_7)_{\mathcal{S}_{(3,5)}} \right|$$

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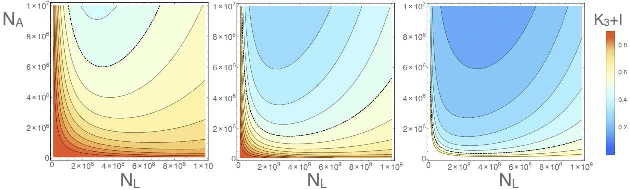

System+Meter are atoms $\vec{J} = (N_A, J_y, J_z)$ and light $\vec{S} = (\frac{N_L}{2}, S_y, S_z)$

$$S_y^{(\mathrm{out})} = S_y^{(\mathrm{in})} + \kappa J_z^{(\mathrm{in})}$$

noise can be parameterized with $\chi = \exp(-\eta N_L)$

$$(\Delta J_z)^2_{(\mathrm{out})} = \chi^2 (\Delta J_z)^2_{(\mathrm{in})} + \chi(1-\chi)\frac{N_A}{2} + (1-\chi)\frac{2}{3} N_A$$

Summary

- We addressed the clumsiness loophole of MR tests by
  1. Exploiting the features of QND measurement
  2. Defining an appropriate clumsiness quantifier
- We extended current MR tests to exclude larger set of theories, including some invasivity
- We showed that **adroit** MR tests are feasible in **macroscopic systems** (e.g. atomic ensembles) with current-state technology

THANKS FOR YOUR ATTENTION!

C. Budroni, GV, G. Colangelo, *et al.*, **PRL 115, 200403 (2015)**

GV, PhD Thesis, **arxiv:1511.08104**

+ in preparation

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