Stability analysis of electroweak-dark strings

Based on 1612.03151 (Phys. Rev. D95 (2017) 035003), 1603.03291 (Phys. Lett. B762 (2016) 271-275), 1608.00021 (Phys. Rev. D94 (2016), 125018) and 1909.07447

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Outline

Cosmic strings

- Line-like objects of cosmic size
- Models: classical solutions of field theories
- Gravitational effect: tension $\mu = E/L$
- Other signatures: microstructure

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- Field content: Higgs and vector boson
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Importance

- GUT: structure formation (?)
- primordial magnetic fields
- accelerator signature

Abrikosov–Nielsen–Olesen strings

Abelian Higgs model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - \frac{\beta}{2}(\phi^*\phi - 1)^2 \,,$$

where $D_\mu\phi = (\partial_\mu - iA_\mu)\phi$, β: Ginzburg-Landau parameter

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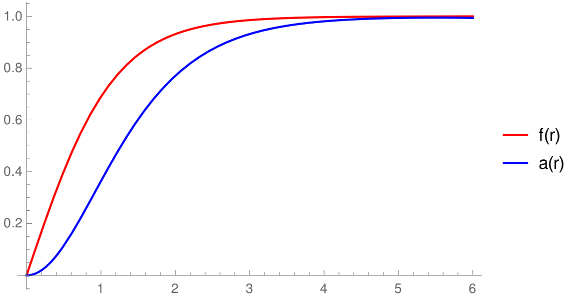
Ansatz:

$$\phi(t, \vartheta) = f(r)\exp(in\vartheta)\,, \quad A_\vartheta = na(r)$$



Abrikosov (1957), Nielsen & Olesen (1973), Kibble (1976)

Main ingredients

- A complex, spontaneously breaking scalar field in 2D

$$\phi(\mathbf{x}) = \phi(r, \vartheta)\,, \quad |\phi(r \to \infty)| = \eta$$

- Winding number (no. flux quanta)

$$\int_0^{2\pi} \phi^{-1} \partial_\vartheta \phi \mathrm{d}\vartheta = 2\pi i n$$

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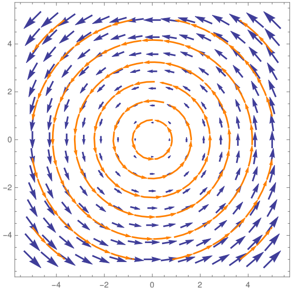
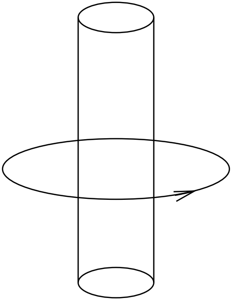
- In three dimension: vortex line (vortex string) or flux tube





Physics of vortices and strings

Cosmic strings

- Strings (flux tubes) in the Higgs field(s) of particle physics

  Abrikosov (1957), Nielsen & Olesen (1973), Kibble (1976)
- Most relevant: energy scale (GUT, electroweak, ...)
- Tension $\mu = E/L$ characterises gravitational effects
- Possible accelerator signatures Nambu (1977), Huang & Tipton (1981)

See also Vilenkin & Shellard (1994), Hindmarsh & Kibble (1995), Vachaspati etal. (2015)

Vortices in condensed matter

- vortex lines in superfluids, BECS
- flux tubes in superconductors Abrikosov (1957)
- multiple order parameter

  Babaev (2002), Babaev & Speight (2005), Catelani & Yuzbashyan (2010), Forgács & Lukács (2016)

See also Pismen (1999)

Analogies: "Cosmology in the laboratory"

Electroweak strings

Physical particles: $W_\mu^3, Y_\mu \to A_\mu, Z_\mu$

Z-string

$$\phi_2 = \eta f(r) \mathrm{e}^{in\vartheta}\,, \quad Z_\vartheta = nz(r)$$

- An embedded ANO vortex
- Stability: at $\theta_W = \pi/2$, semilocal, stable for $\beta < 1$
- Realistic (smaller) Weinberg angle: more unstable ($\beta_\mathrm{s} < 1$)

James, Perivolaropoulos, & Vachaspati (1992), Goodband & Hindmarsh (1995), Achúcarro & Vachaspati (2000)

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Stability analysis: $\phi_a \to \phi_a + \delta\phi_a$, $W^a_\mu \to W^a_\mu + \delta W^a_\mu$, $Z_\mu \to Z_\mu + \delta Z_\mu$
Decoupled blocks:

- $\delta\phi_1$, δW^+_μ instability in this block
- $\delta\phi^*_1$, δW^-_μ
- δZ_i, $\delta\phi_2$, $\delta\phi^*_2$

Possible stabilisation mechanisms:

- Bound states of additional fields Vachaspati & Watkins (1993)
- Quantum fluctuations of heavy fermionic fields

Weigel, Quandt, & Graham (2011), Graham, Quandt, & Weigel (2011)

Dark matter

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Dark sector:

- Assumed to contain particles, as the visible sector
- May contain U(1) Abelian interactions
- U(1) interactions must be Higgsed (avoid long-range interaction)

Arkani-Hamed, Finkbeiner, Slatyer, & Weiner (2009), Arkani-Hamed & Weiner (2008)

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Allowed interactions between dark and visible sectors

- Higgs portal

  $$(\Phi^\dagger\Phi - \eta_1^2)(\chi^*\chi - \eta_2^2)\,,$$

  where Φ: SM Higgs, χ dark Higgs (scalar)
  Silveira & Zee (1985), Patt & Wilczek (2006)

- Gauge kinetic mixing

  $$\frac{\sin\varepsilon}{2} Y_{\mu\nu} C^{\mu\nu}\,,$$

  were $Y^{\mu\nu}$ is weak hypercharge U(1) field strength, $C^{\mu\nu}$ dark sector U(1) field strength

  Holdom (1986)

A theory of Dark Matter

The Standard Model (SM) extended with a Dark Sector (DS)

Glashow-Salam-Weinberg theory

$$\mathcal{L}_{GSW} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + D_\mu \Phi^\dagger D^\mu \Phi - \lambda_1 (\Phi^\dagger \Phi - \eta_1^2)^2 \,,$$

where $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \epsilon_{abc} W^b_\mu W^c_\nu$, $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$, $D_\mu \Phi = (\partial_\mu - igW^a_\mu \tau^a/2 - ig' Y_\mu)\Phi$

Dark sector

$$\mathcal{L}_{DS} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + \tilde{D}_\mu \chi^* \tilde{D}^\mu \chi - \lambda_2 (\chi^* \chi - \eta_2^2)^2 \,,$$

$$\mathcal{L}_{int} = -\lambda' (\Phi^\dagger \Phi - \eta_1^2)(\chi^* \chi - \eta_2^2) + \frac{\sin \epsilon}{2} C_{\mu\nu} Y_{\mu\nu} \,,$$

where $C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$, $\tilde{D}_\mu \chi = (\partial_\mu - i\hat{g} C_\mu/2)\chi$

Couplings: λ' Higgs portal, $\sin \epsilon$ gauge kinetic mixing Holdom (1986)

Arkani-Hamed, Finkbeiner, Slatyer & Weiner (2009), Arkani-Hamed & Weiner (2008)

Parameters

Electroweak parameters

- Higgs mass M_H
- Z-boson mass M_Z

determined to a high precision by LEP

Tanabashi *etal.* (Particle Data Group) (2018)

Dark sector parameters

- Dark scalar mass M_S $> M_H$
- Scalar mixing angle θ_s
- Gauge kinetic mixing $|\varepsilon| \lesssim 0.03$ for $M_X < 200\,\mathrm{GeV}$, $|\varepsilon| \lesssim 10^{-3}$
- Dark gauge boson mass M_X

Largely unconstrained if dark sector heavy enough

Arkani-Hamed, Finkbeiner, Slatyer & Weiner (2009), Arkani-Hamed & Weiner (2008), Hook, Izaguirre, & Wacker (2011), Carmi, Falkowski, Kuflik, & Volansky (2012), Hyde, Long, & Vachaspati (2014)

Construction

Main steps:

- Identify physical fields

$$\begin{pmatrix} Y_\mu \\ W^3_\mu \\ C_\mu \end{pmatrix} = \mathbf{M} \begin{pmatrix} A_\mu \\ Z_\mu \\ X_\mu \end{pmatrix}$$

- Consider scalar mixing
- Choose cylindrically symmetric Ansatz consistent with field equations

Z-string Ansatz:

$$\phi_2 = f(r)\mathrm{e}^{\mathrm{i}n\vartheta}\,, \qquad Z_\vartheta = nz(r)$$

Coupled dark fields:

$$\chi = f_d(r)\,, \qquad X_\vartheta = nx(r)$$

Solve radial equations numerically

Vachaspati (2009), Hyde, Long, & Vachaspati (2014)

Radial equations

Radial equations

$$\frac{1}{r}(rf')' = f\left[\frac{n^2(1 - z(r) - g_{XH}x(r))^2}{r^2} + \beta_1(f^2 - 1) + \beta'(f_d^2 - \eta_2^2)\right],$$

$$\frac{1}{r}(rf_d')' = f_d\left[\frac{n^2(g_{ZS}z(r) - g_{XS}x(r))^2}{r^2} + \beta_2(f_d^2 - \eta_2^2) + \beta'(f^2 - 1)\right],$$

$$r(z'(r)/r)' = 2f^2(z(r) + g_{XH}x(r) - 1) + 2g_{ZS}f_d^2(g_{ZS}z(r) + g_{XS}x(r)),$$

$$r(x(r)'/r)' = 2g_{XH}f^2(z(r) + g_{XH}x(r) - 1) + 2g_{XS}f_d^2(g_{ZS}z(r) + g_{XS}x(r)),$$

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Boundary conditions

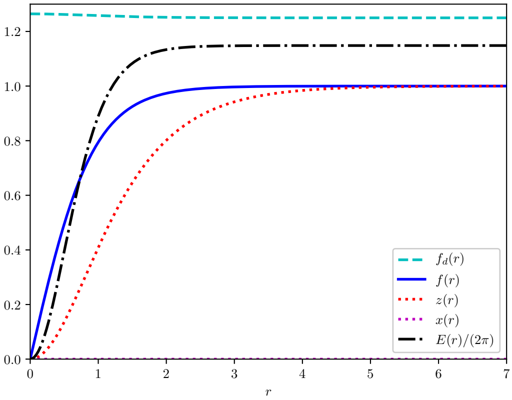
- Regular origin

$$f(0) = 0 , \quad z(r) = 0 , \quad f_d'(0) = 0 , \quad x(0) = 0$$

- Vacuum at infinity

$$r \to \infty : f \to 1 , f_d \to \eta_2 , z \to z_\infty , x \to x_\infty$$

Numerical solution



- Slightly deformed Z-string
- Energy (tension) mostly determined by electroweak scale

Stability analysis

Linearisation:

- Add perturbations $Z_\mu + \delta Z_\mu$, $X_\mu + \delta X_\mu$, $\phi_a + \delta\phi_a$, $\chi + \delta\chi$, ...
- First order gauge fixing: background field gauge

$$F_1 = \partial_\mu \delta W^{\mu +} - \mathrm{i} g W^3_\mu \delta W^{\mu +} - \frac{\mathrm{i}g}{\sqrt{2}} \phi_2^* \delta\phi_1 = 0\,,$$

  and $F_i = 0$, $i = 1, \ldots, 4$.

  Goodband & Hindmars (1995a,b), Baacke & Daiber (1995)

- Use decouplings
  - Time- and translation-independence: Fourier-modes

$$\exp[\mathrm{i}(kx - \Omega t)]$$

  - Rotation invariance: partial waves

$$\exp(\mathrm{i}\ell\vartheta)$$

Decoupled blocks

- time- and z-translation invariance
  vector field 0 and 3 components decouple
- Fixed direction of Ansatz in internal space ($\phi_1 = 0$): blocks
  1. photon field free
  2. δW_i^+, $\delta\phi_1$
  3. δW_i^-, $\delta\phi_1^*$ (conjugate of previous)
  4. δX_i, δZ_i, $\delta\phi_2$, $\delta\phi_2^*$, $\delta\chi$, $\delta\chi^*$

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  4. δX_i, δZ_i, $\delta\phi_2$, $\delta\phi_2^*$, $\delta\chi$, $\delta\chi^*$
- Resulting equations of the formation

$$D^I \Phi^I = (\Omega^2 - k^2)\Phi^I \, ,$$

  $\Omega^2 < 0$ signals instability
  lowest eigenvalue: $k = 0$
- Known instabilities of electroweak strings in block (ii).
- Other blocks: deformation of ANO perturbations (large positive eigenvalues)

Radial equations I

Consider block (ii):

$$\begin{aligned}\delta\phi_1 &= s_{1,\ell}(r)\mathrm{e}^{\mathrm{i}\ell\vartheta}\mathrm{e}^{\mathrm{i}\Omega t}\,,\\ \delta W_+^+ &= \mathrm{i}w_{+,\ell}(r)\mathrm{e}^{\mathrm{i}(\ell-1-n)\vartheta}\mathrm{e}^{\mathrm{i}\Omega t}\,,\\ \delta W_-^+ &= -\mathrm{i}w_{-,\ell}(r)\mathrm{e}^{\mathrm{i}(\ell+1-n)\vartheta}\mathrm{e}^{\mathrm{i}\Omega t}\,,\end{aligned}$$

$[\delta W_+^\pm = \exp(-\mathrm{i}\vartheta)(\delta W_r^\pm - \mathrm{i}\delta W_\vartheta^\pm/r)]$, yielding equations

$$\mathcal{M}_\ell^{(\mathrm{ii})}\Phi_\ell^{(\mathrm{ii})} = \Omega^2\Phi_\ell^{(\mathrm{ii})}\,.$$

with

$$\mathcal{M}_\ell = \begin{pmatrix} D_{\ell,1} & B_{1+\ell} & B_{1-,\ell} \\ B_{1+,\ell} & D_{+,\ell} & \\ B_{1-,\ell} & & D_{-,\ell}\end{pmatrix}\,,$$

Radial equations II

and

$$D_{\ell,1} = -\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}}{\mathrm{d}r} + \left(\frac{[n(g_{Z\phi^+}z(r) + g_{X\phi^+}x(r)) - \ell]^2}{r^2} + \beta_1(f^2-1)\right.$$

$$\left. + \beta'(f_d^2 - \eta_2^2) + \frac{g^2}{2}f^2\right),$$

$$D_{+,\ell} = -\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}}{\mathrm{d}r} + \left(\frac{[\ell - 1 - n(1 + g(\alpha_2 z(r) + \alpha_3 x(r)))]^2}{r^2} + \frac{g^2}{2}f^2\right.$$

$$\left. - 2\frac{gn}{r}(\alpha_2 z'(r) + \alpha_3 x'(r))\right),$$

$$D_{-,\ell} = -\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}}{\mathrm{d}r} + \left(\frac{[\ell + 1 - n(1 + g(\alpha_2 z(r) + \alpha_3 x(r)))]^2}{r^2} + \frac{g^2}{2}f^2\right.$$

$$\left. + 2\frac{gn}{r}(\alpha_2 z'(r) + \alpha_3 x'(r))\right),$$

Radial equations III

and

$$B_{1+\ell} = -g \left(f' - \frac{nf}{r}(1 - g_{ZH}z(r) - g_{XH}x(r)) \right) ,$$
$$B_{1+\ell} = \quad g \left(f' + \frac{nf}{r}(1 - g_{ZH}z(r) - g_{XH}x(r)) \right) .$$

Method of solution:

- $\theta_{\rm W} \to \pi/2$: semilocal limit, decoupling, scalar in potential

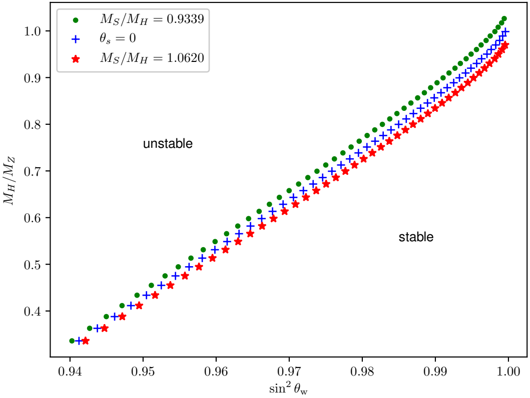
  $$\beta_1(f^2 - 1) + \beta'(f_d^2 - \eta_2^2)$$

  semilocal-dark model

  Forgács & Lukács (2017)

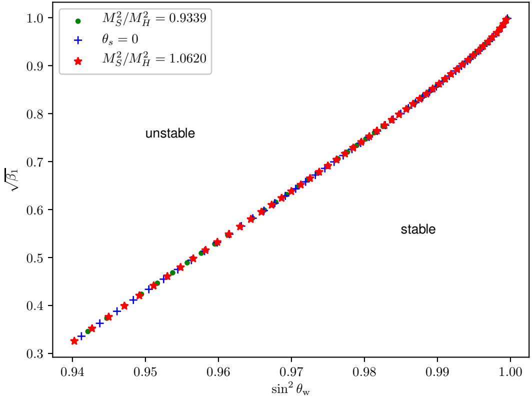
- extend to smaller $\theta_{\rm W}$; physical $\sin^2\theta_{\rm W} \approx 0.22$
- domain of stability on $M_H^2 - \sin^2\theta_{\rm W}$ for different dark sector parameters

Higgs portal



The boundary of the domain of stability, for $\varepsilon = 0$, $\bar{g} = 0.7416$, $\hat{g} = 0.6172$, $\eta_1 = 173.4\,\mathrm{GeV}$, $\eta_2 = 217.4\,\mathrm{GeV}$, and $\theta_s = 0.75$ compared to that of electroweak strings ($\theta_s = 0$).

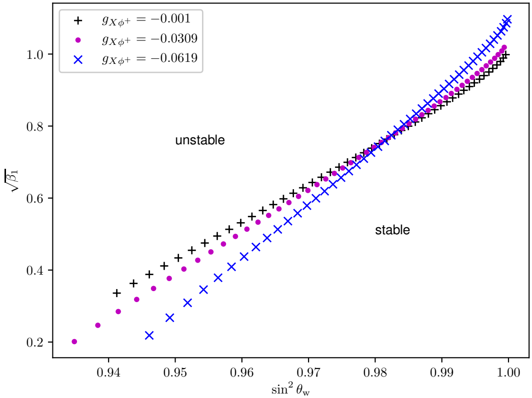
Coincidence



Reasons:

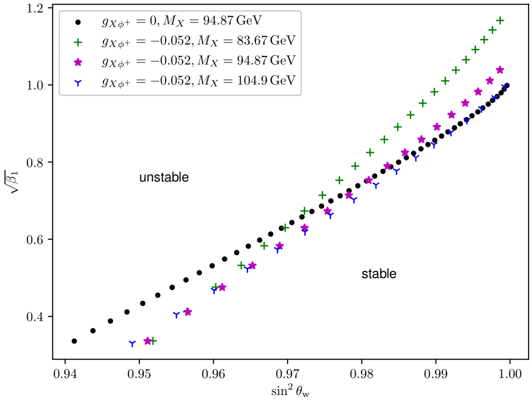
- For smaller θ_W: W condensation
- For Ws: slightly deformed Z-string

GKM



Starting parameters (M_W, M_Z, e, M_H physical and $g_{XS} = e$, $M_S^2 = M_H^2 + 2000\,\mathrm{GeV}^2$, $\theta_s = 0$, $M_X = 94.87\,\mathrm{GeV}$ and $g_{X\phi^+} = -0.001$ and -0.0619) $\Rightarrow \bar{g} = 0.7416$, $\hat{g} = 0.6172$, $\varepsilon = 7.37 \cdot 10^{-5}$, $\eta_1 = 173.9\,\mathrm{GeV}$, $\eta_2 = 217.4\,\mathrm{GeV}$ and $\bar{g} = 0.7362$, $\hat{g} = 0.6406$, $\varepsilon = 0.0446$, $\eta_1 = 175.7\,\mathrm{GeV}$, $\eta_2 = 208.6\,\mathrm{GeV}$.

GKM II



Effect of dark gauge boson mass

Conclusions

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- not stabilised by gauge kinetic mixing

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- If $M_S < M_H$: potential correction destabilising
- Effect of scalar potential and GKM weak: for θ_{W} physical, mechanism of instability is W condensation

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THANK YOU FOR YOUR ATTENTION!

Data

$\sqrt{\beta}_1$	$\sin^2\theta_W$ G & H	electroweak	$M_S/M_H = 0.9339$	$M_S/M_H = 1.0620$
1	1.0	0.9996	0.9995	0.9996
0.9	0.9910	0.9933	0.9933	0.9933
0.8	0.9836	0.9850	0.9849	0.9849
0.7	0.9756	0.9758	0.9758	0.9758
0.6	0.9666	0.9664	0.9664	0.9664
0.5	0.9576	0.9568	0.9568	0.9568
0.4	0.9486	0.9472	0.9472	0.9472

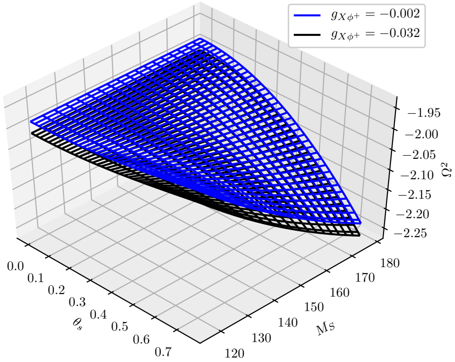
G & H: Goodband & Hindmarsh (1995)

Derivatives

parameter	derivative
$g_{X\phi^+}$	0 (parabolic maximum)
M_X	$-9.02 \cdot 10^{-8}$
g_{XS}	$2.77 \cdot 10^{-5}$
M_S	$-5.57 \cdot 10^{-3}$
θ_s	$-9.87 \cdot 10^{-2}$

Derivatives of the eigenvalue of the stability equation with respect to model parameters at $M_W = 80.4\,\mathrm{GeV}$, $M_Z = 91.2\,\mathrm{GeV}$, $e = 0.3086$, $M_H = 125.1\,\mathrm{GeV}$ (physical values), $M_X = 94.87\,\mathrm{GeV}$, $M_S = 132.8\,\mathrm{GeV}$, $g_{X\phi^+} = 0$, $g_{XS} = 0.3086$ and $\theta_s = 0.75$. Note that $-\Omega^2$ is squared growth rate corresponding to rescaled time, i.e., in units of $1/(g_{ZH}\eta_1)^2$. The units of the derivatives are this $(1/\mathrm{GeV}^2)$ divided by the units of the parameters. Here $g_{ZH} = -0.3708$, $|g_{ZH}|\eta_1 = 64.49\,\mathrm{GeV}$, $1/(g_{ZH}\eta_1)^2 = 2.405 \cdot 10^{-4}\,\mathrm{GeV}^{-2}$

Eigenvalue



The eigenvalue of the stability equation as a function of M_S and θ_s, at M_Z, M_W, e and M_H physical, $M_X = 94.87\,\mathrm{GeV}$, and $g_{XS} = e = 3086$, $g_{X\phi^+} = -0.002$ and -0.032. See also in colour online.